1. Special Nonlinear Forms

* Motivation

Consider LTI system

Which is almost analyzed thoroughly introducing “Controllability and Observability”. Mathematicians are trying these to apply in non-linear system as

* Can we define “Controllability” and “Observability”? They adapted “Lie algebra”, which is related to the differential operator.. For example

Here

is called “Lie Derivative” of h along f. Then, they tried to define “Controllability and Observability” in non-linear system. However, non-linear system is really and really versatile. Moreover, the model are uncertainties are inevitable. **I will skip this chapter**, If you want to study, you need “differential geometry”.

1. State feedback Linearization
   1. Basic concept

To design a feedback ,”static feedback”, or , “dynamic feedback”,such that the origin is an asymp.stable in the closed loop system

Or

* 1. Linearization with state feedback

Consider

Linearized system is

Where

Assume is controllable, design to assign the eigenvalues of

to be symp.stable, with this controller

Has the desired eigenvalues, further more the original system

Linearize this closed loop is

So that the closed original system is an exponential stable at equilibrium point.

%%% Kim’s comment

Remember, to check the stability

1. Linearized it : If is asympt.(exponential) stable, the original is asympt(exponential) stable. However the region of attraction is difficult to estimate. To find the Lyapunov function is a good candidate.(without Lyapunov, you may not get
2. If linearized system is not asympt.stable, nothing can be concluded about the stability for the original system
3. If linearized system is unstable, then the original system is unstable at the origin.
   1. Feedback Linearization

* Example

Consider

To stabilize the system, we may have free variable as an input. Select

Then the closed system is asympt.stable (even exponentially). However,

1. To apply input, , the state should be measured.
2. Even if the measured state is available, the measured is not perfect, i.e.

So that there should be uncertainty to need a bounded.

1. Some non-linear system is not easy to find linearizing control.

-The EnD-

Consider

Suppose, so called change variable, there is a (we call diffeomorphism) on D, such that with

.

Then with the inverse ,

The converted system is

Hence the origin is stabilized by so that

,

the closed loop system is inherited the stability of

Moreover if the region of attraction in is defined

Then in x – domain

Is also the region of attraction in x –domain

* Ex. 9.3

Let a new state variable

the derivative of

1. Controller: Select

Then the closed loop is

Which is asymptotic stable with . so that is asympt. stable at the origin.

1. The Region of attraction.

Since is asymp.stable, the region of attraction is found in a quadratic form,i.e.,

The region of attraction for the original system is

* Complicate in this case
* Feasibility of state feedback linearization

1. Consider ()

Case. 1) Select . Then the closed loop is

Case.2) Select . Then the closed loop is

In case.2) evenif the closed loop system is non-linear,

* It is globally asymp.stable / exponentially so that the the trajectories approach faster than case.1)
* The control is less efforts and is simpler.

`` - The EnD-

1. Consider (Ex.9.16 -passivity)
2. Feedback linearization
3. Passivity

If take , with the positive storage function , so that the system is passive since

Now with the feedback results in

which ensures with zero-state observability, the closed loop system is asymp. stable at the origin. The advantages over feedback linearization are

1. The measurement is only compared to the in case of the state feedback linearization.
2. It may not use a model
3. The flexibility to choose . For example if , select
4. However, it is difficult to check the passivity. Here, if you measure , then the system may not be passive.

**HW.1** : Consider

And

Check the system is passive or not-passive -The EnD-

* 1. Partial Feedback Linearization
* **Lemma 9.2**

The origin of the cascade system (9.9) is asympt.stable(respectively exponentially) stable if the origin of and are asympt. stable(exponentially) stable **locally**

%%% Kim’s comment

Consider a cascade system as (9.9)

The system is asympt. stable if is asympt. stable and is asympt. stable ,

Is asympt.stable at least locally but not globally. %%%The EnD-

* Ex.9.7

Take and are asympt. stable . By **the lemma 9.2** the cascade system is asympt. stable at the origin (**it is not globally, see the textbook**) -The EnD-

%% Kim’s comment: This example 9.7 can be viewed as

Design to stabilize the following non-linear system as

u

The “static controller “ is not good in this case. If is not possible due to the singularity. Now we may design a “dynamic controller “ at least **locally** asympt. stable.

%%% One may easily misunderstand a conjecture which is **not true.**

Consider

Then if is stable(globally), then is stable(BIBO).

However in Non-linear system is not true

If is globally asymp.stable, is globally asymptotic stable. This is not true by the Kokotovic, 1991. %%%

* 1. **Backstepping**

The previous controller is open-loop controller. If we select a feedback controller, then the global stability guarantees?

The idea is That the output of the first component is a function of the following states due to feedback from the following component

* Concept: Ex. 9.12

Consider

which is the same to the Ex.9.7 but the controller is designed by feedback

1. First, from (1), notice , it is stabilized, Select a Lyapunov for (1) as
2. Now the backstep should be governed by (2) and select the closed loop system’s Lyapunov as

The time derivative along the trajectory is

If we select , , implies the closed loop system is globally asympt.stable.

1. The difference between Ex. 9.7 and 9.12 is

* The cascade and feedback stabilization which shows stability locally and globally.
* Theory – Kokotovic, 1990.

Consider a system as

Where, are all smooth, and contain the origin .

1. First, in (9.12), view as a control input. Assume

Is asymp.stable at the origin, so that

1. Then (9.12) of the system can be equivalent to

Define a fictitious state as

1. (Backstepping) With the change the dynamic equation (2) into (9.13)

where

To stabilize , select the Lyapunov function

Then

**Select as**

**So that**

**Which is asymp.stabilizing (9.13)’ - The EnD-**

%%% Kim’s comment: it may not be memories the formula. See the following example %%

* Ex.9.13 (Back-stepping controller)

1. For the first equation, define a fictitious to stabilize it. i.e.,

design to stabilize the first equation. Choose

corresponding a Lyapunov function as . The derivative is

1. (Backstepping, change variable))

Define . Then the original equation is transformed in to

Select a lyapunov equation for the closed loop system as

Its derivative is

Select

Then

So that the closed loop is globally asympt.stable.

-The EnD-