1. Special Nonlinear Forms

* Motivation

Consider LTI system

Which is almost analyzed thoroughly introducing “Controllability and Observability”. Mathematicians are trying these to apply in non-linear system as

* Can we define “Controllability” and “Observability”? They adapted “Lie algebra”, which is related to the differential operator.. For example

Here

is called “Lie Derivative” of h along f. Then, they tried to define “Controllability and Observability” in non-linear system. However, non-linear system is really and really versatile. Moreover, the model are uncertainties are inevitable. **I will skip this chapter**, If you want to study, you need “differential geometry”.

1. State feedback Linearization
   1. Basic concept

To design a feedback ,”static feedback”, or , “dynamic feedback”,such that the origin is an asymp.stable in the closed loop system

Or

* 1. Linearization with state feedback

Consider

Linearized system is

Where

Assume is controllable, design to assign the eigenvalues of

to be symp.stable, with this controller

Has the desired eigenvalues, further more the original system

Linearize this closed loop is

So that the closed original system is an asymp.stable (even exponential stable) equilibrium point.

* 1. Feedback Linearization
* Example

Consider

To stabilize the system, we may have free variable as an input. Select

Then the closed system is asympt.stable (even exponentially). However,

1. To apply input, , the state should be measured.
2. Even if the measured state is available, the measured is not perfect, i.e.

So that there should be uncertainty to need a bounded.

1. Some non-linear system is not easy to find linearizing control.

-The EnD-

Consider

Suppose, so called change variable, there is a (we call diffeomorphism) on D, such that with

.

Then with the inverse ,

The converted system is

Hence the origin is stabilized by so that

,

the closed loop system is inherited the stability of

Moreover if the region of attraction in is defined

Then in x – domain

Is also the region of attraction in x –domain

* Ex. 9.3

Let a new state variable

the derivative of

1. Controller: Select

Then the closed loop is

Which is asymptotic stable with . so that is asympt. stable at the origin.

1. The Region of attraction.

Since is asymp.stable, the region of attraction is found in a quadratic form,i.e.,

The region of attraction for the original system is

* Complicate in this case
* Feasibility of state feedback linearization

1. Consider ()

Case. 1) Select . Then the closed loop is

Case.2) Select . Then the closed loop is

In case.2) evenif the closed loop system is non-linear,

* It is globally asymp.stable / exponentially so that the the trajectories approach faster than case.1)
* The control is less efforts and is simpler.

`` - The EnD-

1. Consider (Ex.9.16 -passivity)
2. Feedback linearization
3. Passivity

If take , with the positive storage function , so that the system is passive since

Now with the feedback results in

which ensures with zero-state observability, the closed loop system is asymp. stable at the origin. The advantages over feedback linearization are

1. It may not use a model
2. The flexibility to choose . For example if , select

-The EnD-

* 1. Partial Feedback Linearization
* Lemma 9.2

The origin of the cascade system (9.9) is asympt.stable(respectively exponentially) stable if the origin of and are asympt. stable(exponentially) stable **locally**

* Ex.9.7

Take and are asympt. stable . By the lemma 9.2 the cascade system is asympt. stable at the origin (**it is not globally, see the textbook**)

%%% One easy, misunderstand a conjecture which is **not true.**

Consider

If is globally asymp.stable, is globally asymptotic stable. This is not true by the Kokotovic, 1991. %%%

Some is

* 1. **Backstepping**
* Concept

Consider to design a controller stabilizing

1. First see the first equation. Since is a function of , **define** a **fictitious control** to stabilize

Select Lyapunov , select as

🡪

Hence with the fictitious , is asympt.stable

1. To meet the condition , define a **fictitious** **state** as

Then, for , , differentiating

Design the **second controller** for , select

If , , i.e.,

1. Transform to state and u

where

1. Analysis

With the state feedback

Is asymp.stable. Since we introduce an integral controller, even if the model has

uncertainties, it may be robust compared to feedback linearization. And it is a constructive way to stabilize a system.

* Theory – Kokotovic, 1990.

Consider a cascade system as

Where, are all smooth, and contain the origin .

1. First, in (9.12), assume

Is asymp.stable at the origin, so that

1. Then (9.12) of the system can be equivalent to

Define a fictitious state as

1. The transformed system (9.13) with ,

where

To stabilize , select the Lyapunov function

Then

**Select as**

**So that**

**Which is asymp.stabilizing (9.13)’ - The EnD-**

%%% Kim’s comment: it may not be memories the formula. See the following example %%

* Ex.9.9 (Back-stepping controller)

1. Design a fictitious controller for the first equation.

Define a **fictitious controller**

Design to stabilize (1) , one of them is

),

Using this ,

So that the origin of (1) is asymp.stable.

1. Select **a fictitious state**  as

so that the

Select a Lyapunov function

Its time derivatives is

Select for to be negative

1. Transform into x

From

The stabilizing input is