<https://www.egr.msu.edu/~khalil/NonlinearControl/>

1. Introduction
   1. Non-linear Model
2. Unforced system
3. Autonomous system -- time invariant system

* Existence and Uniqueness for --Lipschitz

- is piecewise continuous in

- is locally Lipschitz in at a point

If such that

such that

- is locally Lipschitz on a domain

If is locally Lipschitz at every point

- is globally Lipschitz in on

* Kim’s comment : Lipschitz condition

1. 🡪 any two points slope is less than

* If are continuous at , then is Lipschitz at
* If are continuous in , then is locally Lipschitz on the domain
* If are continuous for all ,then is globally Lipschitz iff

are bounded by constants independent of

* Examp

1. is globally Lipschitz
2. is locally Lipschitz but not globally.
3. is locally Lipschitz but not globally.
4. , here is not Lipschitz at .

However it is locally Lipschitz in

* Ex.1.1 :

Is locally Lipschitz but not globally(why? see the textbook).

* Lemma 1.1 (for Uniqueness)

Piecewise continuous in and locally Lipschitz in Then

such that has a unique solution over

* Ex. 1.3

Analyze: is locally Lipschitz not globally. The solution with is

* Terminology: Finite Escape time T:
* Lemma 1.2 : piecewise continuous in and **globally Lipschitz** in for all . Then has a unique solution over
* Too restrictive. are all none globally Lipschitz.
* Lemma 1.3 : piecewise continuous in and locally Lipschitz in on D for all . Let be a compact (closed and bounded), and suppose it is known that every solution of

is in , i.e., . Then there is a unique solution for all

* Ex. 1.4

Is locally not globally Lipschitz. Since ,

* Kim:
* Closed and open set

+ is open, is closed set

+ is a closed set

* Bounded such that
* Closed and unbounded set
* What is and Can you draw
* What is definition of open and closed set?
  1. Non-linear **Phenomena**

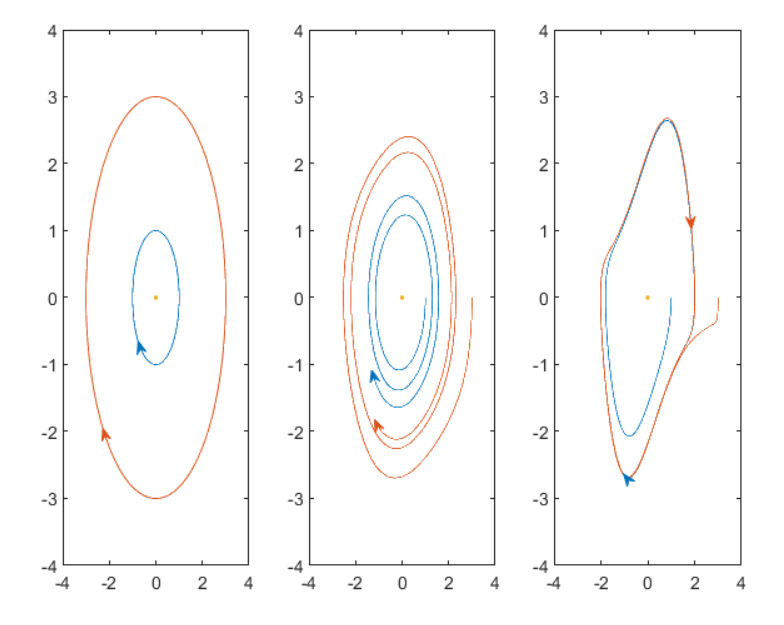
-. Finite escape time :

-. Multiple isolated equilibrium points

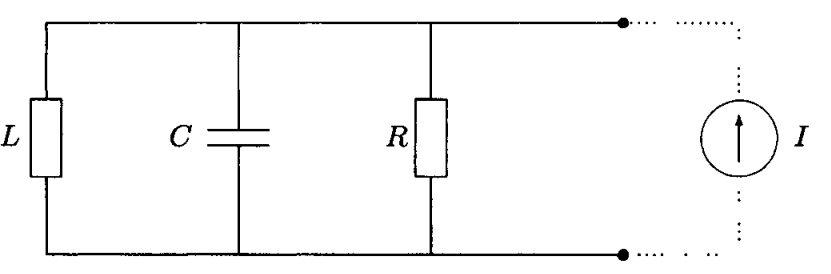
-. Limit cycle: van der Pol equations :

* 1. If , then which is the equivalent to the linearized system
  2. , then the circle of the linearized trajectories is distorted
  3. , regardless of initial points the trajectories converge to only one isolated orbit.

This isolated orbit is called a limit cycle.



+ famous physical example: oscillator with a negative resistance

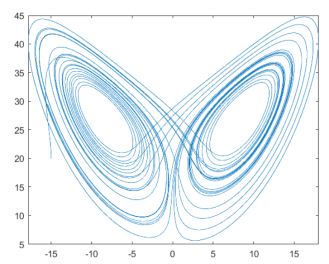
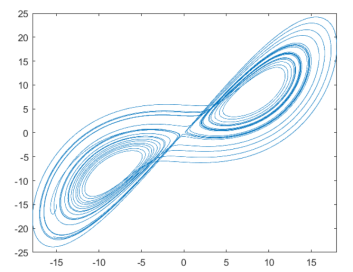
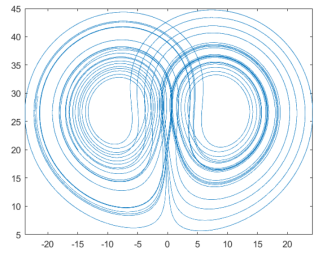


-.Chaos: Lorenz equation(not Lorentz!!)

Consider (called Lorenz equation)

where . Assign the parameters as

Then the trajectory in plane are



* Attractors: butterfly wings
* A non-linear system but a stochastic behavior due to unpredictable
* Bifurcation / Chaos

+ famous physical example : double pendulum, weather dynamics.

* Chaos is happened only in the order is greater than 2
* Unpredictable weather / optical lasers (Jurassic Park – A butterfly flpas its wings in the Amazonian jungle, and subsequently a storm ravages half of Europe:

<https://thedecisionlab.com/reference-guide/economics/the-butterfly-effect>



Jurassic Park

1. Magnetic Suspension System

The parameter depends on the states

1. Inverted Pendulum on a Cart :

Two equilibrium points :In fact dynamics of pendulum is non linear

1. Ball-and Beam System

The steady states are dependents on the initial points

1. Double pendulum

The behavior is similar to a random.

%% Kim’s comment: Non-linear

1. Remember in my point of view, the only linear system are in Computer Algorithms
2. In an electric circuit, the passive elements “R”, “L”, “C” , values of these are to be assumed as a constant in Linear system.Any active elements Transistor, OP amp, are ALL non-linear. Moreover if there is a saturation, the system is non-linear.
3. Any dynamic(kinematic, or follow Newton’s law) systems are non-linear !!
4. Chemical process, biology, economic models are ALL non-linear.

Hence to select a topics for thesis, carefully to look at any simple math models, sometimes it will give a success or drag you into a slump. %%%