Ch.3

Jacobian from Taylor series: Here vector valued function to be expanded into polynomials using Taylor series at

Now for linearization

Hence the Jacobian is

**Not**

1. Stability and Equilibrium

%% Kim: construction of Lyapunov Functions

* Construct a Lyapunov function

1. Linear system

Assume . Here is unknown. Then the derivative of along the trajectory is

Now for the asymptotic stability, we need

Let us choose a positive definite matrix so that is a positive definite function. Now

* If there exists a positive definite matrix satisftying

Then Lyapunov function , which is guarantee the asymptotic stability

* Comments:

1. For different , different may be .
2. For the asymptotic stability of the linear system, it is enough to check the e-values of the system matrix. Why we need Lyapunov equation? There are some reasons.

* Kim’s Ex: Consider a bilinear system with a state feedback

Here I want to design to be asymptotic stable.

Consider

In this case , then

* -which is a positive definite matrix.

Using this one

If ,

Hence if , then the feedback system is asymptotic stable.

* Construct Lyapunov function : one method as **variable gradient method**

1. Concept: construct a Lyapunov function for
   1. should be positive definite. It is a little bit easy
   2. For asymptotic stable (we know it by linearization),
2. Background: assume a unknown function which is the gradient of ,

Find such that is negative definite,

which is positive definite. The equation (a) is the path integral along the path(which is the trajectory)

The gradient for the line integral is independent of the path so that

is positive definite

1. In summary : to find such that

and is positive definite

* Example.3.7(p.65)
* Problem : to find a Lyapunov function for

where is locally Lipschitz,

* Procedure to select

1. Gradient constraint

1. Negative definite constraint
2. Positive definite constraint
3. Assume

From a) constraint

We may select

From the negative definite constraint

To cancel the cross-product terms,

which is done by taking **,** then

So that

From positive definite constraint

Where

Select , is positive definite.

Since , the origin (i.e., ) is globally asymptotic stable.

* Revisit the Example 3.6 The pendulum equation is

Since so that , to find a Lyapunov candidate, we may apply the variable gradient method

1. Previous Lyapunov function
2. Apply variable gradient method,

with . If we select ,

And for positive definite constraint,

%% Kim’s comment

What is the difference between Ex.3.6? Similar but more systematic method to find a Lyapunov function %%

**Test..**

1. Find the equilibrium point
2. Using linearization, at the equilibrium point, check the stability.
3. Find a Lyapunov function using variable gradient method.

-END-

* 1. The Invariance Principle

%% Kim’s comment

Asymptotic stable is a good property in the sense of robustness, i.e., for any perturbation in the initial conditions, the trajectory will be convergent to the equilibrium points. We may find a Lyapunov function whose derivative is negative definite to guarantee asymptotic stable. However, in general, it is difficult to find it, but it is less difficult to find a negative semi-definite of the derivative of it. If it is negative semidefinite, and if the system equation has some properties, then the system is asymptotic stable. Upto now we may guess the trajectory at the equilibrium points. However, one of special features in non-linear system is limit cycle which is not a point but a circle. What is appropriate stability definition? %%%

Consider a pendulum described by

We DO know, at is asymptotic stable. What is an appropriate Lyapunov function whose derivative is negative definite?

One candidate which is not a negative definite of

1. 🡪 which is not negative definite.

The other candidate which is a negative definite

1. , where

So far it is good. But a little bit complicate. We need another concept. Even if is not negative definite, i.e., sometime the system is asymptotic stable.

* Concept

In the previous pendulum Example 3.6, even if is asymptotic stable, a Lyponuv candidate

🡪 which is not negative definite. i.e., it is not guarantee to be asymptotic stable. Now

From the system equation,

* , i.e., if 🡪 asymptotic stable.

In conclusion, the positive definite function is decreasing along the trajectories and converges to zero

1. not only at which is deduces by the Lyapunov method
2. but also by the system equation.
3. Hence even if is negative semidefinite, the origin is asymptotic stable.

* Definition(p.67) : positive limit set

Let be a solution of . A point is a positive limit point of

If , with , such that

The set of all positive limit points is called the positive limit set

Ex: (asymptotic stable) , oscillation circle, stable limit cycle

* Definition(p.67) : Invariant set

A set is an invariant set with respect to if

1. An equilibrium points is an invariant set, i.e., if
2. For autonomous system, for any trajectory is an invariant set
3. A limit cycle is an invariant set
4. The domain of attraction is an invariant set

%% Kim’s comment

1. No analytic description of limit cycle

One of the problems is in case of limit cycle. The asymptotic stable point is

Good. However in the limit cycle case the limit point is not a point

But a closed cycle**, WHICH IS NOT** analytically described in general.

1. In Lyapunov theorem, for asymptotically stable, we need should be negative definite not semidefinite. Here even if semi definite, some additional condition, it may be asymptotic stable. %%

* Theorem 3.5. assume

1. is a locally Lipschitz in ,
2. is continuous differentiable positive definite function in
3. Let
4. In no solution can stay identically except ,

Then

1. the origin is asymptotic stable in
2. If is a compact set which is positively invariant set, then is a domain(region) of attraction.

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