Review Lyapunov theorem

1. 🡪asymp.stable
2. 🡪 stable

* Exm 3.6 ( pendulum equation)

1. Linearize at

* Asymp. Stable, i.e,

But the problem is , this is not said how large .If , then

* , here is called the region of attraction.
* Ex. 3.7 (pp.65) Construct a Lyapunov function – variable gradient

There is a constructive way to find . Consider a Lyapunov function.

The idea is to find such that

1. Define
3. In general

3.4 Invariant Principle

%% Kim’s comment

If and

1. 🡪 asymp.stable
2. 🡪 stable
3. Some system is asymp.stable, but a candidate is is negative **semidefinite.** %%

* Positive invariant set if

%% Kim’s comment:

1. An equilibrium points is an invariant set, i.e., if
2. For autonomous system, for any trajectory is an invariant set
3. A limit cycle is an invariant set
4. The set is an invariant set
5. If the region of attraction set is

* Theorem 3.4 (Lasalle)

Assume

1. is a compact set that is positively invariant w.r.t
2. is continuously differentiable such that in
3. **M is the largest invariant set in E**

Then

%% Kim’s comment

1. Here is not necessary positive definite. i.e., is not necessary “0”, and positive definite.
2. If is a PDF, and , then
3. The set is =0}, i.e.,

which means the level is constant.

1. If and is only the equilibrium point, i.e., , converges to the equilibrium point(i.e., asymptotic stable) even if .

%%%

* Theorem 3.5 (with Lyapunov)

In the Theorem 3.4, is a positive definite, it is same to Theorem 3.4

* Ex\_1(without using Lyapunov equation ) – linear system

Consider

1. Select an invariant set : Let us select , here is a PDF
2. Find set : . Hence
3. Find which is the largest invariant set in

Let then if , then since , i.e.,

1. By Laselle’s theorem, so that it is asymp.stable.

%%% Kim’s comment:

Here, the invariant set then to deduce we do not need to solve Lyapunov equation

But by Laselle’s theorem it is concluded. –The End-

* Ex\_2. non- linear system

1. Select an invariant set : Let us select , here is a PDF in
2. Find set : . Hence
3. Find which is the largest invariant set in

Let then if , then since , i.e.,

1. By Laselle’s theorem, so that it is asymp.stable.

-The End -

* Kim’s comment:

1. To apply Laselle’s theorem, the invariant set should be defined, which is complicated. But a PDF which is negative (not necessary definite), asym. stable is guaranteed.
2. Here still it is difficult to select a candidate ,

Where comes from? You may think

One of the tricks may be memorized. %%%

* 1. Exponential Stability
* Definition 3.3

is said to be exponentially stable if the solution is decreased exponentially , i.e.,

* Theorem 3.6: Let be Lipschitz. Let be continuously differentiable over . such that

Then is said to be exponentially stable.

%% Kim’s comment

In Linear system, asymptotically stable is equivalent to exponentially stable. %%%

* 1. **Region of Attraction**
* Definition 3.2 : Let the origin be an asymptotically stable of

The region of attraction of the origin (or region of asymptotically stable, or domain of attraction, or basin) is the set of all , such that the solution of

will converge to the origin as t tend to infinity.

* Kim’s comment

To find the region of attraction ,which is better is large as possible, is difficult.

1. Linear system : solve Lyapunov equation,

Then in the interested domain , where . Of course is as large as possible at least it is in . How? In this case quadratic form, it is a little bit easy if domain is quadratic ,i.e., . Then the largest “c” will be

1. Procedure

Select

1. Find
2. Select the invariant set