Review Ch.4 : time-varying and perturbed system

* Time varying system

Ex. . If ,🡪 the origin is asymptotic stable. 🡪 **false**

* Define a new functional class as as

1. if for fixed and for fixed

* Uniform stability :

The origin is uniform stable if and , independent of

* Uniform asymptotic stable
* Exponential stable if
* Perturbed system

If the origin of the unperturbed system is exponentially stable, the magnitude of the perturbation is bounded , , then the origin of the original system is asympt. Stable

* Input to state stable (ISS):

is ISS implies

1. For any bounded input the states are bounded
2. If converges to zero as so converges to zero
3. The origin of the unforced system is **globally asymp.stable.**

%% Kim’s comment : Importance of asymp stable at origin . Control engineer point of view. consider

1. If the unique solution exists, the solution is dependent of
2. Control engineer object is to design a controller subject to some constraints. Now If

The system is asymptotic stable, he may not consider the output due to initial points. Moreover it is difficult to estimate the initial points. Asymptotic stability is good assumption to design the controller.

1. Passivity

%% Kim’s comment

Consider a perturbed system with a controller

the nominal plant is asympt. Stable. To stabilizing the perturbed system, there are two types, as **passive control** and **active control**. The difference between them is, roughly speaking, using observer or not. For example, consider a car is on the dumped road. The objective is to keep the level of driver’s seat even if the car’s tires position is vibrating. Two methods are

1. Using spring to damp out to reduce the vibration 🡪 passive control
2. Using actuator (motor) to cancel out the vibration 🡪 active control

In this chapter and the next chapter, we may consider passive control problems

* 1. Memoryless Functions
* Memoryless functions

Kim’s Definition: Consider input and out y is related as

If does not depend on the , then y(t) is memoryless. Otherwise it is a memory function.

Ex.

1. Memoryless

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-Resistor: : as the input the output , it is memoryless

-Force : , as input “a” and the output “F”, it is memoryless

-Probability: , waiting time, white noise

2) Memory

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- Capacitor :

- Probability: , Markov process, Brownian

* Def. 5.1 The system is

-passive if

-lossless if

-input strictly passive if

-output strictly passive if

%% Kim’s comment

1. Here we are dealing with input and output “y”
2. The terminology “Passive” come from the circuit theory, considering the power consumption of a memoryless system. i.e., if an input “” is the voltage across a resistor, the output is current “, then the power consumed in the resister is voltage times current, which is always positive whenever the resistance is positive.
3. If

%%

* 🡪 Sector condition : a function belongs to the sector if

equivalently

To the vector case, ,

where

More general form, if

So that

* Def. 5.1 A memoryless function belongs to the sector

- if

u

y= h(t,u)

- if

- if

- if

%% Kim’s comment

Sector condition is related to globally condition within the sector. Consider the above graph. Then

%%

* 1. State Model

Consider

locally Lipschitz ,

continuous,

* Def. 5.3

The system (5.6) is **passive** if , continuously differentiable **PDF** (here called as the **storage function)** s.t.

Moreover

* Loseless if
* Input strictly passive if
* Output strictly passive if
* **Strictly passive** if -End -

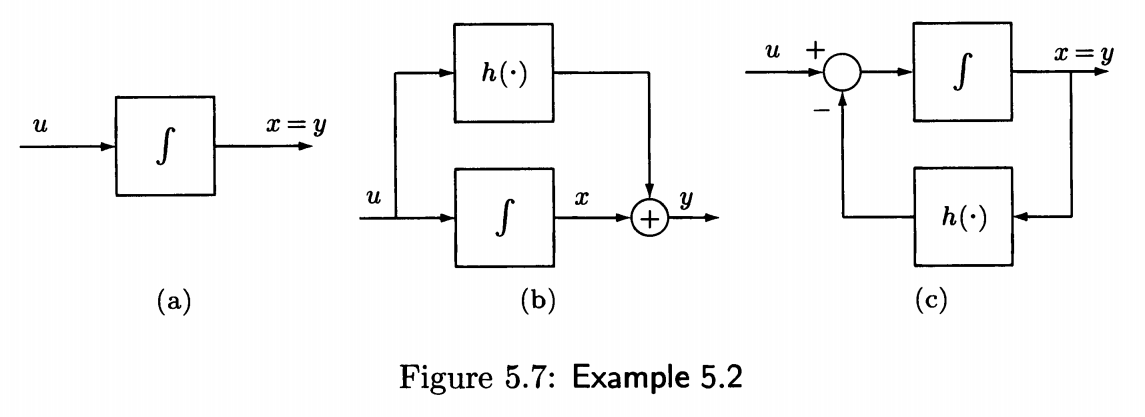
%% Kim’s Comment:

1. Since 🡪 , assume to be power, the input energy is greater than the storage energy, which is internal in the system , then the system is **passive(i.e., no energy is generated by the system).**
2. In order to check the passivity, the variables are input and output , not the state
3. The terminology of “lossless” may be confused to be is passive. However in the definition

passive and **lossless**

🡪 passive %%

* Ex. 5.2



1. For (a), the integrator is

Select a storage function as Then

* Integrator is passive and lossless

1. Feedforward : In b), with a storage function

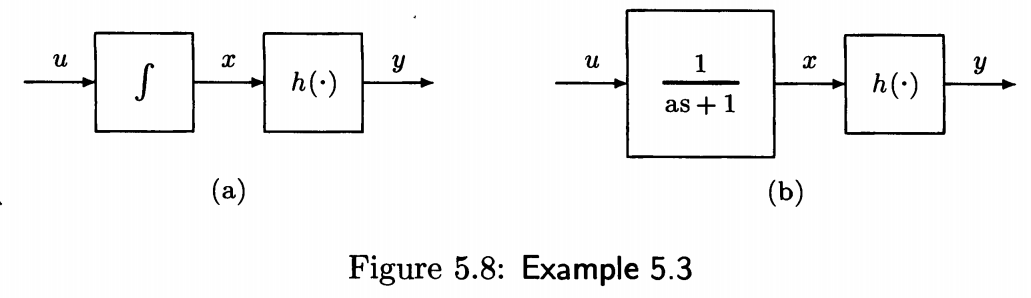
Since

If then the system is

1. Feedback: 1n c)

if it is **output strictly passive**

* Ex. 5.3



1. Case a) assume

Then select a storage function ,

Hence it is lossless passive

1. Case b)

Select , a storage function,

If if

-EnD -

* Ec. 5.4: check the passivity

Select a storage function

where . Then

If where ensures it is **strictly passive**.

–EnD-

* Ex. 5.5

Select . Then

Hence if it is passive, and if it is **strictly passive.**

* 1. Positive Real Transfer Functions
* Def. 5.4 An proper rational transfer function matrix is **(strictly)** **positive real** if

1) poles of are in And

2) is (positive definite) positive semi definite

3) (skip) If is a pole of , it is a simple pole and the residue matrix

%% Kim’s comment

1. What is a proper rational transfer function? If there is a delay component, so its transfer function , this is not rational transfer function
2. Proper rational means ; , the order of is that of
3. If is a positive real transfer function, and a scaler, then

Since is a Hurwitz,

* is positive and real 🡪 Nyquist plot of is in the RHP in complex plane 🡪 %%
* Lemma 5.1

Let , proper rational, and Then is **strictly** real positive if and only if

1) All element of is Hurwitz

2) is positive definite for all

3)(skip) is positive (semi) definite and

* Ex. 5.6

1. is positive real , but not strictly positive real.

Since the pole and

* Positive real transfer function



* is Hurwitz and 🡪 strictly positive

1. is not positive real.

* is not Hurwitz

1. .

* Every element is Hurwitz and
* Strictly positive real
* Every element is Hurwitz and 🡪 strictly positive
* Lemma 5.3 (Kalman –Yakubovich-Popov)

And is controllable, is observable. Then is strictly positive **if and only if** and s.t.

* 1. Connection to the Stability

Consider

* Lemma 5.5 If the system (5.17) is passive with a positive definite storage function, , then the origin **of is stable.**
* Proof: By definition, 🡪 the origin is stable

%% Kim’s comment:

1. for control engineering point of view, the origin stability is not enough. Since

The solution to the dynamic system is decomposed to two components, one is due to internal source(i.e., initial points) and the other is external source. Hence the component due to internal source should be 0, in order to design a controller more easily.

1. Hence without external source, we need asymptotic stability, at least.
2. In the “Passive theory” the input and output are considered. Now WITHOUT input, only with the information on the output, to get the internal state information, we need “Obervability “ similar to the linear system

%%

* Def. 5.5 : (5.17) is **zero-state observable** if no solution of can stay identically in , other than the zero solution

%% Kim’s comment :

1. observable but restricted to **zero-state** observable. In linear system, the criteria for observability is well known, however, in non-linear a little bit complex.
2. We may use “Laselle’s Invariance theorm” %%

* **Lemma 5.6** : in (5.17), the origin of is asymptotic stable if the system is

-**strictly passive** or

-output strictly passive and **zero-state observable**.

%% Kim’s comment

1. Turn to the beginning , Week\_4, in theorem 3.3 to insure the asymptotic stability is

such that . What is difference?

In theorem 3.3

In here , so that we **only know/ measure the output y.** Hence **Given “y”, how to guarantee the internal state stability.**

1. In the linear system, the unforced system’s solution is

If system is observable, i.e., is invertible, then given , the unique is determined. Hence implies . Since the system is observable, if , then the only one , Or we may induce using Lasell’s invariant theory,

1. Remember the definition of output strictly passive. %%.

* Ex.5.7 Consider

Suppose , continuously differentiable positive semidefinite such that

Then,

* The system is stable.

If

Then

which implies it is strictly positive with , In addition, if it is zero-state observable, the origin is asymptotic stable. – EnD -

* Ex. 5.8

1. Select
2. 🡪 output strictly passive
3. For zero-state observability

For the output By Laselle’s theorem, the invariant set includes 🡪 🡪 Zero-state observable 🡪 with 2) the origin is asympt.stable.

-EnD -