1. Input-Output Stability
   1. Stability

The magnitude of a function is defined using “norm”, norm of . The operator should be satisfied

* Ex.
* Piecewise continuous function space : norm space
* Square integral function space: norm space
* Def. 6.1 A , defined for is a gain function if it is non decreasing and
* Def. 6.2 A mapping is stable if and non-negative constant such that

or finite –gain stable if such that

* 1. Stability of State Models - skip
  2. Gain – skip

1. Stability of **Feedback System**

%% Kim’s comment: Aizermann’s Conjecture and Kalman’s Conjecture

1. Aizermann’s conjecture

If is asymp.stable ., then the origin is asymptotic stable?

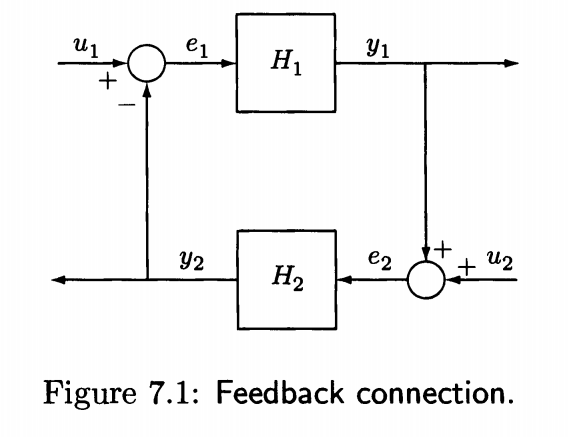
This conjecture is false

1. Kalman’s Conjecture

With the same system but

This conjecture is also false. However, With some modifications Aizermann’s conjecture, it is true with new definition “Absolute stability” %%

* 1. Passivity Theorems



%% Kim’s comment: for simple understanding the Fig.4.1

1. Here are mapping as
2. : sensor noise
3. : controller dynamics

%%%

%% Kim’s comment passivity and stability

In Lemma 5.6

1. If is **strictly passive**, i.e. ,
2. If and zero-state observable

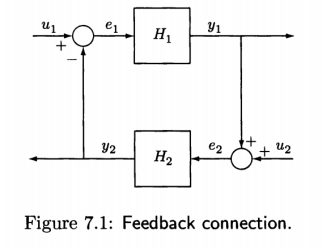
then the origin of the system is asymptotic stable.

Ok… Let us check the meaning of these. The condition 1) means

Since is a PDF 🡪 the unforced system is asympt.stable. Now consider condition 2) with , . Here maybe semi-definite, by Lasalle’s theorem, zero-observable implies if . Hence the origin is asympt.stable.

1. Is it complicate compared to Lyapunov method directly? True. But you **connect many passive systems**, to prove asymptotic stable is a little bit difficult. Here, we may consider only the output to guarantee the origin’s asymptotic stable. %%%

* **Theorem 7.1** The feedback connection of two passive system is **passive.**

**Proof:** Let be the storage functions for , i.e.,

Then

Take as a storage function,

Implies the feedback system is passive.

* **Theorem 7.2** Consider

which is for passivity analysis

The origin of the unforced system i.e.,, is **asymp. stable** if one of the following conditions is satisfied

1. Both feedback components are **strictly passive**
2. Both feedback components are **output strictly passive and zero-state observable**
3. One component is **strictly passive and the other one is output strictly passive and zero-state observable.**

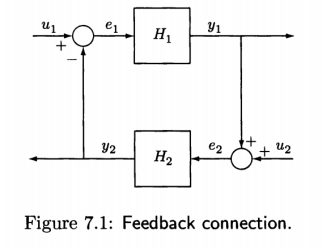
Proof:

Case 1) Since two are strictly passive, the storage function are PDFs.

Select for the closed loop system, then s.t.

Implies negative definite 🡪 the closed loop(unforced) is asymp.stable.

Case 2) since two are **output strictly passive**

Which is only negative **semi** definite(because may not be zero when . )Now by Lasalle’s theorem, since .

Firstis zero-state observable,

Second is zero-state observable,

Hence with are zero-state observable, ,

The closed loop system is zero-state observable,by Lemma 5.6, it is asymp.stable, Case 3) assume strictly passive

Hence

And b zero-state observability of 🡪 🡪 the origin is asympt.stable -EnD-

* Ex.7.1

Where a, b, k > 0

1. For select 🡪

* 🡪 **output strictly passive**

by definition of zero-state observable,

* is zero-state observable

1. For select 🡪

* 🡪 output strictly passive.

by definition of zero-state observable,

* is zero-state observable

1. From **Theorem 7.2 the second condition**, The closed loop is asymp.stable.

* Ex. 7.2 (Combined )

Here the system is same to Ex.7.2 but

1. For , if we select

* : **passive but cannot conclude strict passivity or output strict passivity**.

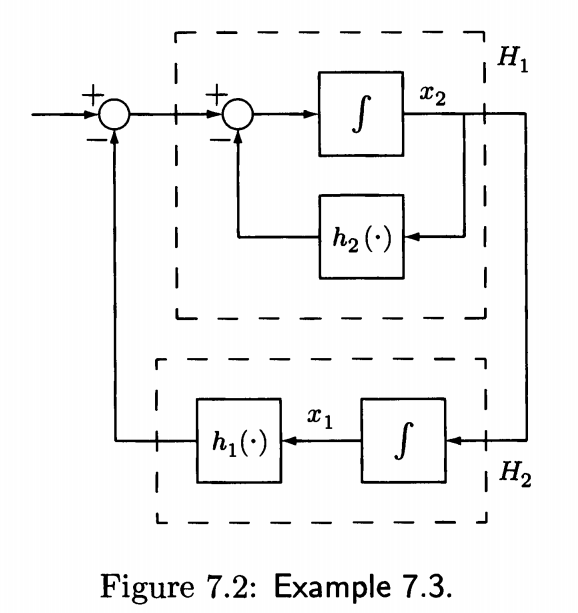
1. Select . Then

It is negative semi definite

1. Lasalle’s theorem :

Therefore the origin is asymp.stable

* Ex.7.3

and locally Litschitz

(Ex. 3.8: Pendulum equation

)

* In Blcok

1. : output strictly passive
2. lossless with passive 🡪 not strictly passive

nor output strictly passive.

1. Select

* Negative semi definite 🡪 passive

For aympt.stable, zero-state observability is needed

By lasalle’s theorem (invariant theorem),

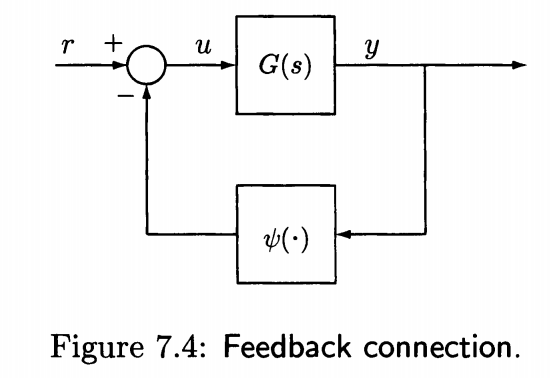
* From
* Zero state observable 🡪 the system is asymp. stable

%%% Kim’s comment

See Ex.3.6 Ex.3.7(variable gradient), Ex.3.8 and Ex.7.3 What are the differences? %%%

* 1. The small gain theorem 🡪 skip
  2. Absolute Stability

1. Problem formulation : Lure’s Problem

Assumption: controllable, observable.

* is memoryless and may be time varying.
* In Tut\_5, the feedforward and the linear feedback is a strictly positive real, then the closed loop is strictly positive 🡪 asymp.stable
* Definition 7.1 :

The system (7.21) is absolutely stable if the origin is globally uniformly asymp.stable for any non-linearity in the sector.

* + 1. Circle Criterion - See Aizemann’s Conjecture in Week\_8
* Theorem 7.8 The system (7.21) is absolutely stable if

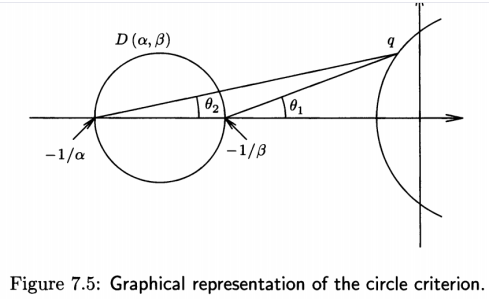
1. and is strictly positive real , **or**
2. and is

strictly positive real.

%%% Kim’s comment : If SISO case

is strictly positive real if is a Hurwitz and Re[H(s)] >0 %%

* Consider single Input single output case . To check the absolute stability,. there are 3 cases,

Case a)

a.1) Consider the second condition first

* Consider 🡪 **the circle at the boundary**
* In order if is outside the Disk

a.2) is Hurwitz,

🡪 is Hurwitz

* is Hurwitz if and only if encircles times , where is the number of poles of in RHP, in the counter clock wise direction.

a.3) In conclusion: ( In SISO case)

The Nyquist plot of does not enter the disk and encircles it times in the ccw direction,

Case b) ,

a.1) By the theorem,

🡪

🡪 Nyquist of should be in the right of the vertical line

a.2) is Hurwitz,

🡪 is Hurwitz

a.3) In conclusion

is Hurwitz and the Nyquist plot lies to the of the vertical line

Case c) ,

c.1) For using a.1) method

🡪

🡪

🡪The Nyquist plot should be inside of

c.2) is Hurwitz,

🡪 Since the Nyquist of is inside of , is Hurwitz

🡪 Due to c.1), The Nyquist plot of can not encircle ,

-> to be Hurwitz, should be Hurwitz.

c.3) In conclusion

is Hurwitz

And the Nyquist of lies in the interior of the Disk

In all cases, the conclusion are in the following Theorem

* Theorem 7.9

In SISO, where a minimal realization of the (7.21) . Then the system is absolutely stable if one of the following conditions is satisfied

1. If , the Nyquist plot of does not enter the disk and encircles it times in the counter clock wise direction, is the number of poles of with positive real parts.
2. If , is Hurwitz and the Nyquist plot of lies to the right of the vertical line
3. If , is Hurwitz and the Nyquist plot of lies in the interior of the Disk

-the EnD -

* 1. Popov Criterion-skip

Similar to the Absolute Stability, however, the sector condition is .