HW\_3

1. Check the following functions whether positive definite or not

Sol:

Sol:

Sol:

Sol:

Sol: and , 🡪 PDF

1. Exercises 3.2

Sol:

1. a.1)

* Is a PDF
* Asymptotic stable

Another one

a.2)

-

-

- asym. stable

1. Similar to a)
2. Global asym. stability

c.1)

c.2) for global asym.stability in theorem 3.3 is

Hence global asym stable, but if ) does not converge to infinity, is not appropriate Lyapunov function. Now find such that

- and . Here should be I and III quadrant passing through zero. And to be finite, there are several function you may select.

One of them is Gaussian. Let

which satisfies

And

whose value is finite. Can you guess the value?

1. Exercise 3.3

The procedure is : unstable 🡪 stable but not asym stable 🡪 asym stable not global 🡪 global asym stable.at the origin.

1. It is stable but not asym stable since if
2. It is unstable

%% Kim’s comment : here 1) and 2) linearization at is same to each other even if it is not defined at at due to piecewise continuous at . Since the eigen value of each case is “0”, it is not concluded the original system is stable.

1. This problem is not appropriate. The stability should be defined at the equilibrium point, in this case, . Let us define the origin at

At 🡪

At 🡪

1. Similar to 3)

At

At

1. . There are no equilibrium point. This problem is wrong.
2. Exercises 3.4

Problem

, ,

1. Pick as

which is a PDF the derivative is

Hence at , it is stable

1. With the equivalent

Hence it is asym.stable.

1. Ex.3.6

Assume

1. Proof of a positive definite function :

For equilibrium point and

i.e.,

For positive : The integrand of

then so that

then

Hence

i.e.,

The reminder part is

Since

even if , but ,

so that is a positive definite function.

Proof: For radially unbound: Since

Hence

i.e.,

which means radially unbounded.

1. For asmp.stable

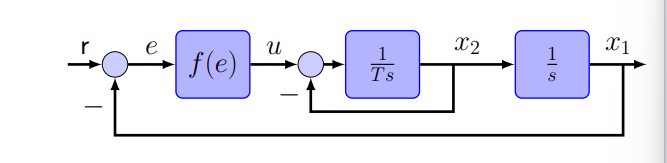
Since

And

So that

imples at the system is asmp.stable.

1. Consider the following feedback system



The state space model is,

where

Choose a Lyapunov candidate as

1. Is it stable?

Since is a PDF, the time derivative along the trajectory is

which is negative (but not definite!), so that at the system is stable

1. If r = step function, plot the

3.1) when

3.2) when

3.3) when

-The End-