* **HW\_4\_1**. Consider

1. Find the equilibrium point

Sol:

Since

1. Using linearization, at the equilibrium point, check the stability.

Sol: at , linearizing results in

Which is asymp.stable if

1. Find a Lyapunov function using variable gradient method. –End –

Sol: let us assume

3.1) select

3.3) For the negative,

Assume

2. 🡪 since

Since . Select then

From i)

1. From iii)

3.4) For positive using gradient integral

Since , the only equilibrium point is .

3.5) In conclusion

Is a PDF and -The End -

* **HW\_4.2** Consider the minimal problem( static optimal problem)

Assume . Find

subject to .

Then prove that the minimum value is -End-

Sol: The Lagrange equation is

The necessary condition at the optimal point is

Hence

implies is an eigenvalue and is an eigen vector. The minimum value of is

* **HW\_4.3(Laselle’s Theorem)**

**%% Review Laselle**

Theorem 3.4 (Lasalle) : sufficient conditions for For asymp. Stability

1. is a compact set that is positively invariant w.r.t
2. is continuously differentiable such that in
3. **M is the largest invariant set in E**

Then

%% Kim’s comment

1. Here is not necessary positive definite. i.e., is not necessary “0”, and positive definite.
2. If is a PDF, and , then
3. The set is =0}, i.e.,

which means the level is constant. Here is not necessarily definite.

If and is only the equilibrium point, i.e., , converges to the equilibrium point(i.e., asymptotic stable) even if .

%%

Consider

Select

1. What is the equilibrium points?

Sol:

1. Using the equilibrium point is stable or asym.stable?

Sol: 🡪 stable.

1. Find the invariant set

Sol: By definition, , hence

1. What is the largest invariant set ?

Sol: Since is the largest set of , i.e.,

Hence

1. Using the Laselle’s theorem, is it asymp.stable?  
   By Laselle’s theorem, given the invariant set , , where . Since has the element of as well as the other points, it is not asymp.stable.

* HW\_4.3 : Exercise 3.15 (pp.85)

Sol:

1. The equilibrium points

From 🡪 , from

Since

Let , If then since and , since , hence for the only point is

If , Since , there is no such that .

Hence the only equilibrium point is the origin.

* HW\_4.4: Exercise 3.20 (pp.85)

Sol:

Let a select .

Then

So far it is not good. But I want to cancel . Select the another one as

Then

Then the disturbing terms are . Ok. Now take the another one as

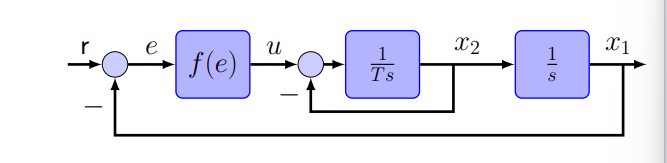
Then

Now we may refine the last candidate Lyapunov function.

For positive definite, from (1) . Let select the region of attraction as

* HW\_4.5

Consider the following feedback system



The state space model is,

where

Choose a Lyapunov candidate as

1. Is it stable?
2. If r = step function, plot the

3.1) when

3.2) when

3.3) when

-The End-