* Ex. 7.4

1. Problem

Is the origin of the unforced closed loop system globally uniformly asymp.stable?

1. Solution

The unforced closed loop system is

Since , i.e.,

Or

implies

Or

1. Globally asympt.stable.

Define a candidate Lyapunov function

and , and

so that it is a radially unbound positive definite function.

Its derivative is

Hence at the origin of the closed loop system is globally stable.

1. Uniformity:

Since is time-invariant system, i.e., by the theorem 4.1 if

the origin of is uniform stable, hence take ,it is uniform stable.

%%% Kim’s comment;

Uniform stable is regarding an initial time in the time varying system. If the system is tine invariant, always it is uniform.

%%%

* Ex. 7.5 (a)

1. Problem

is globally exponentially stable?

1. Solution

To find equilibrium point,

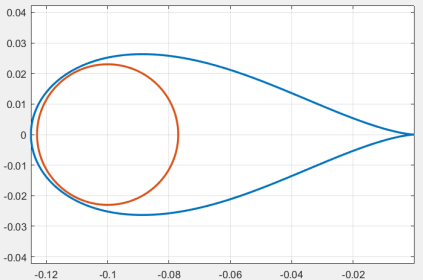
There are another equilibrium point different from , so that has no global stable point. Moreover, the linearized system is not asympt. stable, which implies is not asympt.stable.

* Ex.7.10 (Circle criteria)

Given a transfer function with a non-linear , the closed loop system is absolute stability or not.

1. case

Since in RHP, there is a one pole. To be absolute stable, select a circle with radius 0.023 at the center -0.1, which Nyquist encircle in the CCW direction one time.



Hence the non-linear is in the sector = S(8.13, 12.987)

1. : Since is not Hurwitz, the circle criteria can not insure the absolute stability.
2. : Since is not Hurwitz, the circle criteria can not insure the absolute stability.

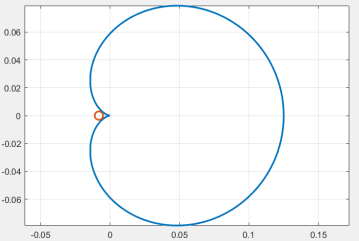
%%% Kim’s comment

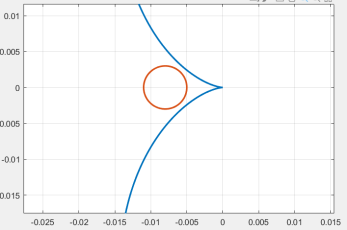
Let us see the characteristic equation of the closed loop assuming is a constant,

Hence if 8, the closed loop is asymptotic stable. However if is nonlinear in a Sector,

1. The circle criterion is sufficient condition
2. Moreover for any non-linear in the sector, the closed loop is asymptotic stable. %%

The circle is with 0.003 radius at the center -0.008





If 🡪 , , i.e.,

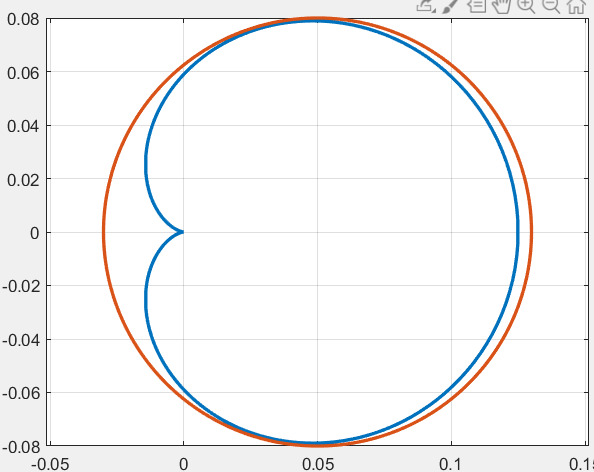
the , by the circle criteria the system is absolute stable.

The minimum real value of the Nyquist is -0.0143, hence

The will ensure the absolute stability.

1. . Since the transfer function is a Hurwitz, may be a negative. The circle is

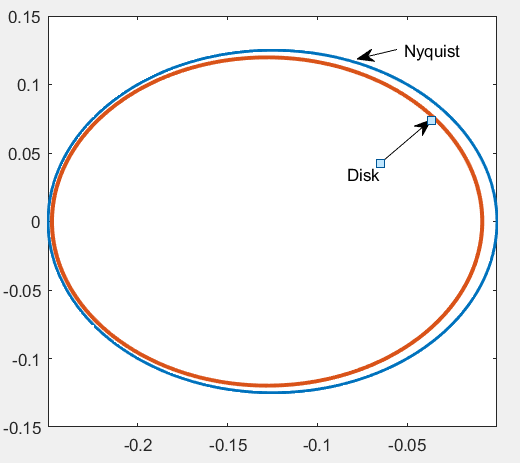
with radius 0.08 at the center 0.05 Hence



Hence the closed loop is absolutely stable with

-33, 7.7)

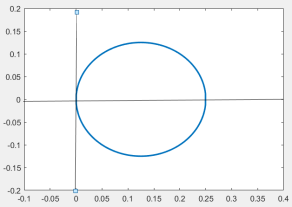
Since is not Hurwitz, by the circle criterion only is considered.

One pole of is RHP, the Nyquist encircles a disk in the CCW, select a Disk with at

Hence

Since there is no poles of G(s) in RHP, for the absolute stability, the Nyquist plot does not enter a Disk, We may select a disk with a radius large enough so that

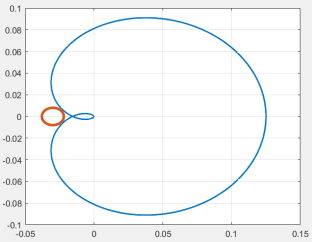
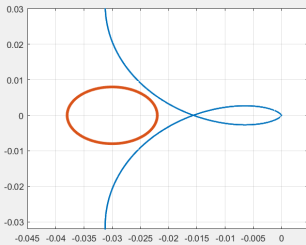
i.e., ,



Since is a Hurwitz, this is the same condition to a)

The Nyquist plot should be inside a disk, assume a disk with radius 0.125 at the center 0.125 which is equivalent of the Nyquist plot except the direction. Hence and , , it is absolute stable.

Since is a Hurwitz and no pole on RHP, we may consider all three cases.

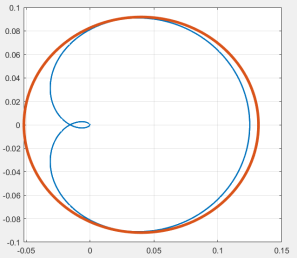


The Disk with radius =

At the center -0.03. Hence

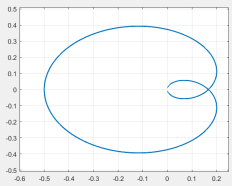
Hence

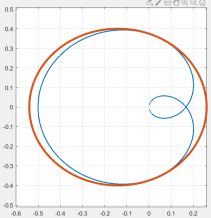
The minimum of the real of part Nyquist plot is -0.0309, so that

Hence , the closed system is absolute stable

The circle with radius 0.092 at 0.04 is encircled the Nyquist,

Hence the system is absolute stable.

The min of real Nyquist is -0.497, hence 🡪 the system is absolute stable.

1. In this case
2. 

The radius of the circle is 0.4 at the center -0.14 .Hence

* -1.85, 3.84)

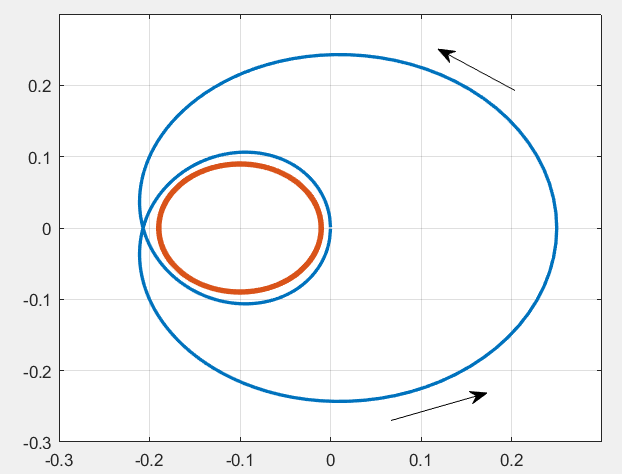
-Since is not a Hurwitz, by the circle criterion, only in is considered.

- is not a minimal realization, the circle criterion can not be applied.

- Let be the minimal realization, , it is not Hurwitz. Hence is considered. One pole of is in RHP, the Nyquist should be circled in the direction of CCW, however, is always a real number . Hence it is NOT absolute stable for any -The EnD-

Since is not a Hurwitz, by the circle criterion, only in the case of is considered for absolute stability.

The Nyquist plot should encircle two times of the disk in the CCW direction.

Select a disk with a radius 0.09 at the center -0.1

Hence

i.e., with it is absolute stable.

* Ex. 9.1

1. Problem

Pick a candidate

Select , then

Hence with is globally asympt. stable

1. Problem

Pick a candidate

,

Select as

Then

By Lasalle’s Theorem, (Why?), the system is globally asymp. stable

1. Problem:

Pick a candidate

,

Select as

Then

Hence globally asympt. stable

1. Problem:

Using backstepping

1. First define . The first equation is

Select

Define . Then the system is

Select

)

Design

Then

Hence with and u, the system is globally asym. Stable.

1. Problem

Using back stepping

1. Define . The first equation is

Select a ,

Choose

Select

Then the system is

Select a

Design

so that

Hence the original system is asym. Stable.

1. Problem

Sol:

First assume as an input so that

Define . The first equation is

Select

Define . Then the system is

Select

Design as

to result in

The next backstepping,

Define the additional system is

And

Select where , where ,

The time derivative

Since

And design as

So that

* Example.1 – the backstepping procedure.

1. Problem
2. Fictitious controller as at the first equation

Select a Lyapunov equation

So that

To be negative

so that

1. Define fictitious state at the second equation

Its dynamics is with , the first equation is

The second equation is

Now select a lyapunov function

Select as

Then

The final controller in terms of is with

* **9.3 (2) : backsteeping**

,

1. first fictitious controller

Pick 🡪

1. Second fictitious state with

The first equation is

The second is

Pick a Lyapunov function

Then

Hence if the controller is

The time derivative of

1. The controller in terms of the states with

* **Ex.9.3) (3) : backstepping**

1. The origin is not at , but (0,-2).
2. first define a fictitious as

The first equation is

Take a Lyapunov function as

So that

Take for to be negative definite

1. Second , define a fictitious state

The first equation is

The second is

Select a lyapunov function for the total system as

The time derivative is

To be negative,

which results in

Then at the system is asymptotic stable, , i.e,

Since , 🡪

1. Now , the controller is calculated in terms of

with

%%% Announcement

As a reference, the answers to the Home assignment is up-loaded except exercise 9.2

**I have to exclude Ex. 9.2. The problem Ex. 9.2 is not well defined.**

**Final Exam will be related to this HA problems, (70%),**

**the others (30%)**

* **Stabilization by Linearization and comparison with state feedback**
* **Linear observer.**

--The EnD--%%%