Nonlinear Control Mid term Exam. May. 2023.

1. (Lipschitz) Answer the following function is Lipschitz or not
2. in

Sol: Lipschitz. Since

1. in

Sol: Not Lipschitz since at , the slope is infinite

1. in

Sol: Lipschitz

1. in

Sol: Lipschitz

1. (Positive definite). Verify that the following functions are positive definite or not

%% Kim’s comment on integral : See HW\_3\_Solution

Sol: Not Positive Definite. Since

Not PDF.

Sol: PDF since

-

- if .

And

Hence , 🡪 PDF

%% Kim: Integral See HW\_3\_Solution

Hence

* 1. if

Hence

In conclusion,

Then

Ex.

Sol: positive semidefinite

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1. (Lasalle’s Theorem) Consider

Consider the system is defined in

Assume

2.1) Define . Find the largest set of

Sol:

Hence

3.2) Find

Sol:

1. Variable gradient

Consider a following system

To construct a Lyapunov function , applying the variable gradient method. (Hint )

Sol:

1. Negative

Select as

1. Positive

Check is a positive definite.

Hence is a PDF for asymptotic stability at the origin

1. The statement is true or not if the solution to a dynamic system is unique.
2. if eigenvalues of has negative real numbers, then is asymptotic stable

Sol : False

1. If the linearized system is asymptotic stable, then the original system is asymptotic stable

Sol: True

1. If the linearized system is unstable at the origin, then the original nonlinear system may be stable

Sol: false. unstable linearized system guarantees the original system instability.

1. If the equilibrium of a linear system is asymptotic stable, then it is also exponentially stable. Sol: True
2. If the equilibrium of a non-linear system is asymptotic stable, then it is also exponentially stable. Sol: False the example in the materials.

%% Team’s grade

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