1. Consider
2. Is the linearized system stable or asymptotic stable at the origin? prove it

Sol:

Hence

which is asymptotic stable.

1. Find a Lyapunov function to check the stability

Sol : Select

So that at the origin is asymptotic stable.

1. Check the equilibrium point is exponentially stable

Sol: it is exponentially stable

1. Consider
2. Find equilibrium point and linearized the system at the equilibrium point to check the stability.

Sol:

If

* 🡪

🡪 not possible

Hence only

If , also only

Hence the equilibrium point =

Linearize:

* unstable

1. Define , and an invariant set . Find the largest set of

Hence

1. With find

Since as x(t) converge to ,

1. Euler equations for a rotating rigid spacecraft are given by

Where is the angular velocity vector along the principle axes, is the vector of torque inputs applied about the principle axes, and to are the principle mpments of inertia.

1. Show that with is stable. Is it asymptotically stable?

Sol :

**The equilibrium point is**

Assume , Change the new variable .Then

and the equilibrium point .

Then the equilibrium point is implies

Select

Hence, at the origin is stable 🡪 is stable.

It is not **asymptotic stable**, since for any constant , since

1. Let , where to are positive constants. Show that the origin off the closed loop system is globally asymptotically stable.

Sol : with , 🡪 asymptotic stable

1. Consider
2. Is the origin globally uniform asymptotic stable using ?

Sol: Select

By the theorem 4.2

And select a positive definite

And are radially unbounded, hence the origin is globally uniform asymptotic stable.

1. If is selected, is it globally exponentially stable?

By the Theorem 4.3 , if

And

However it is impossible to find to meet the inequality

it is not guaranteed the globally exponential stable.

%% consider to compare the magnitude of

If is near the origin, , implies is too small to be convergent to the origin.

%%

1. Consider

Prove the origin is globally exponentially stable.

Sol: Taking , select and then

The time derivative is

Where

Since all of the principle minor are positive,

Where

Due to the Theorem 4.3 the origin is globally exponentially stable.

1. Consider

The system is considered as a perturbed system with a nominal system ,

Since the nominal system is globally asymptotic stable, we would like the perturbed system is also asymptotic stable at least locally. In order to find a domain of attraction of the system, it may follow the following procedure.

Select a Lyapunov function as

1. Regarding the nominal system, find satisfying
2. Regarding the perturbed system, the derivative of the Lyapunov function along the perturbed trajectory,

Select a domain find a the maximum value of so that

1. so that in , is a negative definite, which implies is a domain of attraction of the perturbed system .

%% see example 4.5