* Least Square Estimator(LSE)
  1. Linear Static Systems
* Definition: linear regression
* (1) linear regression model
* is an observed output, regressor, an unknown parameter
* Why regression? The opposite of regression is (advancement, development, evolution)
* Linear system solution:

1. One and unique solution
2. Infinite many solution
3. Non-solution

* In this case we may estimate the solutions in some sense:

++ Pseudo-inverse:

At both side of the equation multiply , then

* 1. Least Squares Estimation
* Def: the prediction error(or residuals)
* If the measurement N, if , is as small as possible. 🡪 Least square estimator.
* Problem statement:

find the least square error estimator of

* Solution

Let (3) in the matrix form as

Hence

The gradient of is zero iff , such that

We call as LSE as

++ Pseudo-inverse:

-. If exists;

where is a NxM , N>M matrix. How to get the LSE?

At both side of the equation multiply , then

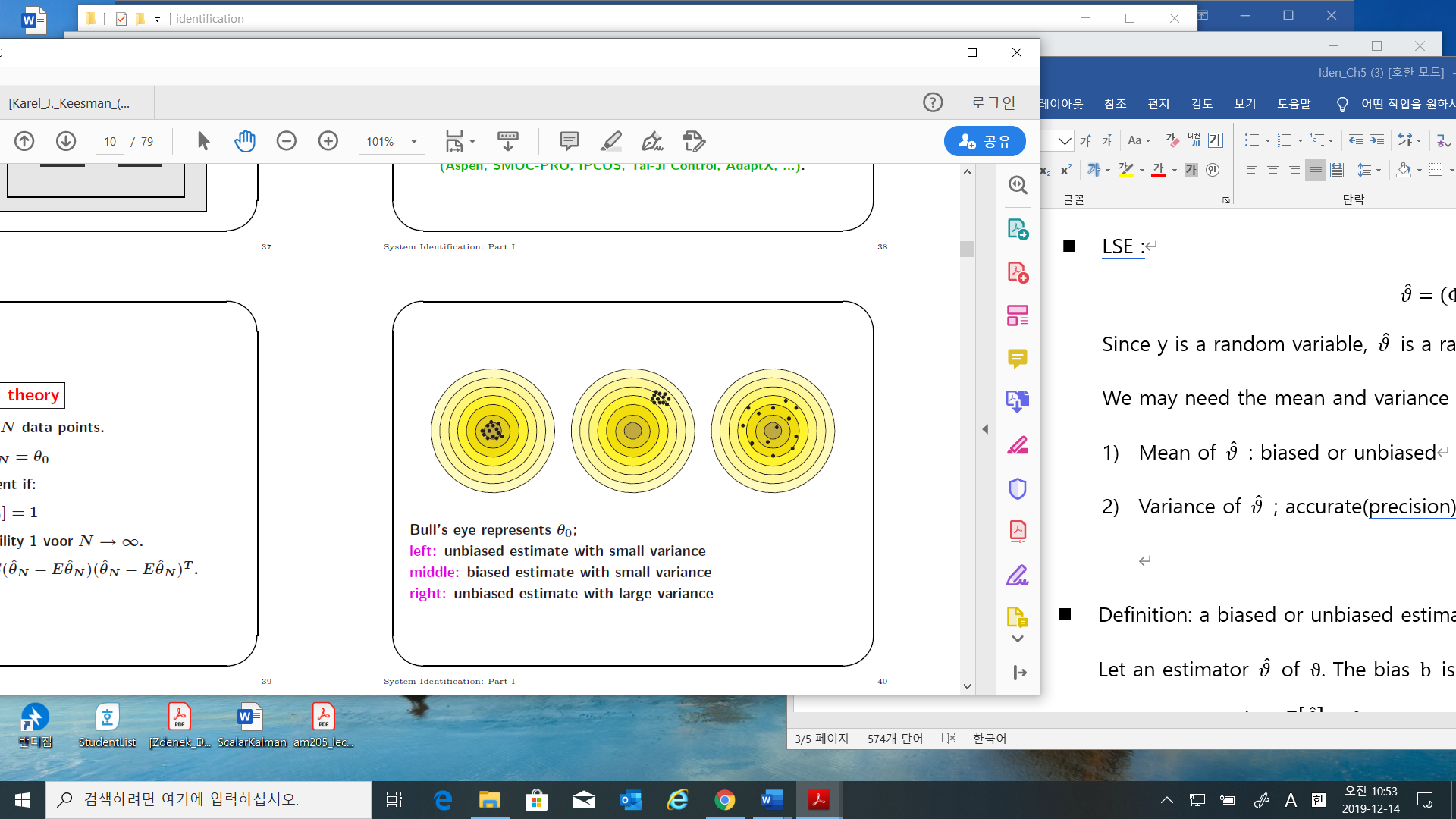
* 1. Performance of LSE
* Characteristic of a Random variable:

The solution of LSE

Since y is a random variable, is a random variable.

We may need the mean and variance of Random Variabe.

1. Mean of : biased or unbiased
2. Variance of ; accurate(precision) or not

\*\* in googling.

* Definition: a biased or unbiased estimator

Let an estimator of . The bias is defined as

The unbiased estimator if , otherwise the biased estimator.

* The unknown of the covariance of

(6)

* Sample mean and variance in scalar case
* Sample mean
* Unbiased Sample variance(standard deviation)

Where M is the number of unknowns

* Check the biased and accuracy
* 🡪 the sample error is zero

If is not zero,

1. Increase the number of measurements and check. If not
2. Change model, then check the bias

* Check the accuracy

1. Calculate the unbiased sample variance

This will be used to in (6)

Calculate

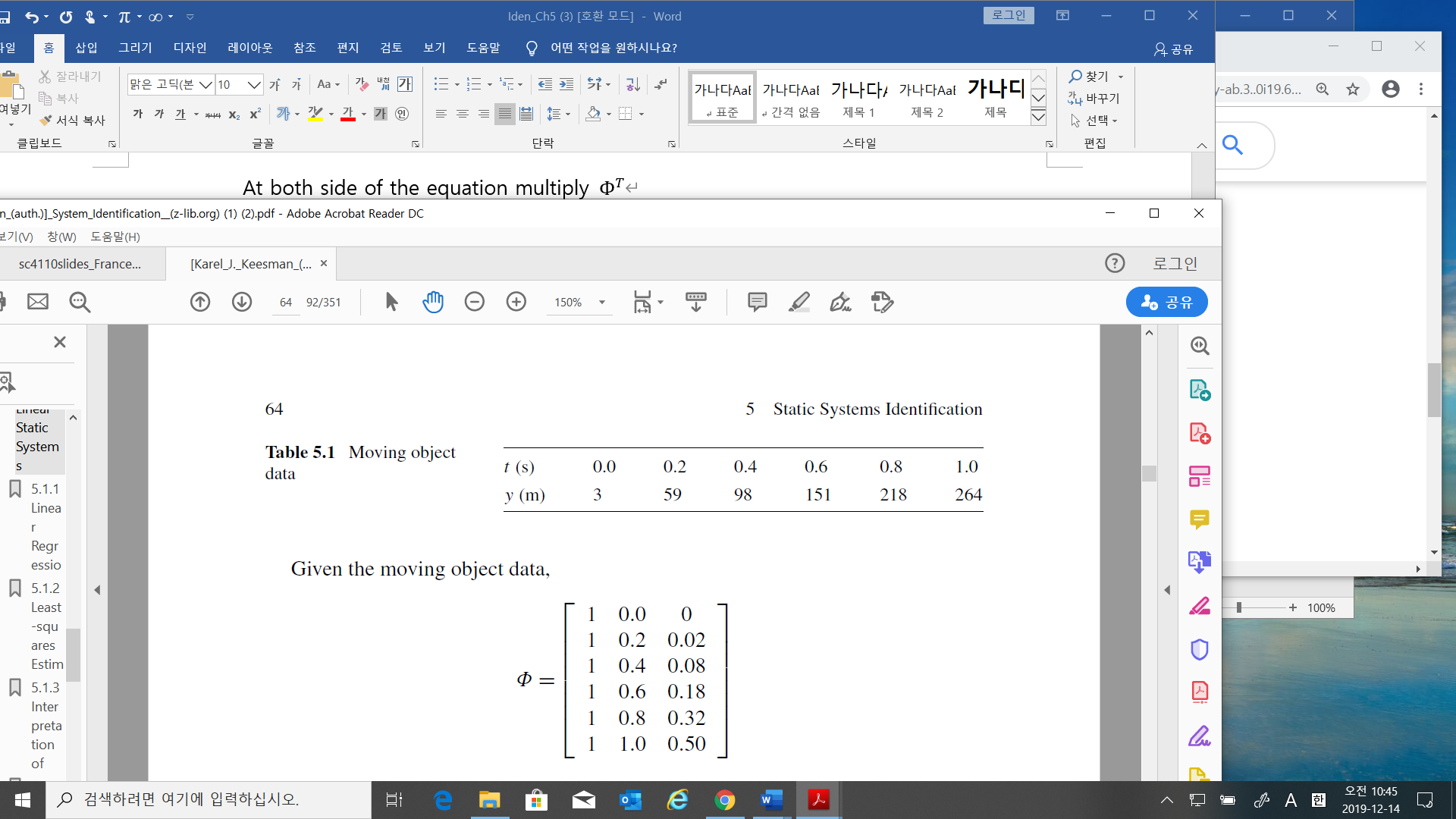
Calculate the variance of

Check the accuracy with and the variance of

* Example

Model: the distance

Observations of the distance



1. Matlab program: the first section of Simulation\_LSE.m

-mean of the residual =

-MSE(the variance) of the residual

* Ex.5.6: This estimator is not the LSE but an unbiased estimator)

Define an estimator is

Then

Hence

Hence

- If then the estimator is unbiased.

- Or if is large, the estimator converges to be unbiased

- Or if is large, the estimator converges to be unbiased

* Ex.5.7: The estimator is not unbiased
* The LSE is unbiased in the system

Hence the LSE is unbiased if

1. is independent and
2. , where

Or if is deterministic, i.e.,

Hence, which implies the estimator is always unbiased.

* + 1. Accuracy (Variance) of LSE
* The unknown of the covariance of

If is a white noise with constant variance , then .

(5.31) is

Since is unknown, the variance of the prediction error

The prediction error sequence

Thus in the variance of the estimator in (5.32) will be replaced by