* System Response Methods
* Frequency Response Method
* Correlation Method

1. Static System Identifications
   1. Linear Static Systems
      1. Linear Regression

* Definition: linear regression
* (5.1) linear regression model
* is an observed output, regressor, an unknown parameter
* Why regression? The opposite of regression is (advancement, development, evolution..)
* Ex. 5.2: Moving object

Model: the distance

Observations of the distance

* + 1. Least Squares Estimation
* Def: the prediction error(or residuals)
* Problem statement:

find the least square error estimator of

* *In stochastic control minimum variance error estimator*:

So in order to get the best MV we need to have the conditional PDF. In the least square estimator, we do not need the conditional estimator.

* Solution

Let (5.3) in the matrix form as

Hence

The gradient of is zero iff , such that

We call as LSE as

In the case of the outliers: the weighted LSE, the weight =

* Ex.5.3

Ex. 5.3 / 5.4 : Moving object

Model: the distance

Observations of the distance

-mean of the residual =

-MSE(the variance) of the residual

* *Statistic: see the sample random variable*.
* The outlier: the weighted least square estimator:

-the cost function

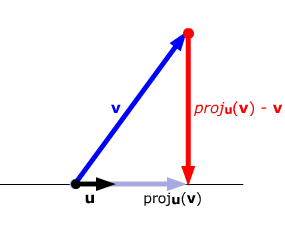
-the solution:

* + 1. Interpretation of LSE

1. Orthogonal property between error and estimator

* Ex. 5.5 Orthogonal projection

<https://www.math.uh.edu/~jiwenhe/math2331/lectures/sec6_3.pdf>



=the orthogonal projection vector of on vector

* Inner product : In Euclidean space

Then the inner product of is

* Orthogonal basis : , then is orthogonal to

Let pick up an orthogonal basis as

Then

* The orthogonality between the error and the estimator

The inner product of

They are orthogonal, hence,

* Definition:

-Projection matrix:

-Orthogonal projection matrix:

-Ex. Let then and hence P is an orthogonal projection matrix. Since

Which implies the estimator is the projection of on the space of

y

* Cross-correlation of least square estimator

Implies

* + 1. Bias of LSE
* Definition: a biased or unbiased estimator

Let an estimator of . The bias is defined as

The unbiased estimator if , otherwise the biased estimator.

* Ex.5.6: This estimator is not the LSE but an unbiased estimator)

Define an estimator is

Then

Hence

Hence

- If then the estimator is unbiased.

- Or if is large, the estimator converges to be unbiased

- Or if is large, the estimator converges to be unbiased

* Ex.5.7: The estimator is not unbiased
* The LSE is unbiased in the system

Hence the LSE is unbiased if

1. is independent and
2. , where

Or if is deterministic, i.e.,

Hence, which implies the estimator is always unbiased.

* + 1. Accuracy (Variance) of LSE
* The unknown of the covariance of

If is a white noise with constant variance , then .

(5.31) is

Since is unknown, the variance of the prediction error

The prediction error sequence

Thus in the variance of the estimator in (5.32) will be replaced by

* Comparison of Minimum mean-square error

where

The variance of the least square error is in (5.32) and (5.33)

* Constrained least squares

%% do not confuse followings:

1. The notation :

* : the estimator . : the mean
* ,

1. If the regressors in is deterministic, The LSE is the MS estimator.

But not vice versa, since is a Random variable,

* + 1. (skip)Identifiability
    2. ~ 5.3 (skip)