1. Linear Optimal Control Theory for discrete time system
   1. Optimal Linear Discrete-Time State Feedbak Control systems

Last Materials I introduce the discrete-time domain linear systems, which is modelled as a

linear difference equation. I summarized it.

1. Consider a difference equation

* The solution of the equation:
* The stability condition:

  2. Lyapunov equation

For a there exists a unique solution which satisfies

1. Consider a difference system,

* The controllability condition:
* The observability condition:

1. For stochastic discrete linear system

* Consider

Then

where,

* In the steady state

So far these are already known to you. Now we may consider “ To design an **optimal controller** in a discrete time linear system”.

%%%%%%%%%%%---------------some comments transpose – Lyapunov equation

I am always confused by the formula. Let’s consider

for stability (1)

and for variance (2)

So there is and . Is that interchangeable to to ?

If you consider the stability, it is possible since

eigen-value of = eigen value of (In linear algebra!)

But, if you calculate the variance, it is not. For example in matlab

clear all;clc;

A = [ 0 1; -1 -2];

B = [ 1;1];

X = lyap(A',B\*B') % stability

Y = lyap(A, B\*B') % variance

In this case , the variance is different! So you are not allowed to interchange and .

Hence we should use the right formula to your problem. By the way in matlab

>> help lyap

*X = lyap(A,Q) solves the Lyapunov matrix equation:*

*A\*X + X\*A' + Q = 0*

The Lyapunov equation is interchanged the original Lyapunov equation, be careful.

--------------------------------%%%%%%%%%%%%%%%%

* + 1. Stability -skip
    2. The Linear Discrete -Time Optimal Regulator Problem

The discrete optimal controller formula has several different forms. In the text book, one of the formula. I introduce another formula which is more understandable. In

<https://stanford.edu/class/ee363/lectures/dlqr.pdf>.

* Definition 6.16: Discrete Optimal Regulator problem

Consider

and the cost function as

where

Find the optimal controller to minimize the cost function (similar -6-233).

* (Similar to) Theorem 6.28: The discrete optimal regulator controller

The optimal linear regulator controller is

where

and

with the terminal condition

* Theorem 6.30

If the discrete system is controllable, then the steady state solution of (6-248) is unique as

And

%%%%%%%%%%%%%%%% ---------------------comments on Theorem 6.28, 6.30

1. The (6-248) has the terminal condition instead of the initial condition. Therefore to get the solution the iterative way is backward. Moreover the solution is time varying, which is difficult to calculate and need the whole trajectory of the solution so that you need big memory capacity.
2. In general, the Riccati solution is constant in time except near the final time. Hence the rule of thumb the constant values of the solution, i.e., the steady state, is used for the following reasons.

* The solution is relatively easy
* The memory size is smaller

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

* Example 6.14: Digital position system (continue Example 6.2 / 4.4 / 2.4)
* First we need discretization of
* Using symbolic math, check the result with the textbook

clear all; clc

syms a dT k

A =[0 1; 0 -a];

B = [0 ; k];

expm(A\*dT);

Ad = expm(A\*dT)

Bd = int(Ad,[0, dT])\*B

% substitute the value

clear all;clc

a = 4.6;

k = 0.787;

dT = 0.1;

A =[0 1; 0 -a];

B = [0 ; k];

Ad =expm(A\*dT)

Bd = [k/a\*(dT - 1/a + 1/a\*exp(-a\*dT)) ; k/a\*(1 - exp(-a\*dT))]

* Hence we may get the discretization system( You may check with the textbook)
* Now design the discrete optimal regulator controller. The controlled output is defined as

The weighting matrices are

Now for the steady state case, in matlab “dare(discrete arithmetic riccati equation)”

will give the answers as

Cd = [1,0];

Q = Cd'\*Cd;

R = 0.00002;

Qf = 0;

[P,L,F] = dare(Ad,Bd,Q,R)

Hence in matlab

P = the solution of steady state Riccati Equation

L = the close loop e-values of

F = the optimal control gain matrix, i.e.,

-The results (check it with textbook)

The closed loop e-values are

0.2288 + 0.3186i

0.2288 - 0.3186i

Of which the absolute values **are less than one** to guarantee the system stable

%%%%%%%%%%% ---------comments

If you have to design the optimal controller in a real environment, You may code to get the solution of your system. But if you use Matlab, you don’t need it. As you see the matlab command ‘dare’ which gives the solution of the controller, is not simple if you try to code it by yourself. There are many numerical algorithms. So please don’t do code it by yourself. Just use matlab.

---------------------------------------%%%%%%%%%%%%%%%%%%%%%%%%%%

* + 1. skip
    2. The stochastic Discrete-time Optima Regulator

Now we are dealing with a stochastic model.

* Definition 6.17: The stochastic discrete-time linear optimal regulator problem

(<https://stanford.edu/class/ee363/lectures/stoch_lqr.pdf>)

Consider

Here

And

The cost functional is

* Theorem 6.33: The solution to the previous Def.

The optimal feedback law is, which is the same to the deterministic law as,

and

If the final time goes to infinity, if the controllability is satisfied, the solution of (6-248) converges to the unique steady state as

And

Those are the same to the deterministic, discrete optimal regulator solution.

%%%%%%%%%%%%%%%%%----------- comments

1. The stochastic optimal law is independent of the noise , and the variance of the initial point.

i.e.,

1. So far deterministic and stochastic discrete time regulator problems, the two optimal control laws are identical to each other.
2. Why do we need this stochastic model if it has the same solution to the deterministic model?

**The estimator** as in the continuous case will be good.