* Contents: Parameter optimization problems

“Applied Optimal Control: Optimization, Estimation and Control”, E.Bryson,1975.

1. Problems without constraints
2. Problems with equality constraints:
   1. necessary conditions for a stationary point
   2. sufficient conditions for a local minimum
3. Numerical method : Skip

* Gradient method / Linear programming

1. Problems with inequality constraints :Skip

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1. Parameter optimization problems
   1. Problems without constraints

-decision vector as

-the performance index

-problem

Find s.t.

-**The stationary points**: the necessary conditions for the optimality

* Exam.1
* Exam.2
* Exam.3: A positive definite matrix
* Exam.4: not positive definite matrix

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* 1. Problems with constraints
* Decision vector
* State vector
* Constraint vector
* Performance Index
* Problem:

Find s.t. with the constraints

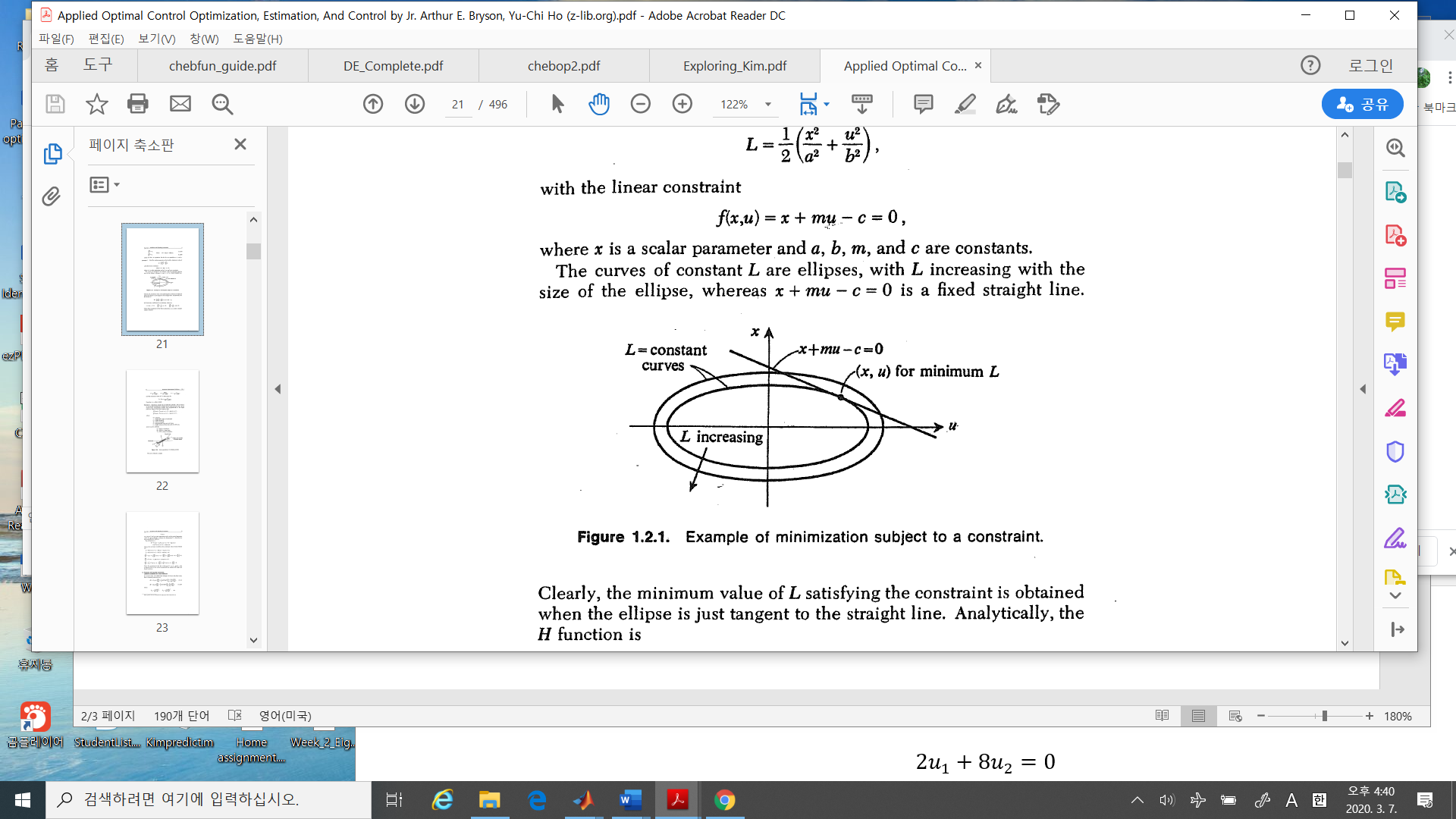
* Exam.5
* **Lagrange Multiplier:**

Define is to “adjoin” the constraints to the index as

* Necessary conditions:

1. Constraints:
2. Stationary points: differential changes in

* Exam.1



* Procedure to Solution:

1. Define adjoined index
2. Necessary conditions)
3. : the constraint condition
4. : the stationary condition w.r.t
5. : the stationary condition w.r.t
6. Find solutions satisfying a), b) and c)
   1. Problems with equality constraints: sufficient conditions
7. Together with the necessary conditions
8. The second derivatives

* Ex.4 (prob.4. page 12)

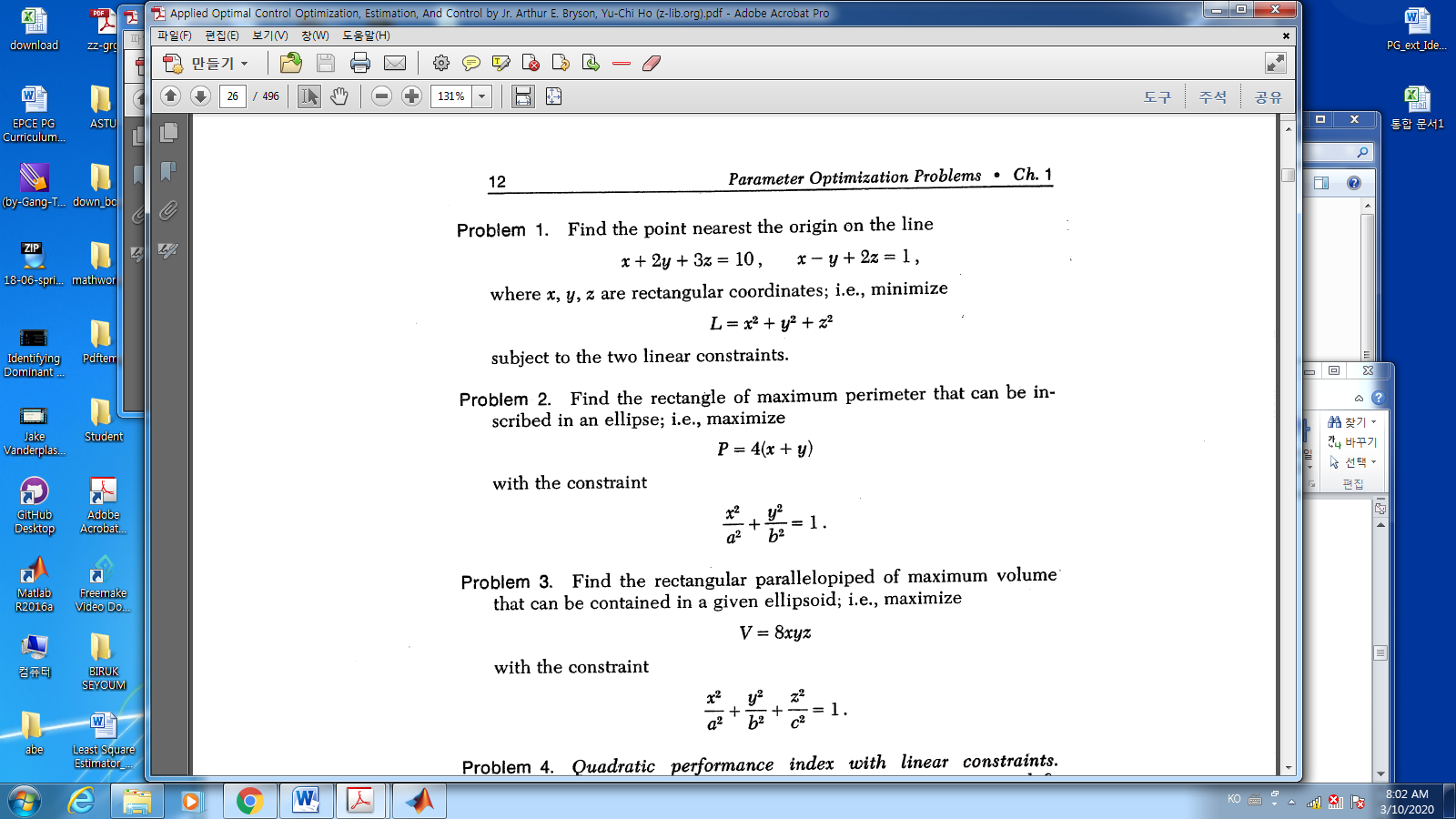
with constraints

Sol:

1. Define as
2. Find the necessary conditions:
3. Constraints(
4. The stationary points
5. Find the optimal
6. The others, you may substitute into equations.

%%%%%%%%---------- home assignment (Bryson, page 12)

Prob. 1,2,3



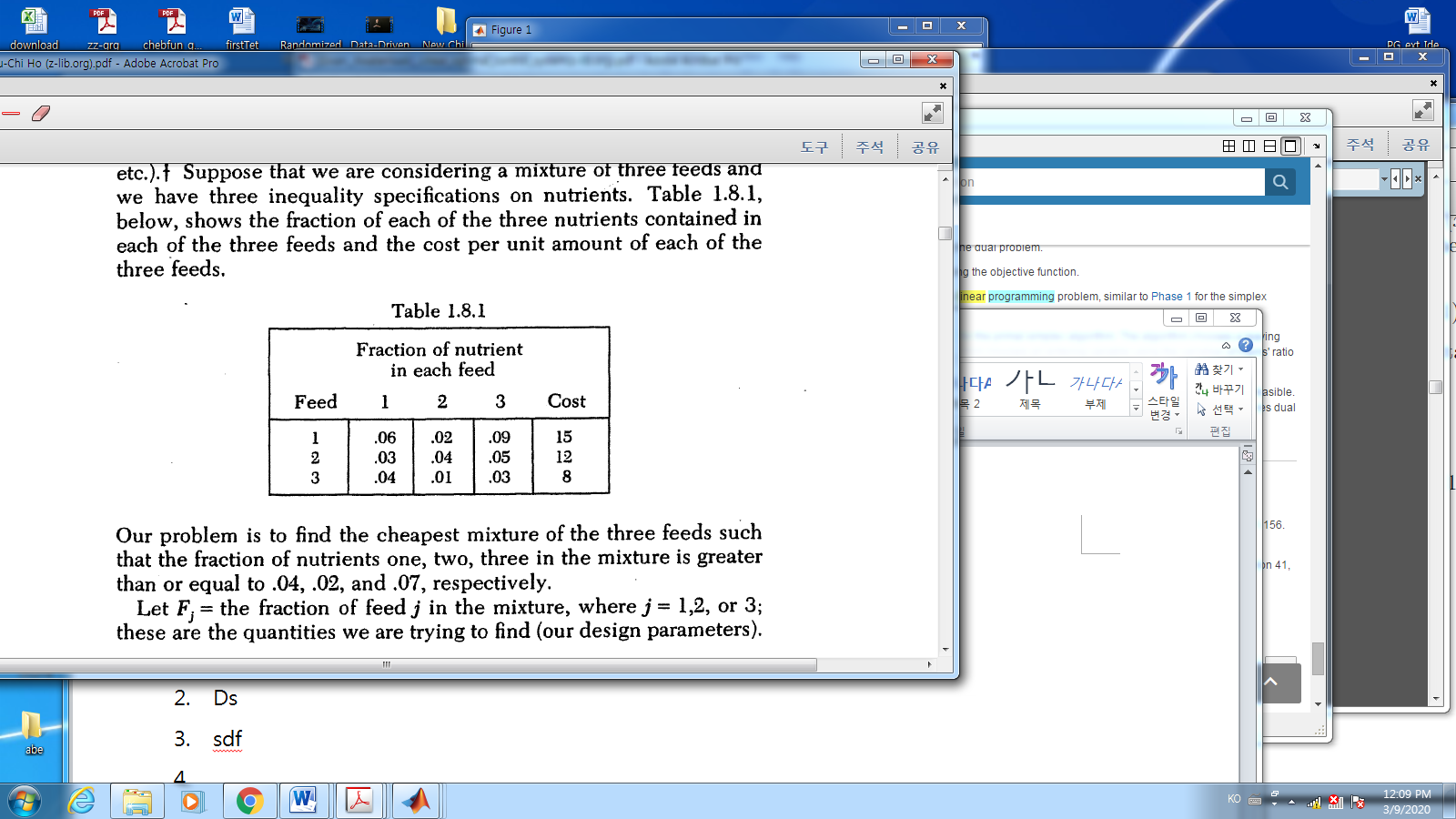
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* 1. Inequality constraints
* Problems

subject to

1. Kuhn-Tucker conditions
2. In general, using numerical method for solutions.

* Linear programming
* Few analytic solutions…
* But important, especially in economics 🡪 many, many papers..
* Example(A bending problem)
* Make a fertilizer with 3 materials.
* Each material is known of its nutrients (ex., protein, fat, vitamins…) and its unit costs.
* Find the fraction of each material to be cheapest price and satisfy the nutrient fraction.



-Mathematical Model:

The fraction of material

The fraction of nutrients of i’s materials:

The unit cost of material

The cost function

* Solution: Tricky and tricky…

1. Calculus of Variations

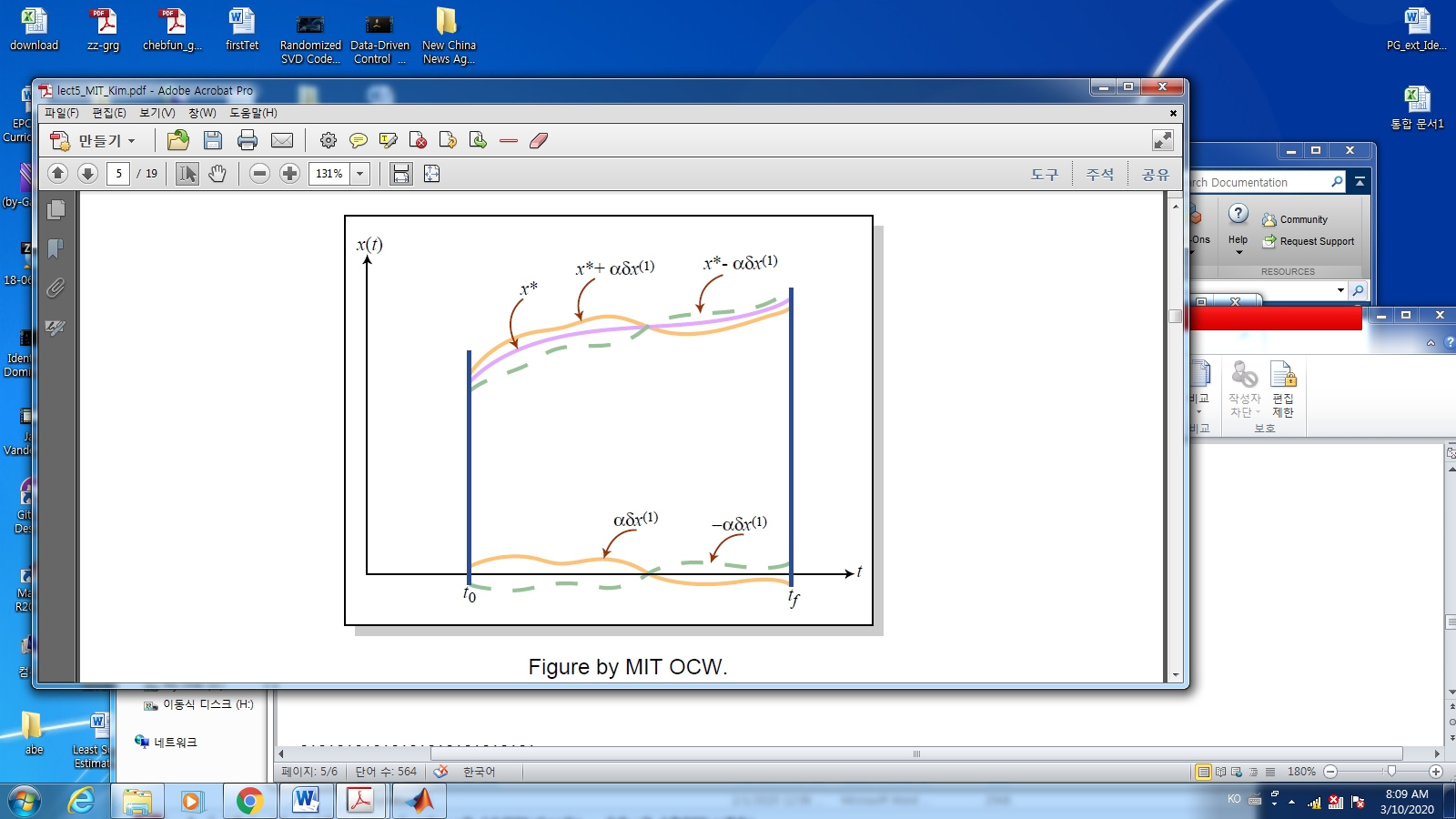
“lec5\_MIT\_Kim.pdf”, Spring, 2006

“lec4\_MIT\_Kim.pdf”,Spring, 2008

* 1. Problem Concept

Subject to

* : a functional.



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* Functional: it is a function but it’s value is a scalar.
* Example

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* 1. Some definitions and facts
* Maximum and minimum: functional

A functional has a local minimum at if

For all admissible in

* Minimum can occur at (i) stationary point, (ii) at boundary..
* An increment of a functional:
* A **variation of the functional** is a linear approximation of the increment
* Fundamental theorem of the calculus of variations

If is an extremal function, then the variation of must vanish on for all admissible ,

* 1. **Euler Equation**: Without path constraints , scalar case
* The cost is
* By the fundamental theorem of the calculus of variations, the necessary condition for an extreme is

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Why the integrand has ? If are independent, then Euler equation is

In our case looks like..

But are not independent. Hence the term is appeared.

* Ex. Find the curve that gives the shortest distance between 2 points I a plane and
* Solution:

Using Euler’s condition,

Since is independent variable, substitute , then and

* if and 🡪
  1. Vector case:

Then Euler condition is

* 1. boundary conditions
* There are several constraints about boundaries. It may be free or fixed.

At each boundary condition, there are necessary conditions for the extreme.

If you are interested in these conditions, which is called as “transversality condition “

at page 12, lec5\_Kim.pdf, or other reference books. I will skip them. But in this

chapter, remember the “Euler’s condition”.

1. Hamiltonian Jacobi Bellman condition: with path constraints

Subject to

* 1. Hamiltonian

Define

* 1. Hamiltonian
  2. Hamiltonian-Jacobi-Bellman condition: necessary and sufficient condition for optimality

1. Continuous LQR
   1. problem
   2. Hamiltonian
   3. For optimal control

* Since 🡪 a global minimum
  1. Solve for

Assume ,

Then

The HJB is

Substituting these into HJB:

* With terminal condition as
  1. Riccati equation
  2. The optimal control
* A linear state feedback!!