SEE Week\_5\_Optimal\_Linear\_State\_Feedback Ex.1.1.

* **Home assignment(W6.1)**:

From (1.15),(1.16),(1.17) and (1.18), derive (1.26)

* **Example 1.4 / Example 1.5 / Example 1.6** (Revisit Example 1.1 – **the solution of Ex.1.1** )

From (1.26) with

The e-values are

The e-Vectors are

Hence from (1.26) the solution is a linear combination as

One of the e-values is positive, hence the system is unstable .

* **Home assignment(W6.2)**:

Find e-Values and e-Vectors (1.86), (1.87) using symbolic math.

* **Example 1.16 (Revisit Examle.1.1** – Closed loop system. Some output feedback: only angular position is measured)

Without control input, **it is unstable** as U see Example 1.5. U may notice many examples in real life. We design a controller to stabilize this example. But before to stabilize, let us consider a simple controller.

Let us **measure only the angular position** at the pivot, . Then with linear output feedback controller, , the closed loop is stabilizable? Let us design a output controller.

1. Find the with respect to the state variables in Ex.1.1 .

In the Ex.1.1

1. Output
2. Linear output control with a negative feedback as
3. The closed loop state space equation

From (1.26)

Hence the closed loop transfer function is

%%%%%%%%%%-------- comment… **textbook (1-252) is not correct.** The sign of is negative but positive in the textbook

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* Pole-zero cancellation

The denominator of the closed loop transfer function(1-252.1) has the 3rd order of the “s” polynomial, which is not consistent , since the closed loop system is 4th order differential equation. Hence there should be pole zero cancellation.

We should check the determinant of , which is

Determinant of = .

Hence there is a pole –zero cancellation as “s”. The correct closed loop transfer function is

1. Stability check.

Since the roots of the denominator of the closed loop transfer function (2.16.3) has at least a positive real.

* **Home assignment(W6.3) :** why does the roots have at least one positive real?
* For any feedback gain , it is not stabilized.--> the only measurement of the angular position is not sufficient to stabilize the inverted pendulum.
* **But If you see the MIT OCW**, <https://www.youtube.com/watch?v=D3bblng-Kcc>,

**He consider a different inverted pendulum which is stabilized using PD controller with the angular position feedback!!**

%%%%%%%%%%---- Matlab has an error in symbolic math.

Consider

clear all;clc

syms M

[1;M]

[ 1 M]'

>> ans:

ans =

1

M

ans =

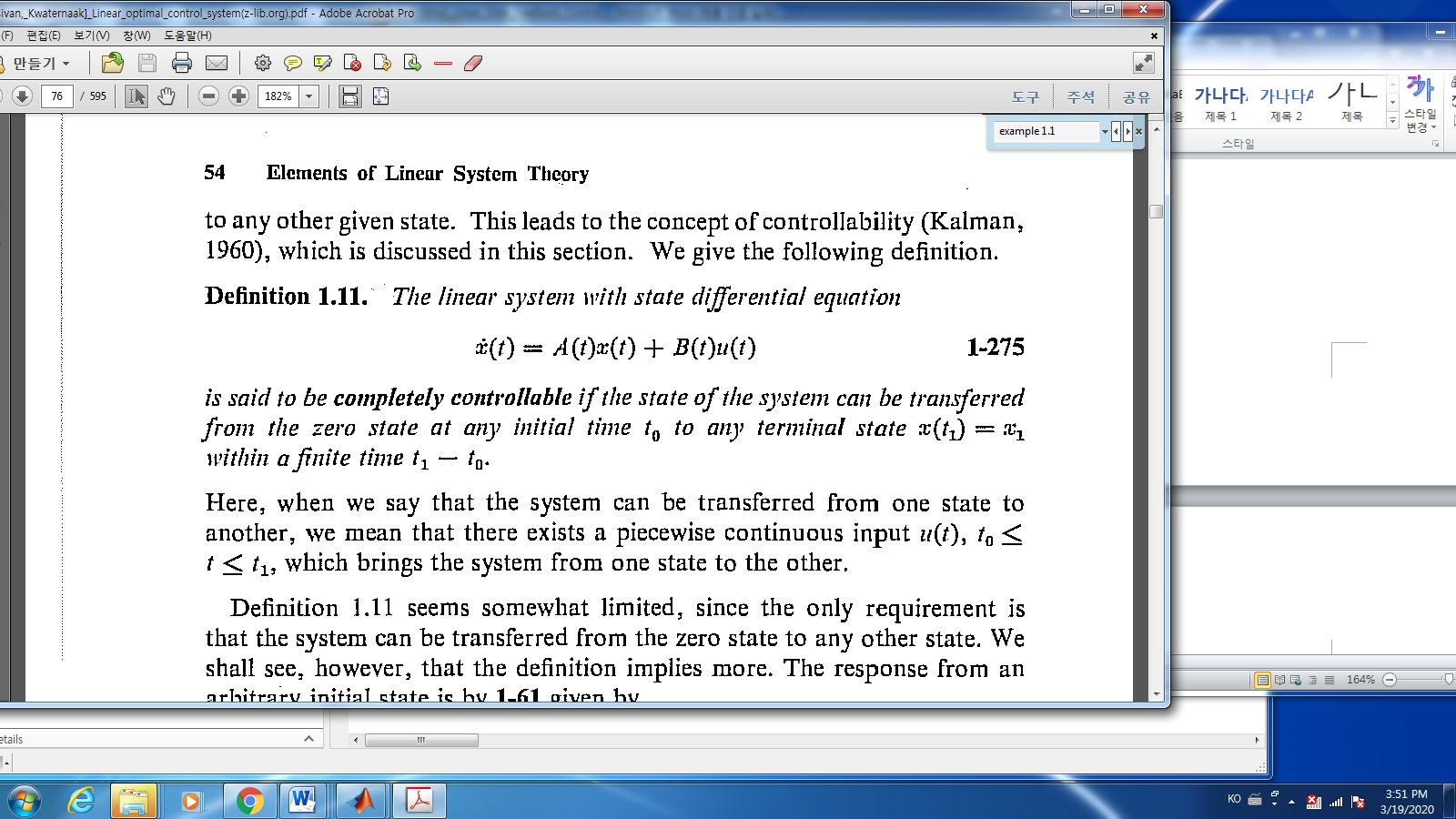
1

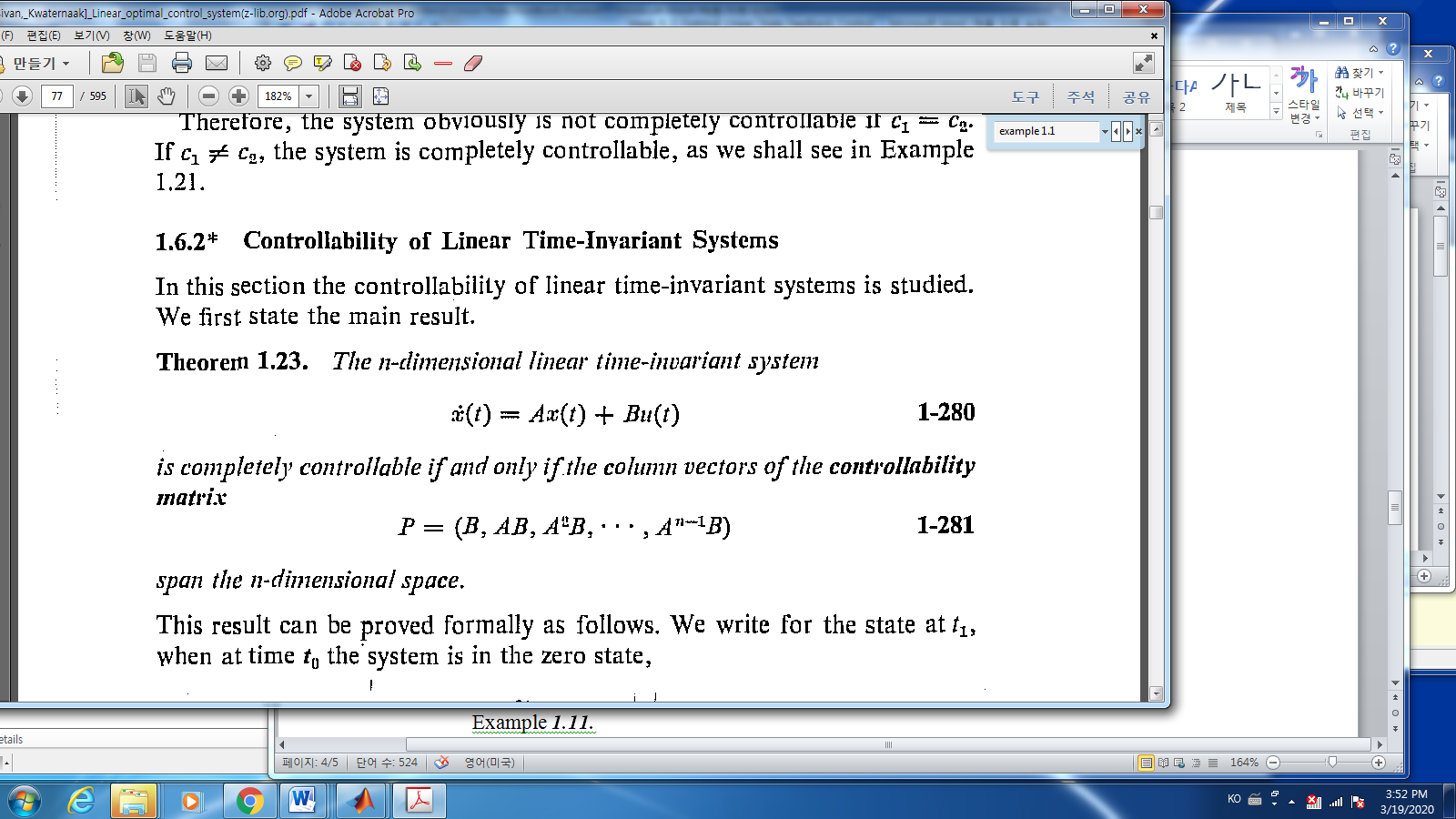
conj(M)

in matlab conj(M) means “M is a complex number, conj(M) is the conjugate of the number”. That is incorrect since “M” is a symbol (as a character) not a number. Hence when I have to transpose a symbolic matrix, I do not use the transpose command.

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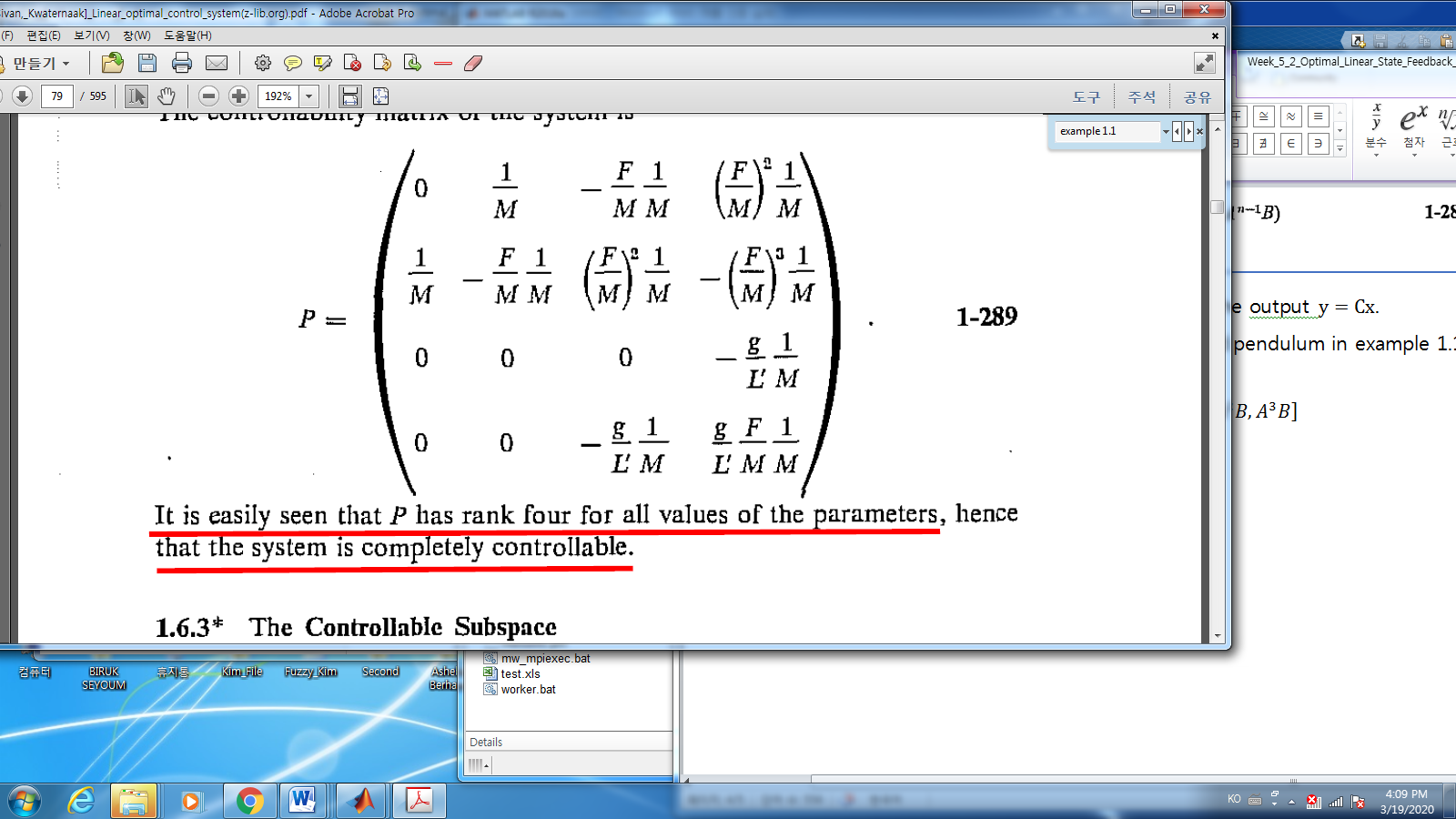
1.6 Controllability





* To be controllable, there is No constraints on the output .
* Example 1.20 (inverted pendulum): the inverted pendulum in example 1.1 is controllable.

Since the controllability matrix ,



%%%%%%%%%%------- it is said “it is easily seen,…, P has rank four for all values of the parameters. Really?

* **Home assignment:** W6.4 Why does P have rank 4?

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