* Review the last lecture

The main topic is “controllability”, which was introduced Kalman at first. As you see the linear system

Now in order to assign any pole as you like, mathematicians thinks the sufficient condition of the controllability is **the input matrix is full rank,**

Yap. That’s a common sense. But the drawback is You need the number of the controllers(i.e., actuators, in mechanics in general) is “n” , **which is TOO many controllers**.

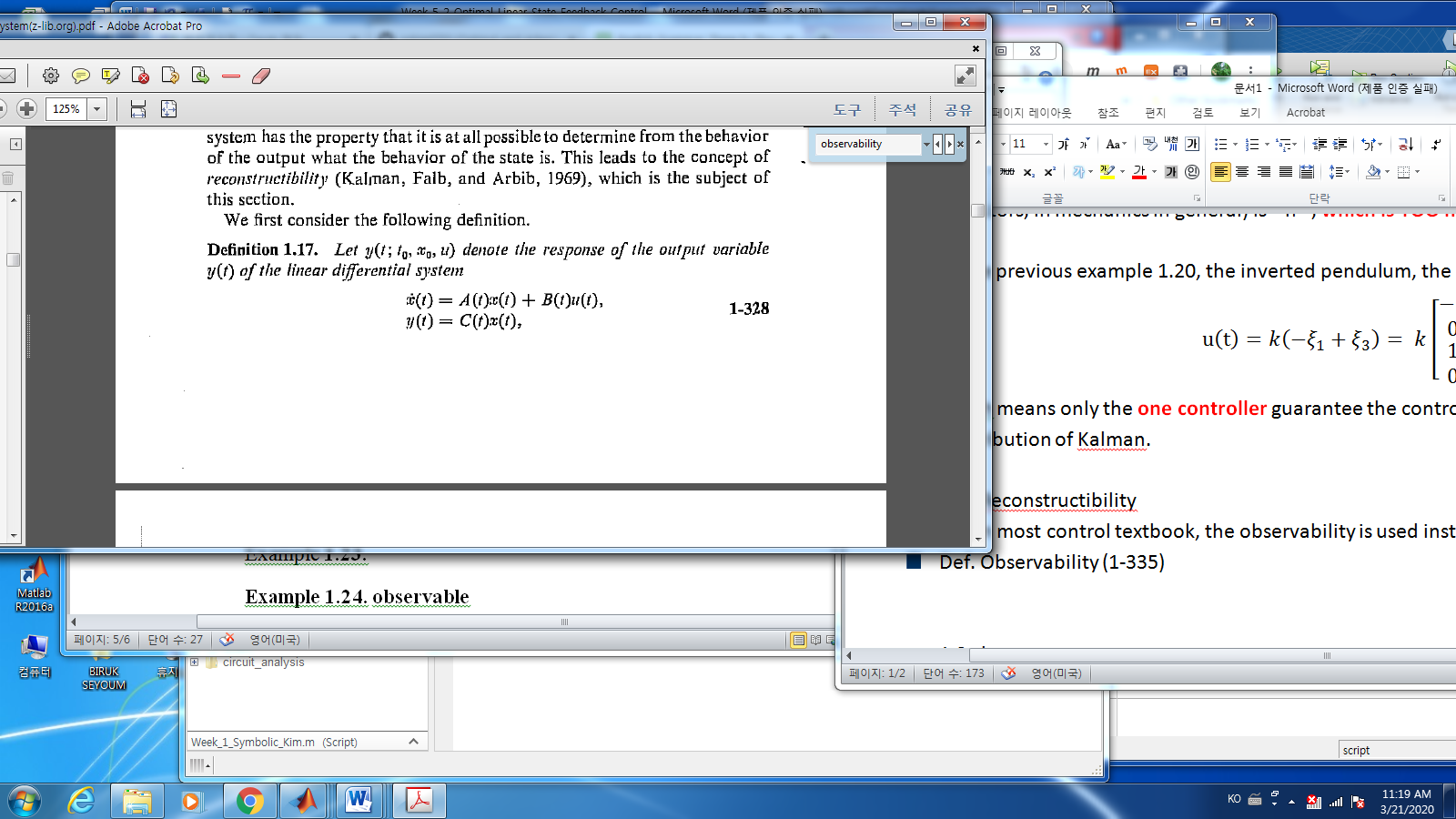
In the previous example 1.20, the inverted pendulum, the controller is

which means only the **one controller** guarantee the controllability !, which is one of the main contribution of Kalman.

* **Home assignment:** W6.5, If , is the system is controllable?
  1. Reconstructibility

In most control textbook, the observability is used instead of the reconstructability.

* Def. Observability (1-335)



The system (1-328) is said to be completely observable if for all there exists a such that

for all implies that

%%%%%%%%%%%--------- what is meaning on the observability?

It is more common to deal with time-invariant systems in this course, i.e. are constant w.r.t. time. Then the system is observable,

If the system is time-invariant, i.e. are constant w.r.t time, for any

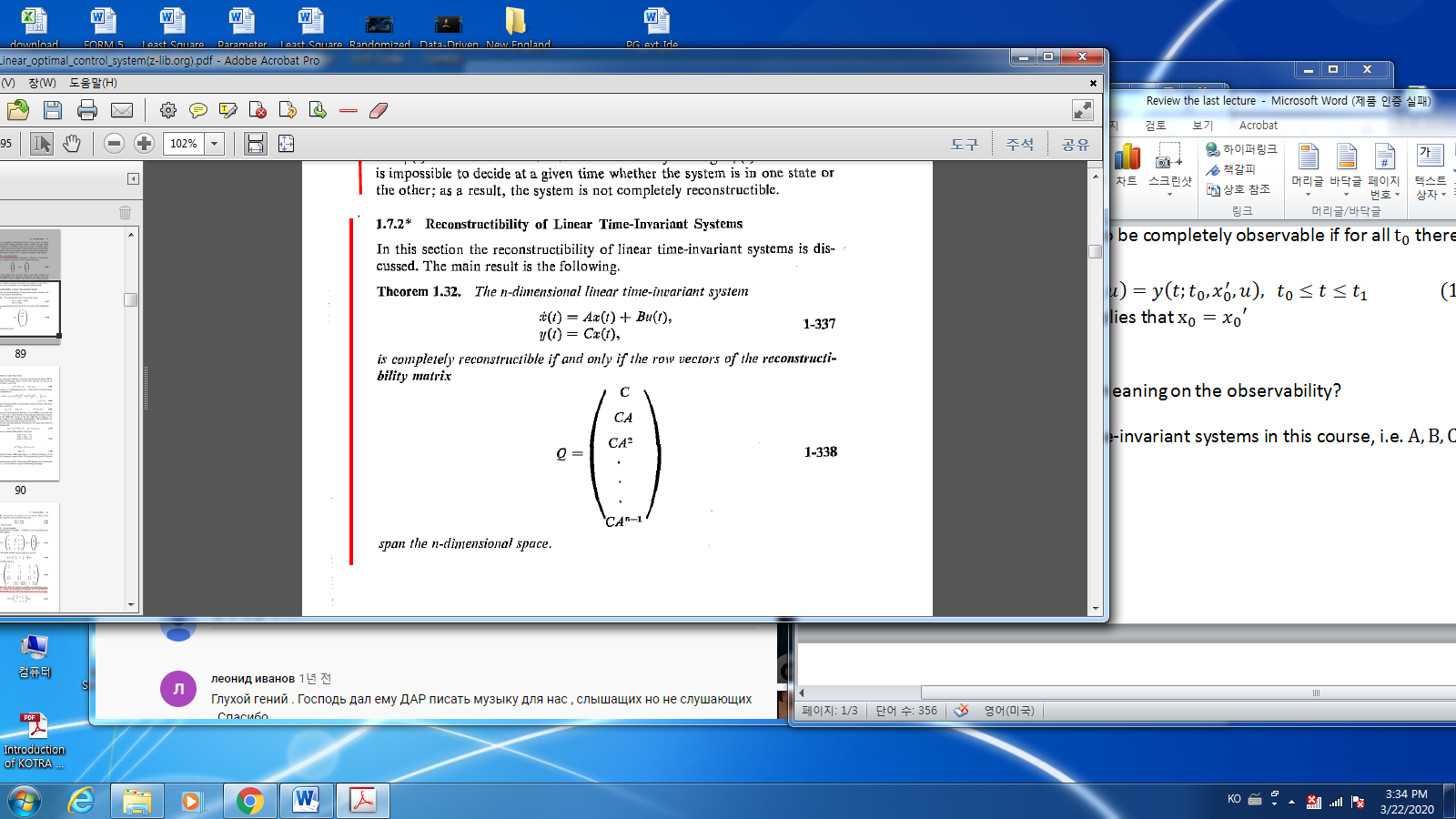
What does it mean? Let and . Then if the system is observable, for any t

which implies, measured output is uniquely determine . You should remember

.

Hence given you may calculate **the exact value of** , not estimate of .

* Theorem 1.32 (The Reconstructibility (the observability) condition)



%%%%%%%%%--------------- example. Let the position of a vehicle with a constant acceleration is modelled as

The corresponding state space model may be defined as

1. Case \_1 : If your measurement is the position, i.e.,

Hence the reconstructibility(observability) matrix is

Its rank is 2 🡪 observable, which means if you know the input in this case , and your measurement is the position, in this case , You may determine not only but also

1. Case\_2: If your measurement is the position, then , the observability matrix is

It’s rank = 1, which is not observable. As U see, in this case, u may not determine the value of

1. Case \_3 : If your measurement is then , the observability matrix is

It’s rank = 2.which is observable. As U see, in this case, U measure not independently of and , the u may determine the value of and

* **Home assignment:** W6.5. In the above case\_3) at the measurement

Determine and

-----------------------------------%%%%%%%%%%%%%%%%%%%

* Example 1.24 (Inverted pendulum: Observability)

1. The measured output is the angle (Continue Ex.1.1 / 1.10 / 1.11/ 1.16 /1.20

The measured output is the angle , then

The Reconstructibility (Observability) matrix is

%% Example 1.24 - Observability Check

clear all;clc

syms F M g L s

% L = (J + mL^2)/ mL

A = [ 0 1 0 0; 0 -F/M 0 0; 0 0 0 1; -g/L 0 g/L 0];

C =1/L\* [ -1 0 1 0];

Q = [C; C\*A; C\*A\*A; C\*A\*A\*A]

rank(Q)

>> ans

Q =

[ -1/L, 0, 1/L, 0]

[ 0, -1/L, 0, 1/L]

[ -g/L^2, F/(L\*M), g/L^2, 0]

[ 0, - g/L^2 - F^2/(L\*M^2), 0, g/L^2]

And rank(Q) = 3 . Hence it is unobservable system.

1. The measured outputs are

The angle and the position of the carriage . Hence the output is a vector as

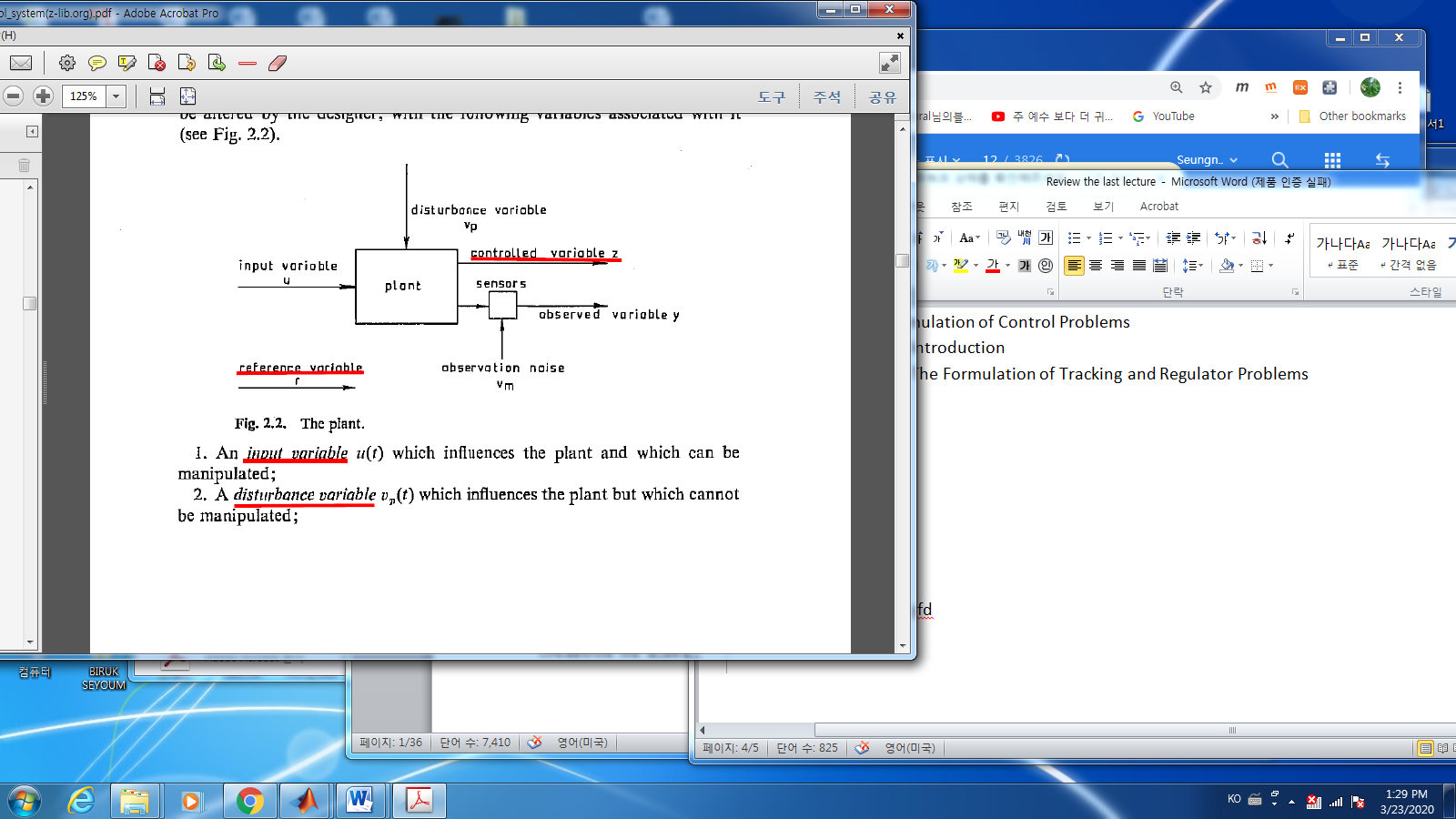
Then the observability matrix has a rank =4, implies the system is observable.

* 1. Skip
  2. Skip
  3. Vector Stochastic Process.

One of the major developments in 20 century is the analysis of a Brownian Process, i.e., independent random increments, or independent random walk, which is adapted by Einstein to find “The Relative Theory”. As you know, the famous law is

To derive this law, we need the stochastic process, U may remember this topic was dealt in “Stochstic Control” very little, in the last semester. I will teach this topic once again late of this semester as “Linear Quadratic Gaussian” problem, until then I may **skip** it.

1. Analysis of Linear Control System
   1. Introduction
   2. The Formulation of Control Problems
      1. Introduction
      2. The Formulation of Tracking and Regulator Problems



1

* An input variable : ex. motor rotation voltage
* A disturbance variable : ex. your finger to touch at the top of a rod
* An observed variables : ex. a sum of position -
* A controlled variable ex. an angle of a rod
* A reference variable : controlled variable : ex. the angle of a rod.
* Tracking problems: follows a prescribed value. i.e. an angle may be increased
* Regulation problems: is remained constant in a long time, i.e., an angle may be zero degree.
* Control design considerations

1. The disturbance: It is unpredictable. in general modelled as a random variable
2. The plant parameters: is not knows precisely. i.e., the friction
3. The initial state may not known
4. The observed variable may be contaminated by the noise. In general modelled as a random variable
   1. Closed – loop controllers : The basic design objective
   2. The stability of control systems

* Design Objective 2.1: The control system should be asymptotically stable

As U see Example 1.4 (Week\_5\_2), without feedback, the system is unstable. To stabilize the system, U may design a state-feedback so that the closed loop system is stabilized.

* Example 2.6 (Inverted pendulum: a state feedback continue the Ex. 1.1 / 1.16 )

From 1-26

Substitute the parameter values (Ex.1.1) to give

And the state feedback gain by the textbook

Then the closed loop e-values are

(-1.9070 +/-3.4229i, -4.6980 +/- 1.3784i)

Since all e-values have negative real parts, the closed loop system is stable.(Check matlab! There are small difference between the values of text and those of matlab. Which one is correct? I think Matlab is correct!)

* Important issue!!

Since the the equation 1-26 is derived from linearization from (1.16) ~(1.18) at the stationary point,.i.e., , if the point different from the stationary, this state feedback may not be stabilized.

%%%%%%%%%%%%%--------------

The actual system and the linearized system may be different, sometimes totally different. In this example, linearized at . Since only if

The textbook gives the results different , the closed loop system may be unstable..

Looks nice!! How the authors got this graph without these days fancy computer? Someday I will plot it. Promise. ---------------------------------%%%%%%%%%%%%%%%%%%%%%%%%