Unit 5(a) Lecture Notes for MAT224

§1 5.1 Complex Numbers

The set of complex numbers $\mathbb{C} = \{a + bi \mid a, b, \in \mathbb{R}\}$. Is 2 a complex number? Why or why not?
We know that 2 is a complex number with $\mathrm{Im}(2) = 0$ and $\mathrm{Re}(2) = 2$. Furthermore, we know that all real numbers $a \in \mathbb{R}$ are complex numbers with $\mathrm{Im}(a) = 0$.

Compare and contrast the definitions of "vector space" and "field"? How are they similar? How are they different?

Although vector spaces and fields both satisfy the usual operations of addition and multiplication, a field satisfies that every polynomial with coefficients in F have n real roots in F.

Which arithmetic operations can we do with complex numbers? List all of the operations described in section 5.1. (Example: we can divide $\frac{a + bi}{c + di}$)

The set of complex numbers, denoted $\mathbb{C}$ is the set of ordered pairs of real numbers (a, b) with the operations of addition and multiplication.

What does it mean for a field to be algebraically closed? Is $\mathbb{C}$ algebraically closed?

A set F is said to be algebraically closed if every polynomial $p(z) = a_n z^n + \ldots + a_1 z + a_0$ with coefficients in F, $a_i \in F$ for $i = 0 \ldots n$ has n roots in F. By the fundamental theorem of algebra, we know $\mathbb{C}$ is closed.

§2 4.6 Spectral Theorem

Let $M \in M_{n\times n}(\mathbb{R})$ be a symmetric matrix. What can we conclude about the eigenvalues of M?

Since eigenvectors correspond to distinct eigenvalues of a symmetric mapping, we know that the eigenvalues of M are distinct.

Is M diagonalizable?

By Theorem (4.6.1), we know that M is symmetric and has an orthonormal basis consisting of eigenvectors of M, and thereby diagonalizable.