Ad Exchange Auction Theoretical Report

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1. Problem Definition

We wish to first model the game that we are playing. Let κ be the set of all campaigns, and let $\mathbb{K}$ be the set of all possible sets of campaigns. For a given day d, where $1 \leq d \leq 10$, a campaign will be said to be "active" if d is between the campaign's start and end dates (inclusive), and "inactive" otherwise. We can then define α_d to be the set of active campaigns on day d that our player currently owns (has either won or has been randomly given), β_d to be the set of active campaigns on day d that belong to other players, Υ_d to be the set of inactive campaigns on day d that our agent owns, and ζ_d to be the set of inactive campaigns on day d that other players own.

1.1. Modeling the campaign auction

On a given day d, we are given a set of 5 campaigns $C_d \in \kappa^5$ that we can bid on. Let q_d be our quality score on day d. Then our strategy for the campaign auction involves assessing the campaigns we have access to, the campaigns we know of that we do not have access to, our quality score, and the campaigns we are bidding on, and outputting a bid for each of those 5 campaigns; formally, we want to implement a function $f_d(C_d, \alpha_d, \beta_d, q_d) : \kappa^5 \times \mathbb{K} \times \mathbb{K} \times \mathbb{R} \to \mathbb{R}^5$ to compute our bids.

We are making the assumption that, for any given bidder, their bids on campaigns (and on users) do not depend on any campaigns that they (or other bidders) previously had and are now inactive. In other words, Υ_d and ζ_d are not used in our formalization of f_d.

In addition, it may be important to know, for a given campaign, which bidder it was given to if we were not the bidder who won that campaign. This could be useful, for example, because given the bidding tendencies of other agents, knowing which agent won the campaign can be used to more accurately predict the competition for the campaign's target segment. However, we do not have access to that information as an agent, so we simply group together all campaigns that we do not win. Another group of information that we do not have is the set of randomized campaigns that are given to other bidders based on their quality score, so β_d only includes a subset of the campaigns we know of from the previous campaign auctions. On the other hand, we know the details of the randomized campaigns we are given, so these campaigns should be included in α_d.

1.2. Modeling the ad (user) auction

When we bid on users, we have access to the campaigns that we have, α_d, and a subset of those that we don't have, β_d. Then, for each element in α_d, we output a spending limit in $\mathbb{R}$ and a pair `(per_user_bid, spending_limit)` for each of up to 8 atomic segments (where an "atomic" segment is one that cannot have more specifications added to it), so our output is in the shape of an $\mathbb{R}^{17}$ vector for each campaign we are holding. So, we can represent our user-auction function for each day d as $g_d(\alpha_d, \beta_d) : \mathbb{K} \times \mathbb{K} \to (\mathbb{R}^{17})^{|\alpha_d|}$.

1.3. Modeling the objective

Our goal is to maximize our profits across all 10 days. Let ϕ_d be our profit on day d, let $\vec{f} = (f_1, f_2, \ldots, f_{10})$ be a vector of functions representing our strategy in the campaign auction collectively across all 10 days, and let $\vec{g} = (g_1, g_2, \ldots, g_{10})$ be our collective user auction strategy. Then, formally, our objective is to find $(\vec{f}, \vec{g})^* = \underset{(\vec{f},\vec{g})}{\arg\max} \sum_{d=1}^{10} \phi_d(f_1, f_2, \ldots, f_{d-1}, g_d)$.

For any given day, the resulting profit is equal to the sum, across all campaigns ending on that day, of the campaign's cost subtracted from its effective budget; in other words, $\phi_d = \sum_{c \in \alpha_d, c_r = d} \rho(c) \cdot b(c) - k(c)$, where:

- α_d is a result of the bidding we did in campaign auctions from previous days as according to our previous bidding functions $f_1, f_2, \ldots, f_{d-1}$.
- Given a set S with a total order, $S_{(k)}$ refers to the k^{th} minimum order statistic of S and $S_{[k]}$ refers to the k^{th} maximum order statistic of S.
- $U_a(c, d; g_d)$ is the set of users that we win on day d associated with campaign c in atomic segment a, given that we bid on a according to our function $g_d(\alpha_d, \beta_d)$
- A is the set of 8 atomic segments (i.e., [`Female_Old_HighIncome`, `Female_Old_LowIncome`, `Female_Young_HighIncome`, ...]).
- $\rho(c)$ is our effective reach for campaign c, equal to $\frac{2}{y}\left[\tan^{-1}(y(\frac{\sum_{a \in A} x_a(c,d;g_d)}{c_R}) - z) - \tan^{-1}(z)\right]$, where $x_a(c, d; g_d) = |U_a(c, d; g_d)|$, $y = 4.08577$, $z = 3.08577$, and c_R is the reach of campaign c.
- $b(c)$ is the budget that is assigned to c from its campaign auction. If $E(c)$ is the set of effective bids on c, c_d is the property of c representing the day on which it was auctioned (i.e., the day before c begins), and c_i is the property of c representing its index among the 5 campaigns being auctioned on day c_d, then $E(c)_{(1)} = \frac{(f_{c_d}(\alpha_{c_d}, \beta_{c_d}, C_{c_d}, q_{c_d}))_{c_i}}{q_{c_d}}$ (because we won that auction), and $b(c) = E(c)_{(2)} \cdot q_{c_d}$. However, if campaign c was received randomly and not through an auction, then $b(c)$ is just equal to the reach of c, or c_R.
- $k(c)$ is the cost we incurred for buying users from the user auction to achieve effective reach $\rho(c)$ for campaign c. If $B(u, d; g_d)$ is the set of bids on day d across all (possibly non-atomic) segments s such that user u is in segment s, and the bid for the atomic segment a (which user u is in) outputted by $g_d(\alpha_d, \beta_d)$ is equal to $B(u, d; g_d)_{[1]}$ (since we won that user), then $k(c) = \sum_{a \in A} \sum_{d \in [c_l, c_r]} \sum_{u \in U(c,d,a;g_d)} B(u, d; g_d)_{[2]}$.

2. Problem Analysis

Given this complicated problem, in order to simplify it, we use the same f and g functions for each day, so $\vec{f} = (f, f, \ldots, f)$ and $\vec{g} = (g, g, \ldots, g)$. Additionally, when bidding on campaigns, we bid on each of them independently in order to simplify our strategy, so that our bidding on any of the 5 campaigns on auction C_d does not impact our bidding on any of the other 4 campaigns in C_d.

2.1. Atomic Segment Demand

We model the competition of buying a user from an atomic segment a on a specific day d by estimating the total demand $D_{d,a}$ from other agents. The total demand for an atomic segment represents the expected number of users that the other agents would like to buy from this segment to fulfill their campaigns. To estimate the demand, we make use of information available to us: the campaigns on auction each day, their target segments, their desired reach, and their start and end dates. If a campaign's target segment is atomic, then we simply add the reach of this campaign to the total demand for that segment distributed evenly across the days of the campaign. If a campaign's target segment is not atomic, then we split the demand generated by this campaign's reach into demand for atomic subsegments, proportional to the expected population of each subsegment.

The formal definition of demand is as follows. For conciseness, let $\sigma(s)$ return the list of 1, 2, or 4 constituent atomic segments a for any market segment s. Then, for each campaign $c \in C_d$ that we do not win, if c's active range of days is $[c_l, c_r]$ and c's corresponding segment c_s can be broken down into atomic segments $\sigma(c_s)_1, \sigma(c_s)_2, \ldots, \sigma(c_s)_k$ where $k \in \{1, 2, 4\}$, then the demand update for c consists of incrementing $D_{d,\sigma(c_s)_i}$ by $\frac{c_R}{c_r - c_l + 1} \cdot \frac{|\sigma(c_s)_i|}{|c_s|}$ for each $a_i \in \sigma(c_s)$ and for each day $d \in [c_l, c_r]$, where $|s|$ is the expected population for some segment s on any day. Additionally, to account for the demand generated by the campaigns that are assigned to other agents through randomization based on their quality scores, we introduce a hyperparameter ω representing the hidden demand for each atomic segment on each day. Therefore, we initialize each $D_{d,a}$ as $\omega \cdot |a|$, rather than 0.

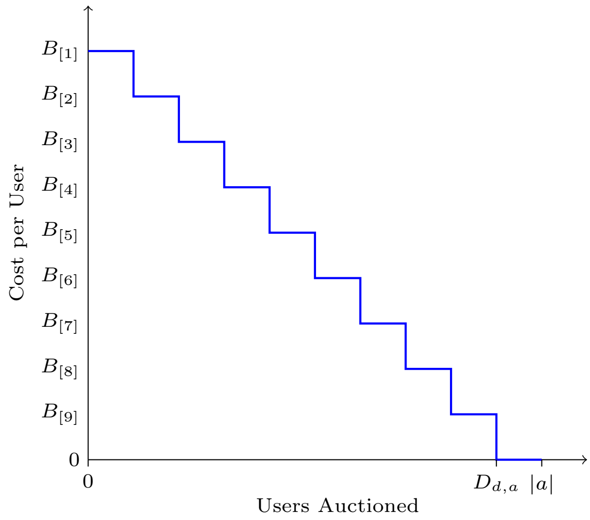

Fig. 1. Price per user curve for user auction when $D_{d,a} \leq |a|$.

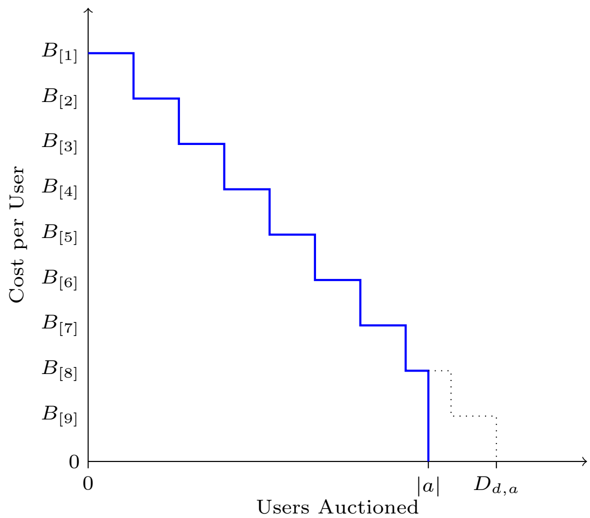

Fig. 2. Price per user curve for user auction when $D_{d,a} > |a|$.

2.2. Distribution Assumptions

Estimating the cost of acquiring each user belonging to an atomic segment accurately is critical for bidding appropriately in the campaign and user auctions. For the campaign auction, this estimate can be used to find an expected price of fulfilling the campaign's reach to an arbitrary extent. For the user auction, this estimate can be used to determine a per-user, per-segment bid that, in expectation, buys out a desired number of users from an atomic segment (and which we later distribute over each campaign that we provide `BidEntry`s for). This depends on the amount of competition, or demand, on users from that segment imposed by other agents, which in turn depends on other users' bidding strategies.

However, if our agent does not win a campaign, it has no knowledge of which agent has won the campaign, and similarly our agent has no knowledge of what bidding strategies other agents will use. Therefore, our agent lacks crucial information to solve for the expected cost of buying a desired number of users from any atomic segment. To get rid of this ambiguity and simplify the problem further, we assume that the demand for a segment is uniformly distributed among the other 9 agents in expectation. We further assume that each of these 9 agents draw their bid from the same distribution, characterized by the probability distribution function h, and cumulative distribution function H. We assume the bids to be uniformly distributed between $[0, 1]$, because agents can't bid below 0, and bidding over 1 is disincentivized. Random campaigns have a budget equal to reach, leading to a truthful valuation of 1 per user, and the campaign on auction will likely have even less. Therefore, even for an agent bidding truthfully (and not trying to acquire users at a lower price like our algorithm), bidding at most 1 is, in the majority of cases, dominant to bidding over 1.

The choice of this bidding distribution function should aim to reflect the general trend formed by other agents' bidding strategies. The uniform distribution represents a balanced mix of aggressive and conservative bidding strategies in expectation. If the strategies among the other agents are skewed to either end, a distribution skewed similarly towards that end will be more accurate. But for now, we focus on deriving an analytical solution for the expected cost of acquiring a desired number of users under the assumption that other bidders bid uniformly.

2.3. Bidding in User Auctions

2.3.1. Cost per User Estimation

To estimate the expected cost of buying a target number of users $t(a)$ from an atomic segment a with an expected population of $|a|$ on day d, we make use of the estimated demand for that segment on day d, previously denoted $D_{d,a}$. We previously assumed that the demand for a was uniformly distributed among the other agents; therefore, we will assume that each agent is uniformly aiming to buy out $\frac{D_{d,a}}{9}$ users from that segment as shown in Fig. 1, and drawing a bid from the distribution characterized by h. Since the population is $|a|$, if $d > |a|$, then only the first $|a|$ users will be allocated to agents, as shown in Fig. 2.

To get the users at the lowest possible price, we aim to buy the last $t(a)$ users of that segment who are on auction. To estimate the bid that will allow us to buy the last $t(a)$ users, we find the rank of the user that we will have to outbid (rank 1 being the highest bidder, rank 9 being the lowest bidder among the other agents). We would like to buy the users $(|a| - t(a), |a|]$, and because demand is uniformly distributed among other agents, we know that users $(0, \frac{D_{d,a}}{9}]$ will be won by bidder with rank 1, $(\frac{D_{d,a}}{9}, 2\frac{D_{d,a}}{9}]$ will be won by bidder with rank 2, and so on. Therefore, the rank that our bid will have to achieve will be given by: $r = \left\lfloor \frac{|a|-t(a)}{\frac{D_{d,a}}{9}} \right\rfloor + 1$.

To achieve this rank with our bid, we would have to outbid the agent which previously had the same rank among the other bidders. The expected bid of the agent with rank r will be the expected value of the r^{th} maximum order statistic among 9 uniform samples, so if $B(a)$ is the set of bids for this atomic segment a drawn from the earlier uniform bidding function h, then this is given by the following formula:

$$\mathbb{E}[B(a)_{[r]}] = 1 - \frac{r}{10}$$

Since the user auction is a second price auction, upon outbidding the previous agent with rank r, now we will be paying their expected bid. Therefore, the total price of acquiring the desired number of users $t(a)$ will be given by:

$$\mathbb{E}[k(t,a,d)] = \left(1 - \frac{r}{10}\right) \cdot t(a) = \left(1 - \frac{\left\lfloor \frac{|a|-t(a)}{\frac{D_{d,a}}{9}} \right\rfloor + 1}{10}\right) \cdot t(a)$$

2.3.2. User Auction Bidding Strategy

Given this cost estimation, we can now put together our strategy g for each day d. From our campaign auction strategy (described below in Section 2.4), we have a target reach for each campaign $c \in \alpha_d$, denoted $t(c)$. Also, we can decompose each campaign's segment c_s into atomic segments and take the union across all campaigns $c \in \alpha_d$, which we will call $S \subseteq A = \bigcup_{c \in \alpha_d} \sigma(c_s)$. We then perform 3 steps, as detailed below.

1. Compute the total target reach $t(a)$ for each of the atomic segments $a \in S$. We already have a target reach $t(c)$ for each campaign c. We can distribute this across all atomic segments that constitute campaign c's associated segment c_s proportionally, so that for each atomic $\sigma(c_s)_i \in \sigma(c_s)$, we have $t(c, \sigma(c_s)_i) = t(c) \cdot \frac{|\sigma(c_s)_i|}{|c_s|}$, where the value of the expected population for each segment per day is given in the original problem document. Then, our total target reach for each possible atomic segment $a \in S$ across all campaigns we are holding is $t(a) = \sum_{c \in \alpha_d} t(c, a)$.

2. Determine the bid $\textsc{Bid}(a)$ for each atomic segment $a \in S$. As described earlier in Section 2.3.1, given the target reach $t(a)$ for any atomic segment a, we can get the rank $r(a, t(a))$ that we have to beat, and the bid that we have to beat will be equal to the expected order statistic $E[B(a)_{r(a,t(a))}]$. Our bid should be a little bit above that, so $\textsc{Bid}(a) = E[B(a)_{r(a,t(a))}] + \epsilon$ for some small $\epsilon \in \mathbb{R}$.

3. For each campaign $c \in \alpha_d$, find the bid $\textsc{Bid}(c, \sigma(c_s)_i)$ and bid limit $l(c, \sigma(c_s)_i)$ for each segment $\sigma(c_s)_i \in \sigma(c_s)$, as well as the spending limit $l(c)$ for the overall campaign. For each campaign c, our bid for each of its constituent atomic segments $\sigma(c_s)_i \in \sigma(c_s)$, or $\textsc{Bid}(c, \sigma(c_s)_i)$, will just be equal to $\textsc{Bid}(\sigma(c_s)_i)$ (which we found in Step 2). However, the spending limits for those atomic segments will (possibly) be different across campaigns. Recall that for any atomic segment a, we have already found the total target reach $t(a)$ across campaigns. We can break this into the maximum number of people that we want to get for that segment for each campaign c proportionally, which is equal to the $t(c, \sigma(c_s)_i)$ that we found earlier in Step 1. Then, our spending limit for that segment and campaign would just be the number of people we buy times the price per person; namely, $l(c, \sigma(c_s)_i) = \textsc{Bid}(c, \sigma(c_s)_i) \cdot t(c, \sigma(c_s)_i)$. For simplicity, our spending limit for the campaign will not place any more restrictions, so then $l(c) = \sum_{\sigma(c_s)_i \in \sigma(c_s)} l(c, \sigma(c_s)_i)$.

2.4. Bidding in Campaign Auctions

2.4.1. Cost per Campaign Estimation

In order to get the expected cost $k(c)$ for fulfilling a campaign c, we can simply sum up the costs of fulfilling each atomic segment of the campaign across each day of the campaign. However, this depends on the demand for each atomic segment on each day, and if we are bidding on campaigns on day d, then for any future day $e \in [c_l, c_r]$ (where $c_l = d + 1$ and $c_r \geq c_l$), then our estimate for the demand $D_{e,a}$ for day e and some atomic segment a may be below the true value (because of future campaign auctions, in which we lose campaigns and the reaches for those campaigns will get added to those days' demand values). Since we assume demand generated by each campaign will be distributed (evenly) across all days of the campaign, the demand values we track for later days can only increase.

To account for this, we introduce another hyperparameter: a scaling factor $\gamma \geq 1$ that accounts for the increase in demand for later days as more campaigns are auctioned off. Using it, we can calculate the modified demand for any day $e \in [c_l, c_r]$ as $\Gamma_{e,a} = \gamma^{e-c_l} D_{e,a}$. Now, we can compute a scaled cost for each day and segment:

$$\mathbb{E}[k_{\text{scaled}}(t,a,d)] = (1 - \frac{r_{\text{scaled}}}{10}) \cdot t(a) = (1 - \frac{\left\lfloor \frac{|a|-t(a)}{\frac{\Gamma_{d,a}}{9}} \right\rfloor + 1}{10}) \cdot t(a)$$

Finally, the expected cost for fulfilling a campaign c up to its target reach $t(c)$ would be the sum of scaled costs over all days and over all constituent atomic segments:

$$\mathbb{E}[k(c)] = \sum_{(c_s)_i \in \sigma(c_s)} \sum_{d \in [c_l, c_r]} \mathbb{E}[k_{\text{scaled}}(t, (c_s)_i, d)]$$

2.4.2. Campaign Auction Bidding Strategy

For each campaign in the campaign auction, we bid on it independently from other campaigns in the auction by (1) first computing a target reach for which fulfilling that target reach will get us an expected maximum profit, then (2) computing a bid to submit for the campaign such that the cost for fulfilling that target reach is no more than some percentage of the payout from fulfilling it (so that we can expect to get a strictly positive profit if we win this campaign). We set this percentage to be 90%. Pricing a campaign based on expected maximum profit from the user auction is an optimistic strategy; it relies on our agent being able to achieve its set target reach perfectly. However, under the assumption that our cost estimation is accurate, this is a reasonable strategy.

Further description of each of these 2 steps are given below.

1. Find the target reach $t(c)$**.** It may be the case that fulfilling only a small portion of the desired reach of a campaign may be the most profitable strategy. If we strictly aim to maximize profit for the current campaign, then in this case the quality score of our agent can decrease drastically, which will in turn reduce our agent's future profits (as it will be less likely to receive random campaigns, and its effective bids on the campaign auctions will be scaled up). To maintain a high quality score throughout the game, we heuristically restrict our search for the target reach $t(c)$ to be between $[\lceil \tau \cdot c_R \rceil, \lceil 1.38 c_R \rceil]$, where c_R is the reach goal of the campaign and τ is a lower-bound hyperparameter that we set to 0.9. By maximizing our profit based on potential target reach values $t(c)$ in this interval, our algorithm will be aiming to fulfill a large portion of the reach goal of every campaign. This is likely to keep our quality score high. If bidding on campaigns on day d, we compute the target reach that yields the maximum profit we can get from a campaign as follows:

$$\underset{t(c) \in [\lceil 0.9 c_R \rceil, \lceil 1.38 c_R \rceil]}{\arg\max} \rho(t(c), c_R) \cdot b(c) - \mathbb{E}[k(c)]$$

where $b(c)$ represents the final budget of the campaign (i.e. second lowest bid times our quality score).

2. Find the target final budget $b(c)$**.** Let q_d be our quality score on day d. While bidding for the campaign, if we are to bid x, then our effective bid becomes $\frac{x}{q_d}$, and if we win, then the second lowest effective bid is $E(c)_{(2)} \geq \frac{x}{q_d}$ which makes our final budget $b(c) \geq \frac{x}{q_d} \cdot q_d = x$. In other words, our bid is the lower bound for our final budget if we win.

We note solving for the optimal value of $t(c)$ requires knowing the value of $b(c)$, which will be the result of the auction, and is therefore unknown. We set $b(c) = R$ (which is the value it is equal to for campaigns c that were not given through auctions) in Step 1 to solve for $t(c)$, and then determine the lowest budget that will give us at least 10% profit given that we are aiming to by $t(c)$ users. Note that the profit function is monotonically decreasing with decreasing b_c, so we can find the lowest budget that will generate 10% profit as follows:

$$\rho(t(c), c_R) \cdot b(c) - k(c) = 0.1(\rho(t(c), c_R) \cdot b(c)) \Rightarrow$$

$$0.9(\rho(t(c), c_R) \cdot b(c)) - k(c) = 0 \Rightarrow$$

$$b(c) = \frac{k(c)}{0.9\rho(t(c), c_r)}$$

Therefore, our expected cost will be at most 10% less than our payout from winning users. If we end up winning the campaign with this bid, our agent will attempt to fulfill the target reach $t(c)$ in the user auction, paced equally over the days of the campaign as described in Section 2.3.2.

Table 1. Average profit per ω value.

ω	vs. Tier 1	vs. Tier 2
0.0	1684.06	4101.24
0.3	1662.78	4199.00
0.5	1383.80	3828.73
0.7	**1727.10**	**4220.00**
1.0	1658.97	4015.81

Table 2. Average profit per γ value.

γ	vs. Tier 1	vs. Tier 2
1.0	**1820.00**	4027.74
1.125	1743.25	4048.54
1.25	1602.20	**4584.15**
1.5	1577.44	3869.13
2.0	1626.69	4190.62

Table 3. Average profit per τ value.

τ	vs. Tier 1	vs. Tier 2
0.3	**1720.72**	4169.19
0.5	1655.52	3801.76
0.7	1418.47	3942.76
0.9	1718.15	**4389.87**
1.1	1681.60	4345.67

3. Experiments

In order to better understand how these stochastic algorithms are impacted by the different hyperparameters involved, we perform independent hyperparameter tuning for the following 3 variables: ω, the initial proportion of atomic segment population that is used to account for demand from randomly-assigned campaigns; γ, the scaling factor of per-day demand for computing per-campaign cost; and τ, the proportion of per-campaign reach that is used for the lower bound of the interval in which we search for per-campaign target reach. For each variable, we test 5 different values, run 100 simulated games against 9 instances of each of the Tier 1 and Tier 2 TA agents, and present the average profits. The results are given in Tables 1, 2, and 3.

We see that for τ, a value of 0.7—in line with δ_3 from the problem, which is one possible value that is used to scale an atomic segment's demand and obtain a campaign's final reach—is on average better over 100 auctions when competing against both the Tier 1 and Tier 2 agents. The other 2 variables do not present any specific patterns across their tested values after running these local simulations; however, all combinations of hyperparameter values result in relatively low rewards on the official auction, so we use a modified version of our strategy as described below in Section 4.

4. Practical Considerations

The theoretical campaign and user bidding strategies established in the previous sections are efficiently computable; however, they perform poorly in the actual Ad Exchange competition. We take a more heuristic approach for the actual competition setting, modifying the following parts of the theory that we have derived to achieve better performance:

- Changes related to computing demand:
  - We set the baseline demand hyperparameter, ω, to 0.5.
  - Instead of evenly spreading the demand generated by a campaign on its target segment over the days of the campaign, we set the demand for the target segment on the start day of the campaign to be equal to the total demand generated by the campaign. We decrement the demand by the population of the segment each day, and if there is still unfulfilled demand remaining, we roll this unfulfilled demand over to the next day. This model captures hasty bidding by other agents, where each agent tries to fulfill their campaign at an acceptable price as soon as possible, rather than pacing their bids evenly across the days of the campaign.
- Changes related to bidding in the campaign auction:
  - Instead of searching through all the possible values of target reaches in a constrained space to find the one that achieves maximum profit, we simply set target reach to 1.05.
  - Instead of pacing the target reach over the duration of the campaign, we pace the remaining budget evenly over the remaining days of the campaign. On each day, we limit our spending to this paced budget, and submit a truthful valuation for acquiring a single user as our bid. We compute our truthful valuation for acquiring a single user by computing the marginal revenue we get by acquiring the user, computed by multiplying the budget of the campaign with the derivative of the effective reach function.
- Changes related to bidding in the user auction:
  - We use the same method for calculating the expected cost of buying users. However, in this calculation, we do not use scaled demands, and instead rely on the roll-over demand. Furthermore, we use a uniform distribution between $[0.5, 1]$ as the bidding distribution used by all the other agents rather than using a uniform distribution between $[0, 1]$.
  - Instead of submitting bids for atomic subsegments for a given segment in the user auction, we calculate the total demand for a segment by summing up the total demand for its subsegments, and compute an acceptable bid for the general segment.

We have observed from our local simulations and the results of the actual competition that the algorithm with the above modifications significantly outperforms the strictly theoretical algorithm, and achieves acceptable performance by placing in the middle of the leaderboard.

5. Appendix 1: Variable Glossary

Below is the notation we use for variables in our theoretical model of, and strategy for, the AdX Auction.

- Agent-related variables
  - q_d: Our agent's quality score on day d
  - f: Our agent's campaign bidding function
  - g: Our agent's user bidding function
  - ϕ: Profit
  - ϕ_d: Profit on day d
- Campaign-related variables
  - General:
    - d: Day
    - c: Campaign
    - e: Day within a campaign's duration
  - Sets:
    - κ: Set of all campaigns
    - $\mathbb{K}$: Set of all sets of campaigns
    - α_d: Set of our active campaigns on day d
    - β_d: Set of other agents' active campaigns on day d
    - Υ_d: Set of our inactive campaigns on day d
    - ζ_d: Set of other agents' inactive campaigns on day d
    - C_d: Set of campaigns on auction on day d
  - Properties of campaigns:
    - c_R: Reach goal of campaign c
    - c_l: Start date of campaign c
    - c_r: End date of campaign c
    - c_d: The day campaign c is auctioned off
    - c_i: The index of campaign c among the 5 campaigns that are on auction on day c_d
    - c_s: Target (possibly non-atomic) segment of c
  - Functions of campaigns:
    - $\rho(c)$: Our agent's effective reach for c
    - $b(c)$: Budget of c
    - $k(c)$: Cost of users bought for c
    - $t(c)$: Computed target reach for campaign c
    - $t(c, a)$: Computed target reach for atomic segment a that we are specifically trying to buy in campaign c
- Segment-related variables
  - s: Any segment
  - a: Any atomic segment
  - $|s|$: Expected population of a segment on each day
  - A: Set of all atomic segments
  - S: Set of all atomic segments that our agent is interested in
  - $t(a)$: Target reach for atomic segment a
  - $U_a(c, d; g_d)$: Set of users that we win on day d associated with campaign c in atomic segment a, given that we bid on a according to our function $g_d(\alpha_d, \beta_d)$
  - $D_{d,a}$: Total demand from other agents on atomic segment a on day d
  - γ: Scaling factor representing the anticipated increase in demand for each segment each day
  - $\Gamma_{d,s}$: Anticipated demand for segment s on day d
  - $\sigma(s)$: List of atomic subsegments of segment s
- Bid-related variables
  - $B(s)$: The set bids of other agents for segment s
  - r: Bidder rank (highest bidder = rank 1, lowest bidder = rank 10)
  - $\textsc{Bid}(a)$: Bid for atomic segment a
  - $\textsc{Bid}(c, a)$: Our agent's bid value in the `BidEntry` for campaign c associated with atomic segment a
  - h: Assumed probability distribution function for bidding used by each opponent agent
  - H: Assumed cumulative distribution function for bidding used by each opponent agent
  - ω: Assumed baseline demand from other agents for each segment due to randomly assigned campaigns
  - $S_{(k)}$: k^{th} minimum order statistic of some sample set S
  - $S_{[k]}$: k^{th} maximum order statistic of some sample set S
  - $E(c)$: Set of effective bids submitted for campaign c
  - $l(c, a)$: Spending limit used in user auction to get users for campaign c in atomic segment a