

Trigram o ngram that . (/s) 'm sorry , ngram do that . I'm sorry , Dave . 'm afraid I can 't do Dave . (/ afraid I c

Trigrams

How many n-grams are there in a seni

Short detour: colorless green ideas

- But it must be recognized that the notion 'probability of a sentence' is an entirely useless one, under any known interpretation of this term. Chowsky (1968)
- \* The following 'sentences' are categorically different Furiously sleep ideas green colorless
   Colorless green ideas sleep furiously
- . Can n-gram models model the difference: Should n-gram models model the difference?

How to test n-gram models?

Extrinsic: improvement of the target application due to the language model:

- Speech recognition accuracy
   BLEU score for machine translation
- Keystroke savings in predictive text applications
  - the higher the probability assigned to a test set better the model. A few measures:
    - Likelihood
    - · (cross) entropy

Intrinsic evaluation metrics: cross entropy

perplexity

Like any ML method, test set has to be different than training set.

 $H(\mathbf{w}) = -\frac{1}{N} \sum_{\mathbf{w}_k} \log_2 \widehat{P}(\mathbf{w}_k)$ 

ode the data coming from

\* The lower the cross entropy, the better the model

Cross entropy is not sensitive to the test-set size

Cross entropy of a language model on a test set w is

Reminder: Cross entropy is the bits required to P using another (approx nate) distribution P

 $H(P,Q) = -\sum P(x) \log \widehat{P}(x)$ 

What do we do with unseen n-grams?

- per with MLE esti Words (and word sequences) are distributed according to the Zipf's law: many words are rare.
  - $\star$  MLE will assign 0 probabilities to unseen words, and sequences containing

  - . Even with non-zero probabilities, MLE overfits the training data
  - One solution is smoothing: take some probability mass from known words, and assign it to unknown words



Laplace smoothing

 The probability of a bigram becomes  $P_{+1}(w_1w_{t-1}) = \frac{C(w_1w_{t-1})+1}{N+V^2}$ 

- and, the conditional probability 
$$P_{+1}(w_i \mid w_{i-1}) = \frac{C(w_{i-1}w_i) + 1}{C(w_{i-1}) + V}$$

 $P_{+1}(w_{t-n+1}^t) = \frac{C(w_{t-n+1}^t) + 1}{N + Vn}$  $P_{+1}(w_{t-n+1}^t \,|\, w_{t-n+1}^{t-1}) = \quad \frac{C(w_{t-n+1}^t) + 1}{C(w_{t-1}^{t-1}) + 1}$ 

Trigram probabilities of a sentence

 $P_{uni}(I~\text{'m sorry}~,~Dave~.~\langle/s\rangle)~=2.83\times10^{-9}$ 
$$\begin{split} &P_{bs}(I~\text{'m sorry , Dave . }\langle/s\rangle) &= 0.33 \\ &P_{bs}(I~\text{'m sorry , Dave . }\langle/s\rangle) &= 0.50 \end{split}$$

What do n-gram models model?

orphosyntax: the bigram 'ideas are' is (much) more likely than

'ideas is \* Some semantics: 'bright ideas' is more likely than 'green ideas' \* Some cultural aspects of everyday language: 'Chinese food' is more likely

than 'British food' more aspects of 'usage' of language

Intrinsic evaluation metrics: likelihood

- . Likelihood of a model M is the probability of the (test) set w given the m  $\mathcal{L}(M \,|\, \boldsymbol{w}) = P(\boldsymbol{w} \,|\, M) = \prod \, P(s)$
- \* The higher the likelihood (for a given test set), the better the model . Likelihood is sensitive to the test set size

Practical note: (minus) log likelihood is used more commonly, because of ease of numerical manipulation

 $\mathsf{PP}(\boldsymbol{w}) = 2^{\mathsf{H}(\boldsymbol{w})} = \mathsf{P}(\boldsymbol{w})^{-\frac{1}{N}} = \sqrt[N]{\frac{1}{\mathsf{P}(\boldsymbol{w})}}$ · Perplexity is the average branching factor

Similar to cross entropy
 lower better
 not sensitive to test set size

one smoothing

Intrinsic evaluation metrics: perplexity

+ The idea (from 1790): add one to all counts . The probability of a word is estimated by

 $P_{+1}(w) = \frac{C(w)+1}{N+V}$ 

Then, probability of an unknown word is:

Bigram probabilities lace smoothing w<sub>1</sub>w<sub>2</sub> C

(s) I I'm 'm sorry 0.188 0.176 0.125 0.133 Davi 0.012 1.000 1.000

### MLE vs. Laplace probabilities

w	1	'n	sorry		Dave		(/s)	
PMLE	1.00	0.67	0.50	1.00	1.00	1.00	1.00	0.33 1.84 × 10 <sup>-6</sup>
$P_{+1}$	0.19	0.18	0.13	0.13	0.13	0.13	0.19	1.84 × 10 <sup>-6</sup>
w		'm	I		sorry	Dave	(/s)	
$P_{\rm MLE}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00 1.17 × 10 <sup>-12</sup>
$P_{+1}$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	$1.17 \times 10^{-12}$
w	1	'm	afraid		Dave		(/s)	I
PME	1.00	0.67	0.50	0.00	1.00	1.00	1.00	0.00 4.45 × 10 <sup>-7</sup>
P+1	0.19	0.18	0.13	0.03	0.13	0.13	0.19	4.45 × 10 <sup>-7</sup>

## Lidstone correction

+ A simple improvement over Laplace smoothing is adding  $\alpha$  instead of

$$P_{+\alpha}(w_{t-n+1}^{t} \mid w_{t-n+1}^{t-1}) = \frac{C(w_{t-n+1}^{t}) + \alpha}{C(w_{t-n+1}^{t-1}) + \alpha V}$$

- With smaller α values, the model behaves similar to MLE, it overfits: it has high variance
- Larger  $\alpha$  values reduce overfitting/variance, but result in large bias
- We need to tune  $\alpha$  like any other hyperparameter.

### Good-Turing smoothing

- ity mass to be reserved for the novel n-grams using the observed n-g
- . Novel events in our training set is the ones that occur once

$$p_0 = \frac{n_0}{n}$$

- where  $n_1$  is the number of distinct n-grams with frequency 1 in the training data
- . Now we need to discount this mass from the higher counts
- \* The probability of an n-gram that occurred r times in the corpus is

$$(r+1)\frac{n_{r+1}}{n_r}$$

- Issues with Good-Turing discounting

  - \* Zero counts: we cannot assign probabilities if  $\pi_{r+1}=0\,$ The estimates of some of the frequencies of frequencies are unreliable
  - A solution is to replace  $n_r$  with smoothed counts  $z_r$
  - + A well-known technique (simple Good-Turing) for smoothing  $\mathfrak{n}_\tau$  is to use linear interpolation
  - $\log z_{\rm r} = a + b \log r$

# Back-off and interpolation

The general idea is to fall-back to lower order n-gram when estimation is unreliable

it is unlikely that

C(black squirrel) - C(black wug) - 0 C(squirrel) - C(was)

in a reasonably sized corpus

# Interpolation

### Intermitation uses a linear combination:

 $P_{int}(w_i \mid w_{i-1}) = \lambda P(w_i \mid w_{i-1}) + (1 - \lambda)P(w_i)$ 

In general (recursive definition),

 $P_{int}(w_i \mid w_{i-n+1}^{i-1}) = \lambda P(w_i \mid w_{i-n+1}^{i-1}) + (1-\lambda) P_{int}(w_i \mid w_{i-n+2}^{i-1})$ 

\*  $\sum \lambda_i = 1$  \* Recursion terminates with

- either smoothed unigram counts
   or uniform distribution 

  →

### How much probability mass does +1 smoothing steal?

Laplace smoothing reserves probability mass proportional to the size of the vocabulary

- . This is just too much for lare vocabularies and higher ord n-grams · Note that only very few of the higher level n-grams (e.g., trigran
  - are possible



Absolute discounting



- probability mass, c, for the unseen e . The probabilities of known events has to be re-normalized
- How do we decide what c value to use?

# Good-Turing example



 $P_{GT}(the) + P_{GT}(a) + \dots$  $P_{GT}(that) = P_{GT}(do) = \dots = \frac{2}{15}$  $P_{GT}('m) = P_{GT}(.) = \frac{3 \times 1}{15 \times 2}$ 

### Not all (unknown) n-grams are equal

- \* Let's assume that black squirrel is an unknown bigram
- How do we calculate the smoothed probability

$$P_{+1}(\texttt{squirrel} \,|\, \texttt{black}) = \frac{0+1}{C(\texttt{black}) + V}$$

- . How about black wug?
  - $P_{+1}(\texttt{black wug}) = P_{+1}(\texttt{wug} | \texttt{black}) = \frac{0+1}{C(\texttt{black}) + V}$
- · Would it make a difference if we used a better smoothing method (e.g. Good-Turing?)

## Back-off

Back-off uses the estimate if it is available, 'backs off' to the lower order n-gram(s)

$$P(w_i \mid w_{i-1}) = \begin{cases} P^*(w_i \mid w_{i-1}) & \text{if } C(w_{i-1}w_i) > 0 \\ \alpha P(w_i) & \text{otherwise} \end{cases}$$

\*  $\alpha$  makes sure that  $\sum P(w)$  is the discounted amount

\*  $P(w_i)$ , typically, smoothed unigram probability

## Some shortcomings of the n-gram language models

The n-gram language models are simple and successful, but ...

- They cannot handle long-distance dependencies:
   In the last race, the horse he bought last year finally \_
- The success often drops in morphologically complex languages
- . The smoothing methods are often 'a bag of tricks'
- They are highly sensitive to the training data: you do not want to use an n-gram model trained on business news for medical texts

### Cluster-based n-grams

Modeling sentence types

- The idea is to cluster the words, and fall-back (back-off or interpolate) to the chister
- For example - a clustering algorithm is likely to form a cluster containing words for food, e.g. {apple, pear, broccoli, spinach}

  - if you have never seen eat your broccoli, estimate

  - $P(\texttt{broccoli} \mid \texttt{eat your}) = P(\texttt{FOOD} \mid \texttt{eat your}) \times P(\texttt{broccoli} \mid \texttt{FOOD})$

Clustering can be

hard a word belongs to only one cluster (simplifies the model) soft words can be assigned to clusters probabilistically (more flexible)

- · Another way to improve a language model is to condition on the sentence types
- The idea is different types of sentences (e.g., ones related to different topics)
- have different behavio · Sentence types are typically based on clustering
- . We create multiple language models, one for each ser
- . Often a 'general' language model is used, as a fall-back

# Structured language models

- - Another possibility is using a ge
  - · Parsers try to explicitly model (good) sentences
  - · Parsers naturally capture long-distance depender
- · Parsers require much more computational resources th . The improvements are often small (if any)

### Neural language models

· Similar to maxent models, we can train a feed-forward network that predicts a word from its context

- (gated) Recurrent networks are more suitable to the task:
- Train a recurrent network to predict the next word in the seque
   The hidden representations reflect what is useful in the history
- Combined with embeddings, RNN language models are generally more
- successful than n-gram models
- In recent years, masked language models, combined with neural network architectures called Transformers became the dominant language models

## Additional reading, references, credits

- Textbook reference: Jurafsky and Martin (2009, chapter 4) (draft chapter for the 3rd version is also available). Some of the examples in the slides come from this book.
- + Chen and J. Goodman (1998) and Chen and J. Goodman (1999) include a detailed comparison of smoothing methods. The former (technical report) also includes a tutorial introduction
- . J. T. Goodman (2001) studies a numb J. T. Goodman (2001) studies a number of improvements to (n-gram) language models we have discussed. This technical report also includes some
- actory material
- Gale and Sampson (1995) introduce the 'simple' Good-Turing estimatis noted on Slide 12. The article also includes an introduction to the basic

## Additional reading, references, credits (cont.)

### Skipping

- boring | the lecture was
   boring | the) lecture yesterday was
  are completely different for an n-gram model
- A potential solution is to consider contexts with gaps, 'skipping' one or more
- We would, for example model P(c | abcd) with a combination (e.g., interpolation) of

   P(c | abc\_)

  - P(e | ab\_d P(e | a\_cd

Caching

- If a word is used in a document, its probability of being used again is high Caching models condition the probability of a word, to a larger context (besides the immediate history), such as
- the words in the document (if document boundaries are marked)
   a fixed window around the word

## Maximum entropy models

- . We can fit a logistic regression 'max-ent' model predicting P(w | context Main advantage is to be able to condition on arbitrary feature.

## Summary

- . We want to assign probabilities to senter N-gram language models do this by
- - estimating probabilities of parts of the sent
     use the n-gram probability and a condition
     estimate the probability of the sentence
- MLE estimate for n-gram overfit
- Smoothing is a way to fight overfitt
- Back-off and interpolation yields better 'sm
- \* There are other ways to improve n-gram models, and language models without (explicitly) use of n-grams Next
  - (?) Neural language models Tokenization / Computational morphology

- Additional reading, references, credits (cont.) The quote from 2001: A Space Odyssey, T'm sorry Dave. I'm afraid I can't do it." is probably one of the most frequent popular culture quotes in the CL literature. It was also quoted, among many others, by Jurafsky and Martir (2009)
  - \* The HAL9000 camera image on page 12 is from Wikipedia, (re)drawn by Wikipedia user Cryteria

