

Light-by-Light Scattering Effect in Light-Cone Supergraphs

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Abstract

We give a relatively simple explanation of the light-cone supergraph prediction for the UV properties of the maximally supersymmetric theories. It is based on the existence of a dynamical supersymmetry which is not manifest in the light-cone supergraphs. It suggests that $\mathcal{N}=4$ supersymmetric Yang-Mills theory is UV finite and $\mathcal{N}=8$ supergravity is UV finite at least until 7 loops whereas the n -point amplitudes have no UV divergences at least until $L = n + 3$. Here we show that this prediction can be deduced from the properties of light-cone supergraphs analogous to the light-by-light scattering effect in QED. A technical aspect of the argument relies on the observation that the dynamical supersymmetry action is, in fact, a compensating field-dependent gauge transformation required for the retaining the light-cone gauge condition $A_+ = 0$.

1 Introduction

The light-cone superfield method for $\mathcal{N}=4$ supersymmetric Yang-Mills theory and for $\mathcal{N}=8$ supergravity was proposed long time ago [1], [2]. It was used immediately to prove the UV finiteness of the $\mathcal{N}=4$ supersymmetric Yang-Mills theory in [2]. The interest to this formalism returned during the next few years when it was discovered that $\mathcal{N}=8$ supergravity has better UV properties than expected [3], [4].

Based on the properties of the linearized dynamical supersymmetry for the asymptotic superfields in a light-cone formalism a prediction was made in [5] for the UV properties of the amplitudes. For $\mathcal{N}=4$ supersymmetric Yang-Mills theory the argument gives an alternative proof of UV finiteness, for and $\mathcal{N}=8$ supergravity it suggest that the theory is UV finite at least until 7 loops whereas the n -point amplitudes have no UV divergences at least until $L = n + 3$. In particular the 4-point amplitude in $\mathcal{N}=8$ supergravity is UV finite at least until 7 loops. This is a possible explanation of the 3-loop and 4-loop finiteness of the theory discovered in [4].

The argument in [5] is based on the helicity formalism for the amplitudes and on the light-cone superfield formalism for $\mathcal{N}=4$ supersymmetric Yang-Mills theory and $\mathcal{N}=8$ supergravity. Without a detailed knowledge of these two formalisms the UV prediction of [5] is best understood by examining the relation between dynamical supersymmetry and gauge symmetries in the light-cone which will be given in this paper.

The light-cone superfield formalism starts with the component formalism where the gauge-symmetry is fixed in the light-cone gauge $A_+ = 0$ for YM theory (and $h_{+\nu} = 0$ for supergravity). In this gauge the unphysical A_- and ψ_- fields are integrated out and the theory in components depends only on physical fields, helicity ± 1 vectors $A_1 \pm iA_2$, spinors and scalars.

The ensuing light-cone action can be rewritten using the scalar superfields which makes half of the original 16 supersymmetries manifest. These 8 supersymmetries are called kinematical, they form an algebra $\{\bar{Q}, Q\} = p_+$. The remaining supersymmetries are called dynamical and they are not manifest, in the same way as the Lorentz symmetry, which was broken from the start by the gauge condition $A_+ = 0$.

The light-cone gauge condition $A_+ = 0$ is not sufficient to pin down the gauge symmetry¹. The existence of such a residual gauge symmetry is best seen in terms of the dynamical supersymmetry transformations of the $\mathcal{N} = 4$ theory. At the free level, they are linear in both the transverse derivatives and the superfields of the theory. Their generalization to superconformal interactions was shown in [6] to be very simple: dynamical supersymmetry transformations become quadratic in the superfields, in such a way that the transverse derivatives are generalized to *covariant derivatives*. This indicates a residual gauge symmetry, which is the remnant of the original gauge symmetry with a gauge parameter independent of x_- . It is enough to suggest that amplitudes must depend on the transverse field strengths, and therefore have a different degree of divergence than naively expected. This situation is analogous to Delbrück scattering, the scattering of light by light in QED, where a four-photon amplitude is actually a dimension eight (irrelevant) operator due to the derivatives acting on the external photon lines.

¹with thanks to D. Belyaev and W. Siegel

As we proceed to show in detail, this argument serves to improve considerably the ultra-violet properties of both $\mathcal{N} = 4$ and $\mathcal{N} = 8$ maximally supersymmetric theories. *On each external leg with the chiral superfield $\phi(p, \eta)$ there is an extra transverse momentum p_\perp due to linearized dynamical supersymmetries.* On vector fields they are, in fact, the compensating field-dependent gauge transformations required for the retaining the light-cone gauge condition $A_+ = 0$.

2 Light-cone gauge $A_+ = 0$

In the Yang-Mills theory without supersymmetry the light-cone gauge $A_+ = 0$ does not fix the symmetry completely. We require that $A_+ = 0$ as well as $\delta_{\Lambda(x)} A_+ = 0$

$$\delta_{\Lambda(x)} A_+ = \nabla_+ \Lambda(x) \equiv (\partial_+ + A_+) \Lambda(x) = \frac{\partial}{\partial x_-} \Lambda(x_+, x_-, x_i) = 0 \quad \Rightarrow \quad \Lambda(x_+, x_i) . \quad (2.1)$$

This condition can be solved by an x_- -independent gauge parameter $\Lambda(x_+, x_i)$, so that on the remaining gauge fields the partially local symmetry is still acting, e. g. the remaining symmetry transformation on the transverse gauge fields A_i is

$$\delta_\Lambda A_i = \nabla_i \Lambda(x_+, x_i) = (\partial_i + A_i) \Lambda(x_+, x_i) \neq 0 . \quad (2.2)$$

Under the restricted gauge transformations (2.1) the theory is still invariant since the gauge symmetry (2.2) with a gauge parameter depending on x_+, x_i remains a partially local symmetry.

In a supersymmetric Yang-Mills theory the situation is somewhat different. Before gauge-fixing there is a local gauge transformation and a global susy transformation

$$\delta_{\Lambda(x)+\text{susy}} A_\mu(x) = \nabla_\mu \Lambda(x) + \bar{\epsilon} \gamma_\mu \psi(x) . \quad (2.3)$$

One can gauge-fix the gauge $A_+ = 0$ and preserve this condition so that $\delta_{\Lambda+\text{susy}} A_+ = 0$ by performing a gauge transformation together with the susy transformation and requiring that

$$\delta_{\Lambda+\text{susy}} A_+(x) = \partial_+ \Lambda(x) + \bar{\epsilon} \gamma_+ \psi(x) = 0 \quad \Rightarrow \quad \Lambda(\bar{\epsilon}, \psi(x)) = -\bar{\epsilon} \gamma_+ \frac{1}{\partial_+} \psi(x) . \quad (2.4)$$

Thus the original gauge symmetry parameter $\Lambda(x)$ is not arbitrary anymore, it depends on a global susy parameter $\bar{\epsilon}$ and on the spinorial field $\psi(x)$, it is denoted $\Lambda(\bar{\epsilon}, \psi(x))$ and it is given in eq. (2.4). On the transverse gauge fields the transformations include this field-dependent non-linear $\bar{\epsilon}$ -dependent transformation preserving the gauge $A_+ = 0$, together with the original supersymmetry

$$\delta_{\Lambda(\bar{\epsilon}, \psi(x))+\text{susy}} A_i(x) = -\bar{\epsilon} \gamma_+ \left(\frac{\nabla_i}{\partial_+} \psi(x) \right) + \bar{\epsilon} \gamma_i \psi(x) . \quad (2.5)$$

In the light-cone gauge, after the non-physical A_- fields and ψ_- are integrated out², one finds that the second term in these transformations presents a kinematical supersymmetry whereas the first one corresponds to a dynamical one. We explain the details below using the two-component notation, which is convenient for the helicity formalism.

²Here the spinorial field $\psi = \psi_+ + \psi_-$ is split into a part ψ_+ which is preserved in the light-cone supermultiplet and a part ψ_- which is absent.

2.1 On kinematical and dynamical supersymmetries

Here we start with the Lorentz covariant (not gauge fixed) action of the $\mathcal{N}=4$ YM theory in usual space in components, depending on the vectors, spinors and scalars, $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \dots$. The *linearized symmetry of the asymptotic Lorentz covariant vector fields* includes the linear part of the non-abelian gauge symmetry as well as 16 supersymmetries. On the vector potentials the symmetries act as shown in eq. (2.3), or, in the momentum space in 2-component notation

$$\boxed{\delta_{\text{cov}} A_{\alpha\dot{\alpha}}(p) = p_{\alpha\dot{\alpha}}\Lambda(p) + \bar{\epsilon}_{\dot{\alpha}A}\psi_{\alpha}^A + \bar{\psi}_{\dot{\alpha}A}\epsilon_{\alpha}^A} \quad (2.6)$$

In particular,

$$\delta_{\text{cov}} A_{2\dot{2}}(p) = p_{2\dot{2}}\Lambda(p) + \bar{\epsilon}_{\dot{2}A}\psi_2^A + \bar{\psi}_{\dot{2}A}\epsilon_2^A \quad (2.7)$$

and

$$\delta_{\text{cov}} A_{1\dot{2}}(p) = p_{1\dot{2}}\Lambda(p) + \bar{\epsilon}_{\dot{2}A}\psi_1^A + \bar{\psi}_{\dot{2}A}\epsilon_1^A. \quad (2.8)$$

To fix the gauge $A_+ = 0$ we require that the supersymmetry transformation of the field A_+ is compensated by the field-dependent gauge transformation

$$\delta A_+ = 0 \quad \Rightarrow \quad \Lambda^{\text{comp}}(\psi, \bar{\psi}) = -\frac{1}{p_+} (\bar{\epsilon}_{\dot{2}A}\psi_2^A + \bar{\psi}_{\dot{2}A}\epsilon_2^A) \equiv \frac{1}{p_+} (\Sigma^{\text{comp}}(\psi) + \bar{\Sigma}^{\text{comp}}(\bar{\psi})). \quad (2.9)$$

Thus the theory requires a compensating gauge transformation with the parameter which is a linear combination of the supersymmetry parameters and spinor fields.

This means that in the light-cone gauge $A_{2\dot{2}} = A_+ = 0$ the physical vector fields $A_{1\dot{2}}$ and $A_{i\dot{2}}$ transform under supersymmetry as shown in eq. (2.6) where the compensating gauge transformation parameter (2.9) has to be used. For example,

$$\delta_{\text{g.f.}} A_{1\dot{2}}(p) = p_{1\dot{2}}\Lambda^{\text{comp}}(\psi, \bar{\psi}) + \bar{\epsilon}_{\dot{2}A}\psi_1^A + \bar{\psi}_{\dot{2}A}\epsilon_1^A. \quad (2.10)$$

This can be rewritten in the form

$$\delta_{\text{g.f.}} A_{1\dot{2}}(p) = -\frac{p_{1\dot{2}}}{p_+} (\bar{\epsilon}_{\dot{2}A}\psi_2^A + \bar{\psi}_{\dot{2}A}\epsilon_2^A) + \bar{\epsilon}_{\dot{2}A}\psi_1^A + \bar{\psi}_{\dot{2}A}\epsilon_1^A. \quad (2.11)$$

Since we are looking at the symmetries of the free asymptotic fields, the fermions satisfy the Dirac equation

$$p_{21}\psi_2^A = p_{22}\psi_1^A. \quad (2.12)$$

This leads to a simplification of the gauge-fixed transformations of the vector field since the first and the third terms in eq. (2.11) cancel! The remaining two terms are

$$\boxed{\delta_{\text{g.f.}} A_{1\dot{2}}(p) = -\frac{p_{1\dot{2}}}{p_+} \bar{\psi}_{\dot{2}A}\epsilon_2^A + \bar{\psi}_{\dot{2}A}\epsilon_1^A = p_{1\dot{2}}\bar{\Sigma}^{\text{comp}}(\bar{\psi}) + \bar{\psi}_{\dot{2}A}\epsilon_1^A} \quad (2.13)$$

The first term in this equation is given by a compensating gauge transformation and the second term we recognize as the kinematic supersymmetry. For the conjugate field we find an analogous expression

$$\boxed{\delta_{\text{g.f.}} A_{i\dot{2}}(p) = -\frac{p_{i\dot{2}}}{p_+} \bar{\epsilon}_{\dot{2}A}\psi_2^A + \epsilon_{iA}\psi_2^A = p_{i\dot{2}}\Sigma^{\text{comp}}(\psi) + \epsilon_{iA}\psi_2^A} \quad (2.14)$$

Now we are ready to compare these transformations of the vector fields with those coming from the chiral scalar on shell superfield.

3 Light-cone path integral and symmetries

The path integral for the generating functional of the on shell amplitudes studied in [5], [7] is given by

$$e^{iW[\phi_{in}(z)]} = \int d\phi \, e^{iS[\phi(z)] + i \int d^8 z \, \phi_{in}(z) p^2 \phi(-z)} . \quad (3.1)$$

Here for the $\mathcal{N}=4$ YM case the Lie-algebra valued off-shell superfield $\phi(p, \eta) = \phi^a(p, \eta) t^a$ depends only on physical degrees of freedom of $\mathcal{N}=4$ SYM theory:

$$\phi = \bar{A}(p) + \eta_A \Psi^A(p) + \frac{1}{2!} \eta_A \eta_B \phi^{AB}(p) + \frac{1}{3!} \epsilon^{ABCD} \eta_A \eta_B \eta_C \bar{\Psi}_D(p) + \frac{1}{4!} \epsilon^{ABCD} \eta_A \eta_B \eta_C \eta_D A(p) \quad (3.2)$$

The chiral scalar superfield $\phi(p, \eta)$, the integration variable in the path integral, is off shell, $p^2 \phi(p, \eta) \neq 0$ whereas the asymptotic field $\phi_{in}(p, \eta)$ is on shell, $p^2 \phi_{in}(p, \eta) = 0$. In (3.1) $z = (p, \eta)$ is the 4+4 momentum superspace. The integration is defined as $d^8 z \equiv \frac{d^4 p}{(2\pi)^4} d^4 \eta$.

For $\mathcal{N}=8$ supergravity the path integral is analogous, see the details in [5]. The difference is that the chiral superfield is not Lie-algebra valued and the number of η 's is 8 instead of 4 as in $\mathcal{N}=4$ YM.

The linearized asymptotic symmetries of the free superfield ϕ_{in} are the following in $\mathcal{N}=4$ YM case. There are 16 supersymmetries, $q_{\dot{\alpha}}^A = \bar{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial \eta_A}$, $q_{B\alpha} = \lambda_{\alpha} \eta_B$,

$$\boxed{\delta \phi_{in}(p, \eta) = \left(\epsilon^{\alpha A} \lambda_{\alpha} \eta_A + \bar{\epsilon}_{\dot{\alpha}}^A \bar{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial \eta_A} \right) \phi_{in}(p, \eta)} \quad (3.3)$$

They form the closed algebra

$$\{\bar{q}_{\dot{\alpha}}^A, q_{B\alpha}\} = \delta^A_B \lambda_{\alpha} \bar{\lambda}_{\dot{\alpha}} . \quad (3.4)$$

In the light-cone formulation 8 supersymmetries are manifest in the path integral, they are called kinematical supersymmetries and are given by $q_{A2} = \lambda_2 \eta_A$ and $\bar{q}_2^A = \bar{\lambda}_2 \frac{\partial}{\partial \eta_A}$. They form the algebra

$$\{\bar{q}_2^A, q_{B2}\} = \delta^A_B \lambda_2 \bar{\lambda}_2 = \delta^A_B p_+, \quad p_{2\dot{2}} = p_+, \quad \lambda_2 = \bar{\lambda}_2 = \sqrt{p_+} . \quad (3.5)$$

The remaining 8 supersymmetries are the so-called dynamical supersymmetries given by $q_{A1} = \lambda_1 \eta_A$ and $\bar{q}_1^A = \bar{\lambda}_1 \frac{\partial}{\partial \eta_A}$ and

$$\{\bar{q}_1^A, q_{B1}\} = \delta^A_B \lambda_1 \bar{\lambda}_1 = \delta^A_B p_- = \delta^A_B \frac{p_{\perp} \bar{p}_{\perp}}{p_+}, \quad \lambda_1 = \frac{p_{\perp}}{\sqrt{p_+}}, \quad \lambda_{\dot{1}} = \frac{\bar{p}_{\perp}}{\sqrt{p_+}} . \quad (3.6)$$

The physical on shell amplitudes, computed via the supergraphs in eq. (3.1) are expected to have all 16 linearized asymptotic symmetries. The prediction about the UV properties of the maximal supersymmetric QFT were made in [5] on the basis of the non-manifest dynamical supersymmetry described above.

4 Covariant symmetries upon gauge-fixing versus light-cone superfield ones

From the light-cone superfield transformations (3.3) we find

$$\delta \bar{A} = \left(\bar{\epsilon}_A^{\dot{\alpha}} \bar{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial \eta_A} \right) \eta_B \Psi^B = \bar{\epsilon}_A^{\dot{\alpha}} \lambda_{\dot{\alpha}} \Psi^A, \quad (4.1)$$

which means that

$$\boxed{\delta \bar{A} = -\bar{\epsilon}_{2A} \bar{\lambda}_1 \Psi^A + \bar{\epsilon}_{1A} \bar{\lambda}_2 \Psi^A} \quad (4.2)$$

Here the first term represent a dynamical supersymmetry with $\bar{\epsilon}_A^1$ and the second one a kinematical one with $\bar{\epsilon}_A^2$. For the conjugate vector field we have to compute $\frac{1}{4!} \epsilon^{ABCD} \eta_A \eta_B \eta_C \eta_D \delta A = (\epsilon^{\alpha A} \lambda_{\alpha} \eta^A) \frac{1}{3!} \epsilon^{ABCD} \eta_A \eta_B \eta_C \bar{\Psi}_D(p)$. It follows that

$$\boxed{\delta A = -\bar{\Psi}_A \epsilon_2^A \lambda_1 + \bar{\epsilon}_{1A} \bar{\lambda}_2 \Psi^A} \quad (4.3)$$

We now see that with³

$$\bar{A} = A_{i2} \quad A = A_{1\dot{2}} \quad \psi_2^A = \sqrt{p_+} \Psi \quad (4.4)$$

and the definition of the $\lambda, \bar{\lambda}$ in (3.5), (3.6) we have identified all symmetries of the space-time fields in a gauge-fixed theory with the symmetry transformations of the chiral light-cone superfield $\phi(p, \eta)$.

Via this identification we have also learned that the dynamical supersymmetry of the light-cone superfield theory is actually a compensating gauge symmetry on the vector potentials preserving the gauge-fixing condition $A_+ = 0$:

$$\boxed{\delta_{\text{dyn}} A(p) = \delta_{\text{gauge}} A(p) = p_{\perp} \bar{\Sigma}(\bar{\psi}) \quad \delta_{\text{dyn}} \bar{A}(p) = \delta_{\text{gauge}} \bar{A}(p) = \bar{p}_{\perp} \Sigma(\psi)} \quad (4.5)$$

5 Implications for UV properties

By observing that the dynamical supersymmetry in the on shell light-cone superfield is just a compensating gauge transformations we see the analogy with the well known concept in QED: scattering of light-by-light. The UV divergences should not depend on A_{μ} , they should depend on the field strength $F_{\mu\nu}$. In gravity case they should not depend on $h_{\mu\nu}$ but on the linear part of $R_{\mu\nu\lambda\delta}$. This means that on each external leg $h_{\mu\lambda}$ there is an extra factor of $p_{\nu} p_{\delta}$ which leads to a better UV behavior of the Feynman graphs.

Note that in the covariant analysis of the UV divergences the straightforward effect of “scattering of light-by-light” is already taken into account. The gravitational analog of this

³This is in a precise agreement with spinor field rescaling suggested in [5] which is required to bring the original light-cone superfield of [1], [2] to the form given in eq. (3.2).

effect is the 3-loop candidate⁴ divergence for the $\mathcal{N}=8$ supergravity, R^4 . The fact that it depends on the space-time curvatures takes care of linearized gauge-invariance of the free graviton in the 4-point amplitude.

In the light-cone supergraph amplitudes the chiral superfield $\phi(p, \eta)$ is not a gauge-invariant object (not invariant under dynamical supersymmetry). The lack of gauge invariance is obvious, one can see from eq. (3.2) that the superfield depends on gauge potentials \bar{A} and A and not on the field strength $p_\perp \bar{A} - \bar{p}_\perp A$.

The symmetry is restored when the external factors of transverse momenta are added for each external chiral superfield. For example, for the 4-point amplitude one finds [5] that a special Grassmann δ -function is required to secure the dynamical supersymmetry. In the $\mathcal{N}=4$ YM case

$$A^{YM}(p_i, \eta_i) = \delta^4 \left(\sum_{i=1}^4 \lambda_1^i \eta_i \right) \mathcal{P}^{YM}(p_i) . \quad (5.1)$$

Since $\lambda_1 \sim p_\perp$ we see that for 4 chiral superfields 4 transverse momenta are necessary, see [5] for details. These 4 transverse momenta mean that there is a non-polynomial dependence on transverse momenta in the supergraph amplitude, by dimension. No non-polynomial dependence on the transverse momenta should appear in the UV divergences. This explains the UV finiteness of $\mathcal{N}=4$ YM theory. This is an effect of “scattering of light-by-light” in the supergraphs when some momenta are extracted from the internal lines as they must be present on the external legs.

In $\mathcal{N}=8$ supergravity the supergraph amplitude respecting dynamical supersymmetry has an 8-dimensional δ -function, which corresponds to adding 2 transverse momenta for each chiral superfield leg:

$$A^{SG}(p_i, \eta_i) = \delta^8 \left(\sum_{i=1}^4 \lambda_1^i \eta_i \right) \mathcal{P}^{SG}(p_i) . \quad (5.2)$$

The difference with $\mathcal{N}=4$ supersymmetric Yang-Mills theory is that there is a dimensionful coupling constant. Therefore with extra κ^8 one can remove the non-polynomial dependence on the transverse momenta: this however, takes place for the 4-point amplitude at the level $\kappa^{2(L-1)}$ which is 4 loops higher than the naive expectation at 3 loops. For every extra leg in $\mathcal{N}=8$ one has to add an extra 2 transverse momenta, therefore for the n -point amplitude the delay of the UV divergences is increasing at least to the level $L = n + 3$. This is again a simple analogy with the “scattering of light-by-light” concept, since every leg should depend on curvature and not on the gravitational field $h_{11} \pm i h_{12}$.

We would like to stress here that the breaking of dynamical supersymmetry means also the breaking of the gauge symmetry, the success of dynamical supersymmetry implies the success of the gauge symmetry. However, it may not work the other way around: if we would only secure

⁴This 3-loop R^4 counterterm for $\mathcal{N}=8$ supergravity, covariant and supersymmetric at the linear level was constructed long time ago in [8]. However, the computations in [3, 4] have shown that the 3-loop divergence is absent. Recently it was also noticed in the linearized covariant helicity formalism analysis of [9] that the candidate R^4 divergence is not ruled out. Therefore, the light-cone supergraph analysis of [5] gives the only known explanation as to why in four dimensions the candidate R^4 divergence is absent, in agreement with [3, 4].

the gauge symmetry, it may be insufficient to claim a success of the dynamical supersymmetry in the supergraphs. Under dynamical supersymmetry all fields in the multiplet transform under the linearized symmetry. Meanwhile, spinors and scalars do not transform under the linearized gauge transformations, for them there is no reason to identify the dynamical supersymmetry with the compensating gauge transformation. This may explain why preserving only the linearized gauge symmetry in the covariant analysis of the counterterms may be insufficient to see why, for example, the 3, 4, 5, 6-loop divergences of $\mathcal{N}=8$ supergravity should be absent according to the light-cone supergraphs analysis. Thus, one should view the analogy with the “scattering of light-by-light” concept in a more general context than just a manifestation of the familiar gauge symmetry.

6 Discussion

In conclusion, we have explained here the UV prediction for maximal supersymmetric QFT’s, $\mathcal{N}=4$ supersymmetric Yang-Mills theory and $\mathcal{N}=8$ supergravity, by an analogy to the “scattering of light-by-light” effect in the light-cone supergraph method. The prediction for the absence of $\mathcal{N}=8$ UV divergences until 7 loops follows in a simple way from the properly generalized to supergraphs “scattering of light-by-light” effect.

It is also interesting to ask about the possible influence of the $E_{7(7)}$ symmetry recently studied in [10] on UV divergences in higher loops. The rule $L = n + 3$ for the delay of the divergences in the n -point amplitudes may be combined with $E_{7(7)}$ symmetry. The chiral light-cone superfield $\phi(x, \theta)$ transforms non-linearly under $E_{7(7)}$ symmetry [10] and the transformation involves an infinite power of fields. Therefore, it has been proposed in [5] that the non-linear nature of this symmetry may require that at any given loop order all n -point amplitudes have to be divergent to support a valid counterterm. This would contradict to the rule $L = n + 3$ since for any L there will be some n for which the amplitude is UV finite. So, the hope was expressed in [5] that perhaps $E_{7(7)}$ symmetry may lead to the all loop perturbative UV finiteness of $\mathcal{N}=8$ supergravity.

More studies will be necessary to clearly understand how the requirement of the non-linear symmetries will affect the UV predictions of the light-cone supergraph method in which so far we used only the linearized symmetries of the asymptotic fields.

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