On two-loop divergences of effective action in 6D, $\mathcal{N} = (1,1)$ SYM theory

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Abstract

We study the off-shell structure of the two-loop effective action in 6D, $\mathcal{N}=(1,1)$ supersymmetric gauge theories formulated in $\mathcal{N}=(1,0)$ harmonic superspace. The off-shell effective action involving all fields of 6D, $\mathcal{N}=(1,1)$ supermultiplet is constructed by the harmonic superfield background field method, which ensures both manifest gauge covariance and manifest $\mathcal{N}=(1,0)$ supersymmetry. We analyze the off-shell divergences dependent on both gauge and hypermultiplet superfields and argue that the gauge invariance of the divergences is consistent with the non-locality in harmonics. The two-loop contributions to the effective action are given by harmonic supergraphs with the background gauge and hypermultiplet superfields. The procedure is developed to operate with the harmonic-dependent superpropagators in the two-loop supergraphs within the superfield dimensional regularization. We explicitly calculate the gauge and the hypermultiplet-mixed divergences as the coefficients of $\frac{1}{\varepsilon^2}$ and demonstrate that the corresponding expressions are non-local in harmonics.

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1 Introduction

Quantum properties of gauge theories in higher dimensions attract a lot of attention for a long time (see, e.g., [1–13]). Although these theories are not renormalizable by power-counting since the degree of divergence increases with a number of loops, the direct calculations sometimes demonstrate miraculous cancelations of some divergences. The situation becomes even more interesting in supersymmetric theories due to additional restrictions imposed by supersymmetry. In particular, some arguments were adduced that the maximally extended $\mathcal{N}=8$ supergravity is finite to all orders [14–16]. In this connection, it would be very interesting to study the ultraviolet divergences of supersymmetric gauge theories in the space-times of the dimension D>4 and reveal possible mechanisms of cancelations of the divergences. One can expect that supersymmetry and, especially, the maximally extended supersymmetry, is capable to improve the ultraviolet behavior in supersymmetric higher dimensional gauge theories.

In this paper we concentrate on 6D, $\mathcal{N}=(1,1)$ supersymmetric Yang-Mills (SYM) theory. This theory is in many aspects similar to 4D, $\mathcal{N}=4$ SYM theory, and so one can expect some similarities in the structure of divergences in both theories. However, they essentially differ in the UV domain. In contrast to $\mathcal{N}=4$ SYM theory, which is finite to all loops [17–21], its 6D counterpart is non-renormalizable by power-counting. Nevertheless, an extended supersymmetry provides the on-shell finiteness of the theory up to two loops [4–6, 22]. Recently we have shown that 6D, $\mathcal{N}=(1,1)$ gauge theory is one-loop finite even off shell [23, 24]. The modern methods of computing scattering amplitudes [22] demonstrate that UV divergences in 6D, $\mathcal{N}=(1,1)$ SYM theory should start from the three-loop level in accordance with the equation D=4+6/L valid for $L\geq 2$ [25–27] (see also [13]), which relates the space-time dimension D to the number of loops L at which divergences appear in the corresponding maximally supersymmetric Yang-Mills theory⁵.

When investigating the quantum properties of supersymmetric theories, it is highly desirable to make use of the formulations and quantization procedure which preserve as many supersymmetries as possible. For example, 4D, $\mathcal{N}=1$ supersymmetry is manifest if a theory is formulated in $\mathcal{N}=1$ superspace (see e.g. [29–31]). Similarly, 4D, $\mathcal{N}=2$ supersymmetry can be kept manifest at all stages of calculations within the harmonic superspace approach [32–35]. This formalism is very convenient for analyzing the structure of quantum corrections. Say, the finiteness beyond the one-loop approximation [18, 21] can be proved much easier in the harmonic superspace [36, 37]. The similar harmonic 6D superspace [38, 39] can be efficiently used for dealing with quantum 6D, $\mathcal{N}=(1,0)$ supersymmetric theories. If this method is applied to (the maximally extended) 6D, $\mathcal{N}=(1,1)$ SYM theory, then $\mathcal{N}=(1,0)$ supersymmetry is preserved even at the quantum level, while $\mathcal{N}=(0,1)$ supersymmetry is hidden and can in general be broken by quantum corrections.

In our previous work [40] we initiated the study of the two-loop divergent contribution to the effective action of 6D, $\mathcal{N}=(1,1)$ SYM theory in the harmonic superfield formalism and calculated the divergent contributions proportional to $\frac{1}{\varepsilon^2}$ in the gauge superfield sector of the model. In the present paper we will analyze the general structure of the two-loop divergent contributions to the effective action in both gauge and hypermultiplet sectors.

The UV properties of various 6D, $\mathcal{N} = (1,0)$ and $\mathcal{N} = (1,1)$ theories in 6D harmonic superspace approach were studied in [23, 24, 41–49]. The $\mathcal{N} = (1,1)$ SYM theory in this formulation amounts to $\mathcal{N} = (1,0)$ Yang–Mills supermultiplet minimally coupled to the hypermultiplet in the adjoint representation of the gauge group. Note that, in general, $\mathcal{N} = (1,0)$ theories are plagued by quantum

⁵For recent advances of the amplitude techniques see [28] and references therein.

anomalies [50–53], but $\mathcal{N}=(1,1)$ SYM theory is not anomalous. It was found that 6D, $\mathcal{N}=(1,1)$ theory is off-shell finite in the one-loop approximation in the Feynman gauge [23,24]. However, the off-shell divergences still reappear in the non-minimal gauges [46] (although they vanish on shell). The two-loop divergences in the hypermultiplet two-point Green functions were shown to vanish off shell [42]. However, the complete two-loop calculation in the harmonic superspace approach has not yet been done. In the present paper we will calculate the leading divergences and demonstrate that this theory in the two-loop approximation is not finite off shell even in the Feynman gauge. However, we will demonstrate that the divergences disappear on shell, as it should be.

It is worth noting that although the harmonic superspace provides the manifest $\mathcal{N}=(1,0)$ supersymmetry at the quantum level, in the Feynman supergraphs it produces harmonic-dependent propagators and integrations over harmonics, the number of which increases with the number of loops. For explicit calculations of such integrals, one needs to develop the special methods which would allow to evaluate the higher-loop harmonic supergraphs, based upon various nontrivial identities for the harmonic distributions.

In the present paper we work out the manifestly $\mathcal{N} = (1,0)$ supersymmetric approach to the calculation of the two-loop divergences of the effective action in the theory under consideration. Using this approach we will evaluate the leading divergences proportional to $\frac{1}{\varepsilon^2}$ in the mixed gauge-hypermultiplet sector and show that the result exactly matches the one derived earlier in [40].

The paper is organized as follows. In Section 2 we recall the formulation of the six-dimensional $\mathcal{N}=(1,1)$ SYM theory in $\mathcal{N}=(1,0)$ harmonic superspace and discuss details of its superfield quantization. Section 3 is devoted to the power-counting analysis of possible two-loop divergent contributions to the effective action. We show that the possible counterterms, being, of course, local in space-time, can generically be non-local in harmonics. In Section 4 we describe the general structure of divergent two-loop harmonic supergraphs, as well as the methods of their evaluations. Section 5 is devoted to applications of the above methods to the explicit calculation of the two-loop divergences in the mixed gauge-hypermultiplet sector. Summary collects the main results and briefly outlines the future directions of the study. Some technical details are passed to the Appendices.

2 The harmonic superspace formulation of $6D, \mathcal{N} = (1,1)$ supersymmetric gauge theories

We start by a brief overview of the $\mathcal{N}=(1,0)$ harmonic superspace formulation of $6D, \mathcal{N}=(1,1)$ SYM theory. In such a formulation, the theory enjoys a manifest off-shell $\mathcal{N}=(1,0)$ supersymmetry and an additional hidden on-shell $\mathcal{N}=(1,0)$ supersymmetry. Together they form the complete $6D, \mathcal{N}=(1,1)$ supersymmetry of the classical theory. After quantization, we arrive at a quantum field theory with preserving the manifest $\mathcal{N}=(1,0)$ supersymmetry at all stages of calculations. We recall the basic concepts of the harmonic superspace following notations and conventions of our previous papers [23, 24].

The 6D, $\mathcal{N}=(1,0)$ harmonic superspace is parametrized by the coordinates $(z,u)=(x^M,\theta^a_i,u^{\pm i})$, where x^M , M=0,...,5, are 6D Minkowski space-time coordinates, θ^a_i with a=1,...,4, i=1,2, stand for Grassmann variables. The extra bosonic coordinates u^\pm_i , $u^{+i}u^-_i=1$ (called harmonics) parametrize the coset SU(2)/U(1) of the automorphisms group $Sp(1)\sim SU(2)$ [38,39]. The analytic coordinates $\zeta=(x^M_A,\theta^{\pm a})$ are defined as

$$x_{\mathcal{A}}^{M} \equiv x^{M} + \frac{i}{2}\theta^{+a}(\gamma^{M})_{ab}\theta^{-b}, \qquad \theta^{\pm a} = u_{k}^{\pm}\theta^{ak}, \tag{2.1}$$

where $(\gamma^M)_{ab}$ are the antisymmetric 6D Weyl γ -matrices, $(\gamma^M)_{ab} = -(\gamma^M)_{ba}$, $(\widetilde{\gamma}^M)^{ab} = \frac{1}{2}\varepsilon^{abcd}(\gamma^M)_{cd}$, with ε^{abcd} being the totally antisymmetric Levi-Civita tensor.

By definition, the analytic superfields are annihilated by the operator $D_a^+ = u_i^+ D_a^i$, where D_a^i are the spinor covariant derivatives. Also we will need the covariant derivative $D_a^- = u_i^- D_a^i$ and the harmonic derivatives

$$D^{\pm \pm} = u^{\pm i} \frac{\partial}{\partial u^{\mp i}}, \qquad D^0 = u^{+i} \frac{\partial}{\partial u^{+i}} - u^{-i} \frac{\partial}{\partial u^{-i}}.$$
 (2.2)

The spinor and harmonic derivatives satisfy the algebra

$$\{D_a^+, D_b^-\} = i(\gamma^M)_{ab}\partial_M, \qquad [D^{++}, D^{--}] = D^0, \qquad [D^{\pm\pm}, D_a^{\pm}] = 0, \qquad [D^{\pm\pm}, D_a^{\mp}] = D_a^{\pm}. \quad (2.3)$$

The integration measures over the full harmonic and analytic superspaces are defined, respectively, as

$$d^{14}z \equiv d^6x_{\mathcal{A}} (D^-)^4 (D^+)^4, \quad d\zeta^{(-4)} \equiv d^6x_{\mathcal{A}} du (D^-)^4, \tag{2.4}$$

where

$$(D^{\pm})^4 = -\frac{1}{24} \varepsilon^{abcd} D_a^{\pm} D_b^{\pm} D_c^{\pm} D_d^{\pm} . \tag{2.5}$$

In the harmonic superspace approach the gauge field is a component of the analytic gauge superfield V^{++} . Also it is necessary to introduce a non-analytic harmonic connection V^{--} as a solution of the harmonic zero-curvature condition [34]

$$D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0. (2.6)$$

Using these superfields, we construct the gauge covariant harmonic derivatives $\nabla^{\pm\pm} = D^{\pm\pm} + iV^{\pm\pm}$. The spinor gauge covariant derivatives can be written as [49]

$$\nabla_a^+ = D_a^+, \qquad \nabla_a^- = D_a^- + i\mathcal{A}_a^-, \qquad \nabla_{ab} = \partial_{ab} + i\mathcal{A}_{ab}, \qquad (2.7)$$

where $\nabla_{ab} = \frac{1}{2} (\gamma^M)_{ab} \nabla_M$, $\nabla_M = \partial_M - iA_M$ and the superfield connections are defined as

$$\mathcal{A}_{a}^{-} = iD_{a}^{+}V^{--}, \quad \mathcal{A}_{ab} = \frac{1}{2}D_{a}^{+}D_{b}^{+}V^{--}.$$
 (2.8)

The covariant derivatives (2.7) satisfy the algebra

$$\{\nabla_a^+, \nabla_b^-\} = 2i\nabla_{ab}, \qquad [\nabla_c^\pm, \nabla_{ab}] = \frac{i}{2}\varepsilon_{abcd}W^{\pm d}, \qquad [\nabla_M, \nabla_N] = iF_{MN},$$
 (2.9)

where $W^{a\pm}$ is the superfield strength of the gauge supermultiplet,

$$W^{+a} = -\frac{i}{6} \varepsilon^{abcd} D_b^+ D_c^+ D_d^+ V^{--}, \qquad W^{-a} = \nabla^{--} W^{+a}.$$
 (2.10)

The classical action of 6D, $\mathcal{N}=(1,1)$ SYM model $S_0[q^+,V^{++}]$ in the harmonic superspace is written as [39]

$$S_{0} = \frac{1}{f^{2}} \sum_{n=2}^{\infty} \frac{(-i)^{n}}{n} \operatorname{tr} \int d^{14}z \, du_{1} \dots du_{n} \frac{V^{++}(z, u_{1}) \dots V^{++}(z, u_{n})}{(u_{1}^{+}u_{2}^{+}) \dots (u_{n}^{+}u_{1}^{+})} - \frac{1}{2f^{2}} \operatorname{tr} \int d\zeta^{(-4)} \, q^{+A} \nabla^{++} q_{A}^{+}.$$

$$(2.11)$$

The SU(2) Pauli-Gürsey indices are raised and lowered by the rule $q_A^+ = \epsilon_{AB}q^{+B}$, $\epsilon_{12} = 1$. The analytic superfields V^{++} and q^{+A} belong to the adjoint representation of the gauge group, i.e,

$$V^{++} = (V^{++})^I t^I, \quad q_A^+ = (q_A^+)^I t^I, \quad [t^I, t^J] = i f^{IJK} t^K, \quad \operatorname{tr}(t^I t^J) = \frac{1}{2} \delta^{IJ}, \tag{2.12}$$

where t^{I} are generators of the gauge algebra with the structure constants f^{IJK} , and the covariant harmonic derivative is defined as

$$\nabla^{++}q_A^+ = D^{++}q_A^+ + i[V^{++}, q_A^+]. \tag{2.13}$$

The classical equations of motion for the theory (2.11) have the form

$$F^{++} + \frac{i}{2}[q^{+A}, q_A^+] = 0, \qquad \nabla^{++}q_A^+ = 0,$$
 (2.14)

where we have defined the Grassmann-analytic superfield

$$F^{++} = (D^+)^4 V^{--}, \quad D^{++} F^{++} = 0.$$
 (2.15)

The action (2.11) is invariant under the manifest $\mathcal{N} = (1,0)$ supersymmetry and an additional hidden $\mathcal{N} = (0,1)$ supersymmetry. The hidden supersymmetry mixes the gauge and hypermultiplet superfields [11],

$$\delta_{(0,1)}V^{++} = \epsilon^{+A}q_A^+, \quad \delta_{(0,1)}q_A^+ = -(D^+)^4(\epsilon_A^-V^{--}), \quad \epsilon_A^{\pm} = \epsilon_{aA}\theta^{\pm a}.$$
 (2.16)

As a result, the action (2.11) is invariant under 6D, $\mathcal{N}=(1,1)$ supersymmetry 6 . It is also invariant under the superfield gauge transformations

$$\delta V^{++} = -\nabla^{++}\lambda, \qquad \delta q_A^+ = i[\lambda, q_A^+],$$
 (2.17)

with a real analytic superfield λ in the adjoint representation as the transformation parameter. The non-analytic gauge connection V^{--} is transformed as

$$\delta V^{--} = -\nabla^{--}\lambda\,,\tag{2.18}$$

The harmonic zero-curvature condition (2.6) is invariant under the gauge transformations (2.17) and (2.18). The gauge transformations of the gauge connections (2.8), of the covariant strengths (2.10) and of the analytic superfield (2.15) can be easily found. In particular, F^{++} transforms homogeneously, like Q^{+A} , which ensures the covariance of the equations of motion (2.14).

For quantizing gauge theories, it is convenient to use the background field method, which allows constructing the manifestly gauge invariant effective action. For 6D, $\mathcal{N}=(1,0)$ SYM theory in the harmonic superspace formulation this method was worked out in ref. [23, 24, 41]. In many aspects it is similar to that for 4D, $\mathcal{N}=2$ supersymmetric gauge theories [35, 54] (see also the review [55]).

Following the background field method we split the superfields V^{++} and q_A^+ into the sums of the background superfields V^{++} , Q_A^+ and the quantum ones v^{++} , q_A^+ ,

$$V^{++} \to V^{++} + fv^{++}, \qquad q_A^+ \to Q_A^+ + fq_A^+.$$
 (2.19)

⁶Though the second supersymmetry (2.16) is an off-shell symmetry of the action (2.11), its correct closure with itself and with the manifest $\mathcal{N} = (1,0)$ supersymmetry is achieved only on shell.

Next, we expand the effective action in a power series in quantum superfields and obtain a theory of the superfields v^{++}, q_A^+ in the background of the classical superfields V^{++}, Q_A^+ , which are treated as functional arguments of the effective action. According to refs. [23, 24, 41] the expression for the effective action can be written as

$$e^{i\Gamma[V^{++},Q^{+}]} = \operatorname{Det}^{1/2} \widehat{\square} \int \mathcal{D}v^{++} \mathcal{D}q^{+} \mathcal{D}b \mathcal{D}c \mathcal{D}\varphi \exp\left(iS_{\text{total}} - i\frac{\delta\Gamma[V^{++},Q^{+}]}{\delta V^{++}}v^{++} - i\frac{\delta\Gamma[V^{++},Q^{+}]}{\delta Q^{+}}q^{+}\right). \tag{2.20}$$

Here

$$S_{\text{total}} = S_0[q^+, v^{++}; Q^+, V^{++}] + S_{\text{gf}}[v^{++}; V^{++}] + S_{\text{FP}}[b, c, v^{++}; V^{++}] + S_{\text{NK}}[\varphi; V^{++}],$$

where

$$S_0[q^+, v^{++}; Q^+, V^{++}] = S_0[Q^+ + fq^+, V^{++} + fv^{++}].$$

The action S_{total} also includes the gauge-fixing term corresponding to the Feynman gauge,

$$S_{\rm gf}[v^{++}, V^{++}] = -\frac{1}{2} \operatorname{tr} \int d^{14}z \, du_1 du_2 \, \frac{v_{\tau}^{++}(1)v_{\tau}^{++}(2)}{(u_1^+ u_2^+)^2} + \frac{1}{4} \operatorname{tr} \int d^{14}z \, du \, v_{\tau}^{++}(D^{--})^2 v_{\tau}^{++} \tag{2.21}$$

(the index τ means the " τ -representation", see eq. (2.29) below), the action for the fermionic Faddeev-Popov ghosts **b** and **c**, as well as the action for the bosonic real analytic Nielsen-Kallosh ghost φ ,

$$S_{FP} = -\operatorname{tr} \int d\zeta^{(-4)} \nabla^{++} \mathbf{b} \left(\nabla^{++} \mathbf{c} + i[v^{++}, \mathbf{c}] \right), \tag{2.22}$$

$$S_{\text{NK}} = -\frac{1}{2} \text{tr} \int d\zeta^{(-4)} \varphi(\nabla^{++})^2 \varphi. \tag{2.23}$$

On the subset of analytic superfields the operator $\widehat{\square} = \frac{1}{2}(D^+)^4(\nabla^{--})^2$ in the expression (2.20) is reduced to the covariant super d'Alembertian

$$\widehat{\Box} = \eta^{MN} \nabla_M \nabla_N + i W^{+a} \nabla_a^- + i F^{++} \nabla^{--} - \frac{i}{2} (\nabla^{--} F^{++}), \tag{2.24}$$

where $\eta_{MN} = diag(1, -1, -1, -1, -1)$ is 6D Minkowski metric.

To remove the mixed terms containing both v^{++} and q_A^+ in the quadratic part of the action,

$$S_{2} + S_{gf} = -\frac{1}{2} \operatorname{tr} \int d\zeta^{(-4)} v^{++} \widehat{\Box} v^{++} - \frac{1}{2} \operatorname{tr} \int d\zeta^{(-4)} q^{+A} \nabla^{++} q_{A}^{+}$$

$$-\frac{i}{2} \operatorname{tr} \int d\zeta^{(-4)} \left\{ Q^{+A} [v^{++}, q_{A}^{+}] + q^{+A} [v^{++}, Q_{A}^{+}] \right\}, \qquad (2.25)$$

we shift the quantum hypermultiplet superfield as

$$(q_1^{+A})^I = (h_1^{+A})^I + \int d\zeta_2^{(-4)} \left(G^{(1,1)A}{}_B(1|2) \right)^{IJ} f^{JKL} (v_2^{++})^K (Q_2^{+B})^L . \tag{2.26}$$

The newly defined hypermultiplet h_A^+ has the propagator

$$G^{(1,1)A}{}_{B}(1|2) = \langle h_{1}^{+A} h_{2B}^{+} \rangle = 2\delta^{A}{}_{B}G^{(1,1)}(1|2), \tag{2.27}$$

where we omitted the gauge group indices. The Green function $G^{(1,1)}(1|2)$ satisfies the equation $\nabla^{++}G^{(1,1)}(1|2) = \delta_A^{(3,1)}(1|2)$ and has the following explicit form [34]

$$G^{(1,1)}(1|2)^{IJ} = \left(\widehat{\Box}_1^{-1}\right)^{KL} (D_1^+)^4 (D_2^+)^4 \left((e^{ib_1})^{KI} (e^{ib_2})^{LJ} \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3} \right), \tag{2.28}$$

where the bridge b is defined as $V^{\pm\pm}=-ie^{ib}D^{\pm\pm}e^{-ib}$ or, in more detail, $-if^{IJK}(V^{\pm\pm})^I=-i(e^{ib})^{JL}D^{\pm\pm}(e^{-ib})^{LK}$ [34]. Using the bridge, one can define the τ -representation for a superfield X according to the prescription

$$X_{\tau} = e^{-ib} X e^{ib}. \tag{2.29}$$

After performing the shift (2.26), the quadratic part of the action S_2 (2.25) splits into few terms, each being bilinear in quantum superfields,

$$S_{2} + S_{gf} = \frac{1}{2} \operatorname{tr} \int d\zeta_{1}^{(-4)} d\zeta_{2}^{(-4)} v_{1}^{++} \left\{ \widehat{\Box} \delta_{A}^{(3,1)}(1|2) - 2Q_{1}^{+A} G^{(1,1)}(1|2) Q_{2A}^{+} \right\} v_{2}^{++}$$

$$- \frac{1}{2} \operatorname{tr} \int d\zeta^{(-4)} h^{+A} \nabla^{++} h_{A}^{+}.$$

$$(2.30)$$

Then the propagator of the gauge superfield in (2.30) is expressed as

$$G_{IJ}^{(2,2)}(1|2) = -2(\widehat{\square}_1^{-1})^{IJ} \delta_{\mathcal{A}}^{(2,2)}(1|2)$$

$$+4(\widehat{\square}_1^{-1})^{IN} f^{NKL_1} (Q_1^{+A})^K (Q_{2A}^+)^M f^{ML_2P} G^{(1,1)}(1|2)^{L_1L_2} (\widehat{\square}_2^{-1})^{PJ} + \dots, \quad (2.31)$$

and that of the Faddeev-Popov ghosts as

$$G^{(0,0)}(1|2) = i\langle b(\zeta_1, u_1)c(\zeta_2, u_2)\rangle = -(u_1^- u_2^-)G^{(1,1)}(1|2).$$
(2.32)

The Nielsen–Kallosh ghosts matter only in the one-loop approximation and do not appear in the two-loop supergraphs considered in this paper.

For calculating the two-loop quantum corrections we will need vertices which are cubic and quartic in quantum superfields. The presence of the non-trivial background hypermultiplet leads to the modification of the interaction vertices compared to ref. [42]. Indeed, the substitution (2.26) obviously modifies the interacting vertices. Due to this substitution, in addition to the ordinary 'local' vertices produced by the corresponding terms in the classical action (2.11) (with the formal replacement $q^+ \to h^+$),

$$S_{\text{hyper}}^{(3)} = \frac{f}{4} \int d\zeta^{(-4)} f^{IJK} (h^{+A})^{I} (v^{++})^{J} (h_A^{+})^{K}, \qquad (2.33)$$

$$S_{\text{SYM}}^{(3)} = \frac{if}{3} \operatorname{tr} \int d^{14}z \prod_{a=1}^{3} du_a \frac{v_1^{++} v_2^{++} v_3^{++}}{(u_1^{+} u_2^{+})(u_2^{+} u_3^{+})(u_3^{+} u_1^{+})}, \qquad (2.34)$$

$$S_{\text{SYM}}^{(4)} = \frac{f^2}{4} \text{tr} \int d^{14}z \prod_{a=1}^4 du_a \frac{v_1^{++} v_2^{++} v_3^{++} v_4^{++}}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)}, \tag{2.35}$$

and those coming from the Faddeev-Popov ghost action (2.22),

$$S_{\text{ghost}}^{(3)} = \frac{f}{2} \int d\zeta^{(-4)} f^{IJK} (\nabla^{++}b)_I v_J^{++} c_K, \qquad (2.36)$$

we also encounter some new 'non-local' vertices involving the hypermultiplet Green function $G^{(1,1)}(1|2)$,

$$S_{\text{non-loc}_{1}}^{(3)} = -f \int d\zeta_{1}^{(-4)} d\zeta_{2}^{(-4)} f^{IJK} G_{IL}^{(1,1)}(1|2) f^{LMN} (Q_{2}^{+A})^{N} (v_{2}^{++})^{M} (v_{1}^{++})^{J} (h_{1A}^{+})^{K}, \qquad (2.37)$$

$$S_{\text{non-loc}_{2}}^{(3)} = -f \int d\zeta_{1}^{(-4)} d\zeta_{2}^{(-4)} d\zeta_{3}^{(-4)} f^{IJK} G_{IL}^{(1,1)}(1|2) f^{LMN}$$

$$\times (Q_{2}^{+A})^{N} G_{KP}^{(1,1)}(1|3) f^{PQF} (Q_{3A}^{+})^{F} (v_{1}^{++})^{J} (v_{2}^{++})^{M} (v_{3}^{++})^{Q}. \qquad (2.38)$$

The presence of such types of vertices leads to additional non-trivial diagrams, which we will consider in detail in the next Sections.

3 Power counting

The superficial degree of divergence $\omega(G)$ for L-loop supergraph G can be found by counting the degrees of momenta in the loop integrals taking into account dimensions of the propagators and the background hypermultiplet lines⁷. The result [41] is

$$\omega(G) = 2L - N_Q,\tag{3.1}$$

where N_Q is a number of external hypermultiplet legs. Below we investigate possible divergent contributions to the two-loop effective action using the expression (3.1).

3.1 Possible two-loop divergences

We begin with the general analysis of possible two-loop contributions to the divergent part of the effective action defined within the background field method. The power counting performed in [24] leads to the following superficial degree of divergence for the two-loop diagrams in the theory under consideration

$$\omega_{2-\text{loop}} = 4 - N_Q \,, \tag{3.2}$$

where N_Q stands for a number of the hypermultiplet legs in the diagrams and it should be even as the form of the classical action indicates.⁸. In the two-loop approximation, N_Q can be equal to 0, 2, 4. This allows us to list all possible two-loop divergent contributions to the effective action.

First of all we note that in the framework of the $\mathcal{N}=(1,0)$ supersymmetric background field method the (dimensionally regularized) divergences of the effective action are automatically gauge invariant and $\mathcal{N}=(1,0)$ supersymmetric by construction. Taking into account the space-time locality of the counterterms together with $\mathcal{N}=(1,0)$ supersymmetry, we conclude that the divergences should be local in the space-time and Grassmann coordinates. However, since the harmonics are dimensionless, the power counting cannot guarantee a locality in harmonics. This means that we could expect the supersymmetric and gauge invariant divergences which contain the harmonic non-localities. We believe that the existence of gauge-invariant functionals that are nonlocal in harmonics is a general property of extended supersymmetric theories formulated in harmonic superspace. An example of such a functional is the initial classical action (2.11). Further we shall consider the consequences of

The number of external gauge lines has no impact on the index $\omega(G)$ because the superfield V^{++} is dimensionless.

⁸In the analysis below we take all superfields in the τ -representation [34].

the relation (3.2).

 $N_Q = 0$, $\omega = 4$. The corresponding divergent contribution to the two-point Green function of the gauge superfield is of the fourth order in momenta and is given by the full $\mathcal{N} = (1,0)$ superspace integral

$$\Gamma_1^{(2)} \sim \text{tr} \int d^{14}z du \, V_{\text{linear}}^{--} \Box^2 V^{++}.$$
 (3.3)

Passing to the analytic subspace in (3.3) and restoring terms with higher powers of gauge superfield on the ground of gauge invariance, we obtain the expression

$$\Gamma_1^{(2)} \sim \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} F_1^{++} F_2^{++},$$
(3.4)

where $F_{1,\tau}^{++} = F_{\tau}^{++}(z, u_1)$ and $[F^{++}] = [m^2]$. In Appendix A we show that the expression (3.4) can be identically rewritten in the simpler form which is local in harmonics,

$$\Gamma_1^{(2)} \sim \text{tr} \int d\zeta^{(-4)} F^{++} \widehat{\Box} F^{++} .$$
 (3.5)

The expression (3.5) is local in the analytic coordinate ζ and harmonic variables u in the analytic superspace. Just such an expression was used in our previous paper [40].

 $N_Q = 2$, $\omega = 2$. The possible divergent contribution to the effective action in the lowest order in V^{++} includes only one d'Alembertian operator, when written in the full $\mathcal{N} = (1,0)$ superspace

$$\Gamma_2^{(2)} \sim \text{tr} \int d^{14}z du \, V_{\text{linear}}^{--} \Box[Q^{+A}, Q_A^+].$$
 (3.6)

There exist three following gauge invariant expressions reduced to (3.6) in the lowest order in V^{++}

$$\mathcal{G}_2^{(2)} \sim \operatorname{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} F_1^{++} [Q_2^{+A}, Q_{2A}^+],$$
 (3.7)

$$\mathcal{G}_3^{(2)} \sim \operatorname{tr} \int d^{14}z \frac{du_1 du_2 du_3}{(u_1^+ u_2^+)(u_1^+ u_3^+)} F_1^{++}[Q_2^{+A}, Q_{3A}^+],$$
 (3.8)

$$\mathcal{G}_{4}^{(2)} \sim \operatorname{tr} \int d^{14}z \frac{du_{1}du_{2}du_{3} (u_{1}^{-}u_{2}^{+})}{(u_{1}^{+}u_{2}^{+})(u_{2}^{+}u_{3}^{+})} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{3A,\tau}^{+}]. \tag{3.9}$$

Note that the hypermultiplet commutators appear because the hypermultiplet is taken in the adjoint representation and they are necessary for ensuring gauge invariance. In the Appendix A we show that the expression (3.7) can be identically rewritten in the simpler form local in harmonics

$$\Gamma_2^{(2)} \sim \text{tr} \int d\zeta^{(4)} F^{++} \widehat{\Box} [Q^{+A}, Q_A^+],$$
(3.10)

The expression (3.8) and (3.9) are gauge-invariant functionals which are non-local in harmonics.

 $N_Q = 4$, $\omega = 0$. The possible lowest degree divergent contribution to the four-point Green function of the hypermultiplet superfield in the full $\mathcal{N} = (1,0)$ superspace has the form

$$\Gamma_3^{(2)} \sim \text{tr} \int d^{14}z du [Q^{+A}, Q_A^+] (D^{--})^2 [Q^{+B}, Q_B^+].$$
 (3.11)

The possible gauge invariant expression in the analytic subspace corresponding to (3.11) can be written as

$$\Gamma_3^{(2)} \sim \text{tr} \int d\zeta^{(-4)} [Q^{+A}, Q_A^+] \stackrel{\frown}{\Box} [Q^{+B}, Q_B^+].$$
(3.12)

However, like in the previous case, one can construct more gauge invariant expressions of the same mass dimensions as (3.12), such that they are non-local in harmonics (see, e.g., a recent discussion in [57]). As an example, we quote the operator

$$\mathcal{G}_{5}^{(2)} \sim \operatorname{tr} \int d^{14}z \prod_{a=1}^{4} du_{a} \frac{[Q_{1,\tau}^{+A}, Q_{2A,\tau}^{+}][Q_{3,\tau}^{+A}, Q_{4A,\tau}^{+}]}{(u_{2}^{+}u_{3}^{+})(u_{4}^{+}u_{1}^{+})}, \tag{3.13}$$

which in the lowest order in V^{++} coincides with (3.12).

3.2 On-shell structure

Here we consider the structure of the divergences for the background superfields satisfying the classical equations of motion (2.14). The classical theory (2.11) possesses the hidden on-shell $\mathcal{N}=(0,1)$ supersymmetry (2.16), while the quantum effective action does not preserve such a supersymmetry since the gauge fixing conditions used for the construction of the effective action preserve only the manifest $\mathcal{N}=(1,0)$ supersymmetry. However, for the background superfields satisfying the classical equations of motion, the hidden supersymmetry is restored. Therefore, studying the on-shell divergences amounts in fact to the study of restrictions on the divergences imposed by hidden $\mathcal{N}=(0,1)$ supersymmetry.

In general, the off-shell two-loop divergent contributions to the effective action may include all possible terms of the form $\mathcal{G}_i^{(2)}$, i=1,...,5. Let us examine the corresponding on-shell contributions. First we have to note that all the contributions $\mathcal{G}_i^{(2)}$ should exactly yield one of the expression $\Gamma_i^{(2)}$, i=1,2,3, if the equations of motion for the hypermultiplet are satisfied (see Appendix A for details). Hence, we can write

$$\Gamma_{\text{div}}^{(2)} = \text{tr} \int d\zeta^{(-4)} \Big(c_1 F^{++} \widehat{\Box} F^{++} + c_2 i F^{++} \widehat{\Box} [Q^{+A}, Q_A^+] + c_3 [Q^{+A}, Q_A^+] \widehat{\Box} [Q^{+B}, Q_B^+] \Big)
+ \text{terms vanishing on e.o.m. for } Q^+,$$
(3.14)

which is in the full agreement with our previous analysis in [40]. This expression involves three arbitrary coefficients, each being proportional to $\frac{1}{\varepsilon^2}$ and to the dimensionful coupling constant f^2 with dimensionless numerical coefficients. Fixing the coefficients requires the explicit calculations of the quantum corrections.

The hidden $\mathcal{N} = (0,1)$ supersymmetry (2.14) additionally restricts the structure of the divergent contribution (3.14) and leads to the absence of two-loop divergences on shell (see e.g. [11, 12], [22]

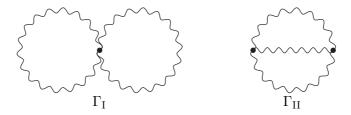


Figure 1: Two-loop Feynman supergraphs with the gauge self-interactions vertices.

and the references therein). As we pointed out, the hidden $\mathcal{N} = (0,1)$ supersymmetry means that the background superfields solve the classical equations of motion (2.14). In this case there arises a connection between the coefficients c_i , i = 1, 2, 3, in (3.14).

Omitting the terms proportional to the hypermultiplet equation of motion, we can rewrite the action (3.14) as

$$\Gamma_{\text{div}}^{(2)} = \text{tr} \int d\zeta^{(-4)} \Big(c_1 E^{++} \widehat{\Box} E^{++} + (c_2 - c_1) i F^{++} \widehat{\Box} [Q^{+A}, Q_A^+] + (c_3 + \frac{1}{4} c_1) [Q^{+A}, Q_A^+] \widehat{\Box} [Q^{+B}, Q_B^+] \Big),$$
(3.15)

where $E^{++} = F^{++} + \frac{i}{2}[Q^{+A}, Q_A^+]$ is the left hand side of the gauge multiplet equation of motion (2.14). The requirement that the expression (3.15) vanishes on-shell, $E^{++} = 0$, leads to the relation between the coefficients

$$2c_2 + 4c_3 = c_1. (3.16)$$

The leading contribution to the coefficient c_1 was calculated in our previous work [40]

$$c_1 = \frac{f^2(C_2)^2}{8(2\pi)^6 \varepsilon^2}, \qquad \varepsilon \to 0,$$
 (3.17)

for the case $Q_A^+ = 0$ that corresponds to the divergent contribution in the pure gauge superfield sector. One of the coefficients c_2 , c_3 in (3.14) remains arbitrary. Hence, to check the relation (3.14) which was obtained solely on the ground of the power-counting arguments, it suffices to calculate only one of these coefficients, e.g., c_2 . The last one will be fixed by eq. (3.16). In what follows, we consider those contributions which include only the term $F^{++}Q^{+A}Q_A^+$ in the divergent supergraphs and explicitly calculate the coefficient c_2 .

4 Analysis of the two-loop harmonic supergraphs

In this section we will discuss evaluation of the leading two-loop divergences in the theory under consideration.

The supergraphs contributing to the two-loop effective action are presented in Figs. 1, 2, and 3. The first two contain the conventional two-loop diagrams of six-dimensional $\mathcal{N}=(1,1)$ SYM theory [40]. Fig. 3 includes a new type of diagrams which are induced by the non-local shift (2.26). In the course of calculations we do not assume any restriction on the background gauge multiplet and hypermultiplet and perform the analysis in a manifestly gauge invariant form.

The graph of ' ∞ ' topology (the contribution $\Gamma_{\rm I}$) contains a vertex corresponding to the quartic interaction. In the theory under consideration it is given by eq. (2.35). The graphs of the ' Θ ' topology



Figure 2: Two-loop Feynman supergraphs with the hypermultiplet and ghosts vertices.

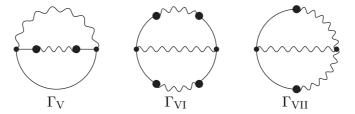


Figure 3: Two-loop Feynman supergraphs with the new 'non-local' vertices.

are generated by cubic interactions. In the considered $\mathcal{N} = (1,1)$ theory they are given by eqs. (2.33), (2.34), (2.36), (2.37), and (2.38). The corresponding contributions are denoted by $\Gamma_{\text{II}} - \Gamma_{\text{VII}}$. Two cubic vertices (2.37) and (2.38) appear due to the non-trivial hypermultiplet background Q^+ and the non-local shift (2.26). These vertices involve the background hypermultiplet Q^+ and the Green function $G^{(1,1)}$.

In the supergraphs presented in Figs. 1 and 2 the gauge propagators $G^{(2,2)}$, the hypermultiplet propagators $G^{(1,1)}$, and the Faddeev–Popov ghost propagators $G^{(0,0)}$ are depicted by wavy, solid, and dashed lines, respectively. The bold dotes in Fig. 3 denote the vertices which include the background hypermultiplet Q^+ .

It is known that the theory under consideration is finite at one loop. Therefore, there is no need to renormalize the one-loop subgraphs in the two-loop superdiagrams.

 $\Gamma_{\rm I}$ diagram. The analytic expression corresponding to the diagram of the ' ∞ ' topology $\Gamma_{\rm I}$ presented in Fig. 1 is written as

$$\Gamma_{\rm I} = -2f^2 \operatorname{tr}(t^I t^J t^K t^L) \int d^{14} z \int \prod_{a=1}^4 du_a
\times \left\{ \frac{G_{IJ}^{(2,2)}(z, u_1; z, u_2) G_{KL}^{(2,2)}(z, u_3; z, u_4)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)} + \frac{1}{2} \frac{G_{IJ}^{(2,2)}(z, u_1; z, u_3) G_{KL}^{(2,2)}(z, u_2; z, u_4)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)} \right\}.$$
(4.1)

The expression (4.1) contains two gauge multiplet Green functions $G^{(2,2)}$ in the limit of coincident z points. Each expression for $G^{(2,2)}$ contains a harmonic δ -function, but the first term in (4.1) contains the expressions $(u_1^+u_2^+)$ and $(u_3^+u_4^+)$ in the denominator which produce a singularity. As was demonstrated in [40] it is this term that contains the divergent contribution of the diagram $\Gamma_{\rm I}$. We discuss it in detail in the next Section.

The second term in (4.1) also includes the gauge multiplet Green function in the limit of coincident points, but the denominator in this case does not vanish. Because of this, the second term in eq. (4.1)

does not contribute to the divergent part of the effective action. To demonstrate this we consider the case $Q_A^+ = 0$ and use the "long" form [34] of the Green function $G^{(2,2)}$ (2.31):

$$G_{IJ}^{(2,2)}(1|2) = -(\widehat{\Box}_{1}^{-2})^{KL}(D_{1}^{+})^{4}(D_{2}^{+})^{4}\Big((e^{ib_{1}})^{IK}(e^{ib_{2}})^{JL}\delta^{14}(z_{1}-z_{2})(D_{1}^{--})^{2}\delta^{(2,-2)}(u_{1},u_{2})\Big)$$

$$= -(\widehat{\Box}_{1}^{-2})^{KM}(D_{1}^{+})^{4}\Big[\Big\{(D_{1}^{-})^{4}(u_{1}^{+}u_{2}^{+})^{4} - \Omega_{1}^{--}(u_{1}^{-}u_{2}^{+})(u_{1}^{+}u_{2}^{+})^{3} + \widehat{\Box}_{1}(u_{1}^{-}u_{2}^{+})^{2}(u_{1}^{+}u_{2}^{+})^{2}$$

$$-iF_{1}^{++}(u_{1}^{-}u_{2}^{+})^{3}(u_{1}^{+}u_{2}^{+})\Big\}^{ML}\Big((e^{ib_{1}})^{KI}(e^{ib_{2}})^{LJ}\delta^{14}(z_{1}-z_{2})(D_{1}^{--})^{2}\delta^{(2,-2)}(u_{1},u_{2})\Big)\Big],$$

where $\Omega^{--} \equiv i \nabla^{ab} \nabla_a^- \nabla_b^- - W^{-a} \nabla_a^- + \frac{1}{4} (\nabla_a^- W^{-a})$. Then, due to the identity

$$(u_1^+ u_2^+)^q (D_1^{--})^k \delta^{(2,-2)}(u_1, u_2) = 0, \quad q > k,$$
(4.3)

the non-zero contribution emerges only for q = k, in particular,

$$(u_1^+ u_2^+)^2 (D_1^{--})^2 \delta^{(2,-2)}(u_1, u_2) = 2 \delta^{(0,0)}(u_1, u_2). \tag{4.4}$$

Hence, we need to collect additional D^{--} operators from the d'Alembertian operator in the denominator in the first and second terms of the expression (4.2). For this purpose, we consider the Green function (4.2) in the coincident point $z_2 \to z_1$ limit and collect the fourth power of D_a^- from the inverse $\widehat{\Box}$ operator to annihilate the Grassmann δ -function $\delta^8(\theta_1 - \theta_2)$,

$$\frac{1}{\widehat{\Box}} = \frac{1}{\Box + W^{+a}D_a^- + \dots} \sim \frac{(W^+)^4(D^-)^4}{\Box^5}.$$
 (4.5)

This implies that the second term in (4.1) gives only finite contributions, and so the divergent contributions can come only from the first term in eq. (4.1).

 Γ_{II} diagram. The analytic expression for the two-loop diagram Γ_{II} (of the ' Θ ' topology) presented in Fig. 1 is constructed using the cubic gauge superfield vertex (2.34):

$$\Gamma_{\text{II}} = -\frac{f^2}{6} \int d^{14}z_1 d^{14}z_2 \prod_{a=1}^{6} du_a f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} \\
\times \frac{G_{I_1 I_2}^{(2,2)}(z_1, u_1; z_2, u_4) G_{J_1 J_2}^{(2,2)}(z_1, u_2; z_2, u_5) G_{K_1 K_2}^{(2,2)}(z_1, u_3; z_2, u_6)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+) (u_4^+ u_5^+)(u_5^+ u_6^+)(u_6^+ u_1^+)}.$$
(4.6)

We substitute the explicit expressions for the Green function $G^{(2,2)}$ and integrate by parts with respect to one of the $(D^+)^4$ factors. After this it is necessary to calculate the harmonic integrals over u_4, u_5, u_6 using the corresponding delta-functions which come from the propagators. As a result, we obtain

$$\Gamma_{\text{II}} = -\frac{f^{2}}{6} \int d^{14}z_{1} d^{14}z_{2} \prod_{a=1}^{3} du_{a} \frac{f^{I_{1}J_{1}K_{1}} f^{I_{2}J_{2}K_{2}}}{(u_{1}^{+}u_{2}^{+})^{2} (u_{2}^{+}u_{3}^{+})^{2} (u_{3}^{+}u_{1}^{+})^{2}} (\widehat{\square}^{-1})_{I_{1}I_{2}} \delta^{14}(z_{1} - z_{2}) \times (D_{1}^{+})^{4} \Big((\widehat{\square}^{-1})_{J_{1}J_{2}} (D_{2}^{+})^{4} \delta^{14}(z_{1} - z_{2}) (\widehat{\square}^{-1})_{K_{1}K_{2}} (D_{2}^{+})^{4} \delta^{14}(z_{1} - z_{2}) \Big).$$
(4.7)

After integrating over θ_2 using the Grassmann delta-function we are left with the coincident $\theta_2 \to \theta_1$ limit in the two remaining delta-functions. As in the previous case, we need to annihilate these Grassmann delta-functions in the coincident θ -point limit, acting on each of them by $(D^{\pm})^8$ -factors. There

are three explicit $(D^+)^4$ factors in the expression (4.7). Therefore, the remaining $(D^-)^4$ factor should be taken from the inverse \Box operators. However, doing so, we will produce an extra operator $(\partial^2)^{-4}$. This implies that the expression for the superdiagram considered can give only finite contribution to the effective action.

 Γ_{III} and Γ_{IV} diagrams. As in the previous consideration for the $Q^+=0$ case [40], the superdiagrams Γ_{III} and Γ_{IV} presented in Fig. 2 exactly cancel each other due to $\mathcal{N}=(0,1)$ supersymmetry (requiring Q^{+A} to be in the adjoint representation) even for the non-trivial Q^+ background⁹. Indeed, the sum of these two superdiagrams is given by the expression

$$\Gamma_{\text{III}} + \Gamma_{\text{IV}} = f^2 \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} \left(1 + (u_1^+ u_2^-)(u_1^- u_2^+) \right)$$

$$\times f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} G_{I_1 I_2}^{(2,2)} (1|2) G_{J_1 J_2}^{(1,1)} (1|2) G_{K_1 K_2}^{(1,1)} (1|2).$$
(4.8)

Applying the identity

$$1 + (u_1^+ u_2^-)(u_1^- u_2^+) = (u_1^+ u_2^+)(u_1^- u_2^-)$$
(4.9)

we cast the contribution (4.8) in the form

$$\Gamma_{\text{III}} + \Gamma_{\text{IV}} = f^2 \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} (u_1^+ u_2^+) (u_1^- u_2^-)$$

$$\times f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} G_{I_1 I_2}^{(2,2)} (1|2) G_{J_1 J_2}^{(1,1)} (1|2) G_{K_1 K_2}^{(1,1)} (1|2) = 0.$$
(4.10)

 $\Gamma_{\rm V}$, $\Gamma_{\rm VI}$ and $\Gamma_{\rm VII}$ diagrams. Now, let us consider the additional superdiagrams $\Gamma_{\rm V}$, $\Gamma_{\rm VI}$ and $\Gamma_{\rm VII}$ in Fig. 3. These diagrams are the modification of the divergent diagram $\Gamma_{\rm III}$ in Fig. 2, where the local cubic vertex (2.33) is replaced by the non-local shifted vertices (2.37) and (2.38). The diagrams $\Gamma_{\rm III}$ and $\Gamma_{\rm IV}$ are divergent but cancel each other due to $\mathcal{N}=(0,1)$ supersymmetry, see eq. (4.10). Hence, the additional diagrams in Fig.3, which appear after the non-local change of variables (2.26), diverge. Since the substitution (2.26) involves only the quantum hypermultiplet and does not act on the ghost superfields, the additional diagrams $\Gamma_{\rm V}-\Gamma_{\rm VII}$ cannot be canceled.

We have to emphasize that we collected only divergent contributions in the mixed gauge-hypermultiplet sector, such as $F^{++}Q^{+A}Q_A^+$. The non-local vertices (2.37) and (2.38) include the background hypermultiplet Q_A^+ , which is depicted by the bold dots in Fig.3 for diagrams $\Gamma_V - \Gamma_{VII}$. However, the diagram Γ_{VI} contains at least four background hypermultiplets and so is beyond our consideration. The remaining diagrams Γ_V and Γ_{VII} include the terms of interest, as implied by the power momentum counting and contribute to the $F^{++}Q^{+A}Q_A^+$ part of the effective action.

5 Calculation of the two-loop divergences

As explained above, we are only interested in the divergent contributions to the effective action of the form $F^{++}Q^{+A}Q_A^+$, which appear solely from the superdiagrams $\Gamma_{\rm I}$, $\Gamma_{\rm V}$ and $\Gamma_{\rm VII}$. In this section we consider the calculation of the $1/\varepsilon^2$ divergences coming from these diagrams in some detail.

⁹See ref. [56], where a similar situation was also observed in 4D, $\mathcal{N}=4$ SYM theory.

Divergent part of $\Gamma_{\rm I}$ diagram. The analytic expression for the superdiagram $\Gamma_{\rm I}$ presented in Fig. 1 is constructed similarly to the case $Q^+ = 0$ considered in [40]. The new peculiarity is that the vector supermultiplet propagator (2.28) now includes a non-trivial Q^+ dependence,

$$\Gamma_{\text{I, div}} = -2f^2 \operatorname{tr}(t^I t^J t^K t^L) \int d^{14}z \int \prod_{a=1}^4 du_a \frac{G_{IJ}^{(2,2)}(z, u_1; z, u_2) G_{KL}^{(2,2)}(z, u_3; z, u_4)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)}.$$
 (5.1)

To calculate the divergent contribution to the effective action, we substitute the expression (2.31) for the vector multiplet Green function into the numerator in (5.1). Then, using the identity

$$\left[\frac{1}{\bigcap_{1}^{2}}, \frac{1}{(u_{1}^{+}u_{2}^{+})}\right] = \frac{1}{\bigcap_{1}} \left(\frac{(u_{1}^{-}u_{2}^{+})}{(u_{1}^{+}u_{2}^{+})^{2}} F^{++}\right) \frac{1}{\bigcap_{1}^{2}} + \frac{1}{\bigcap_{1}^{2}} \left(\frac{(u_{1}^{-}u_{2}^{+})}{(u_{1}^{+}u_{2}^{+})^{2}} F^{++}\right) \frac{1}{\bigcap_{1}} \tag{5.2}$$

and omitting terms that give finite contributions the expression

$$\frac{1}{(u_1^+ u_2^+)} G_{IJ}^{(2,2)}(1|2) = \frac{1}{(u_1^+ u_2^+)} \left(-\frac{2}{\widehat{\Box}_1} \delta_{\mathcal{A}}^{(2,2)}(1|2) + \frac{4}{\widehat{\Box}_1^2} Q_1^{+A} G^{(1,1)}(1|2) Q_{2A}^+ + \dots \right)_{IJ}$$
(5.3)

can be calculated in the limit of coincident points. For this purpose we note that in the leading order in $1/\varepsilon$, where $\varepsilon \equiv 6 - D \rightarrow 0$, in the expression considered one can make the replacement

$$\frac{1}{\Box^3} \delta^6(x - x') \Big|_{x' = x} \to \frac{i}{(4\pi)^3 \varepsilon} \tag{5.4}$$

which is valid in the minimal subtraction scheme. Thus, we obtain

$$\frac{1}{(u_1^+ u_2^+)} G^{(2,2)}(z_1, u_1 | z_2, u_2) \Big|_{z_2 \to z_1} = \frac{2i}{(4\pi)^3} \frac{1}{\varepsilon} \left(F_{1,\tau}^{++} (u_1^- u_2^+) (u_1^+ u_2^+)^2 (D_1^{--})^2 \delta^{(2,-2)}(u_1, u_2) \right) \\
+ 2 Q_{1,\tau}^{+A} Q_{2A,\tau}^+ + \text{finite contributions}, \quad \varepsilon \to 0, \quad (5.5)$$

where τ -representation was introduced in Eq. (2.29).

To reproduce the result of [40], one can put $Q^+=0$ in Eq. (5.5) and substitute it to Eq. (5.1). Then we obtain

$$\Gamma_{\text{I,div}}^{FF} = \frac{f^2(C_2)^2}{8(2\pi)^6 \varepsilon^2} \text{tr} \int d^{14}z \, du_1 du_2 \, F_{1,\tau}^{++} F_{2,\tau}^{++} \frac{1}{(u_1^+ u_2^+)^2}. \tag{5.6}$$

Using the identity

$$F_{1,\tau}^{++} = \frac{1}{2}D_1^{++}D_1^{--}F_{1,\tau}^{++},$$

and integrating by parts with respect to the derivative D_1^{++} , this expression can be rewritten as

$$\operatorname{tr} \int d^{14}z \, du_1 du_2 \, F_{1,\tau}^{++} F_{2,\tau}^{++} \frac{1}{(u_1^+ u_2^+)^2} = -\frac{1}{2} \operatorname{tr} \int d^{14}z \, du_1 du_2 \, D_1^{--} F_{1,\tau}^{++} F_{2,\tau}^{++} D_1^{--} \delta^{2,-2}(u_1, u_2)$$

$$= \frac{1}{2} \operatorname{tr} \int d^{14}z \, du \, (D^{--})^2 F_{\tau}^{++} F_{\tau}^{++} = \operatorname{tr} \int d\zeta^{(-4)} \, du \, F^{++} \, \widehat{\Box} \, F^{++}. \tag{5.7}$$

Thus, the leading $(1/\varepsilon^2)$ divergent part of the two-loop diagram $\Gamma_{\rm I}$ depicted in Fig.1 depending only on the background gauge superfield has the form

$$\Gamma_{\text{I,div}}^{FF} = \frac{f^2(C_2)^2}{8(2\pi)^6 \varepsilon^2} \text{tr} \int d\zeta^{(-4)} \, du \, F^{++} \, \widehat{\Box} \, F^{++}, \tag{5.8}$$

and determines the constant c_1 in Eq.(3.17).

In the general $Q^+ \neq 0$ case, we substitute the whole expression (5.5) into eq. (5.1) and obtain the leading $(1/\varepsilon^2)$ divergent contribution coming from this diagram in the form

$$\Gamma_{\text{I, div}}^{\text{FQQ}} = -\frac{if^2(C_2)^2}{8(2\pi)^6 \,\varepsilon^2} \text{tr} \int d^{14}z \, \frac{du_1 du_2 du_3}{(u_1^+ u_2^+)(u_3^+ u_1^+)} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{3A,\tau}^+] \,. \tag{5.9}$$

The expression (5.9) generalizes the result obtained earlier in [40] for the case $Q^+ = 0$.

Divergent part of Γ_V diagram. The analytic expression for the diagram Γ_V reads

$$\Gamma_{V} = if^{2} f^{I_{1}J_{1}K_{1}} f^{I_{2}J_{2}K_{2}} f^{L_{1}M_{1}N_{1}} f^{L_{2}M_{2}N_{2}} \int \prod_{a=1}^{4} d\zeta_{a}^{(-4)} (Q_{2}^{+A})^{N_{1}} (Q_{4A}^{+})^{N_{2}}$$

$$\times G_{I_{1}L_{1}}^{(1,1)} (1|2) G_{I_{2}L_{2}}^{(1,1)} (3|4) G_{J_{1}J_{2}}^{(2,2)} (1|3) G_{M_{1}M_{2}}^{(2,2)} (2|4) G_{K_{1}K_{2}}^{(1,1)} (1|3).$$

$$(5.10)$$

To single out the divergent contribution in (5.10), we should first substitute the explicit expressions for the Green functions. In the case of the non-trivial hypermultiplet background, the gauge superfield propagator $G^{(2,2)}$ is given by the expression (2.31) involving additional contributions with the hypermultiplet Green function. These terms contain one more d'Alembert operator in the denominator and produce finite contributions to the effective action. Thus, we should consider only the first term in the expression (2.31), which corresponds to the standard propagator for the gauge superfield v^{++} . Then, we integrate over ζ_3 and ζ_4 using the analytic delta-functions $\delta_{\mathcal{A}}^{(2,2)}(1|3)$ and $\delta_{\mathcal{A}}^{(2,2)}(2|4)$ coming from the gauge superfield propagators. After making these steps, we find the following result for the leading divergences

$$\Gamma_{V, \text{div}}^{FQQ} = 4if^2 f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} f^{L_1 M_1 N_1} f^{L_2 M_2 N_2} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} (Q_2^{+A})^{N_1} (Q_{2A}^+)^{N_2}$$

$$\times (\widehat{\Box}_1^{-1})^{J_1 J_2} G_{K_1 K_2}^{(1,1)} (1|2) \Big|_{2=1} (\widehat{\Box}_2^{-1})^{M_1 M_2} G_{I_1 L_1}^{(1,1)} (1|2) G_{I_2 L_2}^{(1,1)} (1|2).$$
(5.11)

Next, we substitute the hypermultiplet Green function (2.28), restore the full superspace measure using $(D^+)^4$ factors from $(G^{(1,1)}_{(1|2)})^{I_2L_2}$, and integrate with the Grassmann delta-function $\delta^8(\theta_1-\theta_2)$ coming from this Green function. Taking the coincident points limit and using the identity $(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)\big|_{\theta_2=\theta_1}=(u_1^+u_2^+)^4$, we obtain

$$\Gamma_{V, \text{div}}^{FQQ} = 8if^{2} f^{I_{1}JK_{1}} f^{I_{2}JK_{2}} f^{I_{1}MN_{1}} f^{I_{2}MN_{2}} \int d^{14}z_{1} d^{6}x_{2} du_{1} du_{2} \frac{(Q_{2,\tau}^{+A})^{N_{1}} (Q_{2A,\tau}^{+})^{N_{2}}}{(u_{1}^{+}u_{2}^{+})^{2}} \times f^{K_{1}K_{3}K_{2}} (F_{1,\tau}^{++})^{K_{3}} \frac{1}{\Box_{1}^{3}} \delta^{6}(x_{1} - x_{2}) \Big|_{x_{2} = x_{1}} \frac{1}{\Box_{2}\Box_{1}} \delta^{6}(x_{1} - x_{2}) \frac{1}{\Box_{1}} \delta^{6}(x_{1} - x_{2}) . \quad (5.12)$$

After calculating the (Euclidean) momentum integrals with the help of the identity

$$\int \frac{d^D k}{(2\pi)^D} \frac{d^D l}{(2\pi)^D} \frac{1}{k^6 l^4 (k+l)^2} \to \frac{1}{(4\pi)^6} \cdot \frac{1}{2\varepsilon^2} + O\left(\frac{1}{\varepsilon}\right)$$
(5.13)

the result for the leading divergences can be written as

$$\Gamma_{V, \text{div}}^{FQQ} = \frac{2if^2(C_2)^2}{(4\pi)^6 \varepsilon^2} \text{tr} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{2A,\tau}^+], \qquad (5.14)$$

where we have also used the relation $f^{ABC}f^{BKL}f^{CLN} = -\frac{1}{2}C_2f^{AKN}$.

Divergent part of Γ_{VII} diagram. The analytic expression for the diagram Γ_{VII} depicted on Fig.3 has the form

$$\Gamma_{\text{VII}} = -f^2 f^{I_1 J_1 K_1} f^{I_2 J_2 K_2} f^{L_1 M_1 N_1} f^{L_2 M_2 N_2} \int d^{14} z_1 \prod_{a=1}^{3} du_a \prod_{b=4}^{6} d\zeta_b^{(-4)} \frac{(Q_5^{+A})^{N_1} (Q_{6A}^+)^{N_2}}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_1^+)} \times G_{I_1 J_2}^{(2,2)}(1|4) G_{J_1 M_1}^{(2,2)}(2|5) G_{K_1 M_2}^{(2,2)}(3|6) G_{I_2 L_1}^{(1,1)}(4|5) G_{K_2 L_2}^{(1,1)}(4|6).$$
(5.15)

We are interested in its divergent part of the form $F^{++}[Q^{+A}, Q_A^+]$. The expression (5.15) already contains the second power of the background hypermultiplet. Hence, we take only the first term from the gauge superfield propagator (2.31). Then we take off the $(D^+)^4$ factors from each Green function $G^{(2,2)}$ to restore the full superspace measures over z_4, z_5 and z_6 . We also integrate over the Grassmann variables θ_4, θ_5 and θ_6 and the corresponding harmonic variables u_4, u_5 and u_6 using the δ -functions coming from the Green functions $G^{(2,2)}$. Then, taking the hypermultiplet Green function $G^{(1,1)}$ in the limit of coincident θ 's, we obtain

$$\Gamma_{\text{VII, div}}^{FQQ} = 8f^{2}f^{I_{1}J_{1}K_{1}}f^{I_{1}I_{2}K_{2}}f^{L_{1}J_{1}N_{1}}f^{L_{2}K_{1}N_{2}} \int d^{14}z_{1}d^{6}x_{4}d^{6}x_{5}d^{6}x_{6}\frac{du_{1}du_{2}du_{3}}{(u_{2}^{+}u_{3}^{+})} \\
\times (Q^{+A})^{N_{1}}(x_{5},\theta_{1},u_{2})(Q_{A}^{+})^{N_{2}}(x_{6},\theta_{1},u_{3}) \\
\times \frac{1}{(u_{1}^{+}u_{2}^{+})} \frac{1}{\widehat{\Box}_{I_{2}L_{1}}}(u_{1}^{+}u_{2}^{+})\delta^{6}(x_{4}-x_{5})\frac{1}{(u_{1}^{+}u_{3}^{+})} \frac{1}{\widehat{\Box}_{K_{2}L_{2}}}(u_{1}^{+}u_{3}^{+})(x_{4}-x_{6}) \\
\times \frac{1}{\Box_{1}}\delta^{6}(x_{1}-x_{4})\frac{1}{\Box_{1}}\delta^{6}(x_{1}-x_{5})\frac{1}{\Box_{1}}\delta^{6}(x_{1}-x_{6}). \tag{5.16}$$

The operator $\widehat{\Box}$ (given by eq. (2.24)) does not commute with harmonic factors due to the harmonic derivative ∇^{--} . In particular, using the identity

$$\left[\frac{1}{\widehat{\Box}}, (u_1^+ u_2^+)\right] = -i(u_1^- u_2^+) \frac{1}{\widehat{\Box}} F^{++} \frac{1}{\widehat{\Box}}, \tag{5.17}$$

we obtain for the divergent contribution

$$\Gamma_{\text{VII, div}}^{FQQ} = 8f^{2}f^{I_{1}J_{1}K_{1}}f^{I_{1}I_{2}K_{2}}f^{L_{1}J_{1}N_{1}}f^{L_{2}K_{1}N_{2}} \int d^{14}z_{1}d^{6}x_{4}d^{6}x_{5}d^{6}x_{6}\frac{du_{1}du_{2}du_{3}}{(u_{2}^{+}u_{3}^{+})} \\
\times (Q^{+A})^{N_{1}}(x_{5}, \theta_{1}, u_{2})(Q_{A}^{+})^{N_{2}}(x_{6}, \theta_{1}, u_{3})(F^{++})^{N_{3}}(x_{4}, \theta_{1}, u_{1}) \\
\times \left(\frac{(u_{1}^{-}u_{2}^{+})}{(u_{1}^{+}u_{2}^{+})}\delta^{K_{2}L_{2}}f^{I_{2}N_{3}L_{1}}\frac{1}{\Box_{4}^{2}}\delta^{6}(x_{4} - x_{5})\frac{1}{\Box_{4}}\delta^{6}(x_{4} - x_{6}) \\
+ \frac{(u_{1}^{-}u_{3}^{+})}{(u_{1}^{+}u_{3}^{+})}\delta^{I_{2}L_{1}}f^{K_{2}N_{3}L_{2}}\frac{1}{\Box_{4}^{2}}\delta^{6}(x_{4} - x_{6})\frac{1}{\Box_{4}}\delta^{6}(x_{4} - x_{5})\right) \\
\times \frac{1}{\Box_{1}}\delta^{6}(x_{1} - x_{4})\frac{1}{\Box_{1}}\delta^{6}(x_{1} - x_{5})\frac{1}{\Box_{1}}\delta^{6}(x_{1} - x_{6}). \tag{5.18}$$

Then we replace the space-time variables x_5 by x_6 and the harmonic ones u_2 by u_3 in the second term

inside the bracket. As a result, the expression considered can be rewritten as

$$\Gamma_{\text{VII, div}}^{FQQ} = 16f^{2}f^{I_{1}J_{1}K_{1}}f^{I_{1}I_{2}K_{2}}f^{L_{1}J_{1}N_{1}}f^{K_{2}K_{1}N_{2}}f^{I_{2}N_{3}L_{1}} \int d^{14}z_{1}d^{6}x_{4}d^{6}x_{5}d^{6}x_{6}du_{1}du_{2}du_{3}$$

$$\times \frac{(u_{1}^{-}u_{2}^{+})}{(u_{1}^{+}u_{2}^{+})(u_{2}^{+}u_{3}^{+})}(Q^{+A})^{N_{1}}(x_{5},\theta_{1},u_{2})(Q_{A}^{+})^{N_{2}}(x_{6},\theta_{1},u_{3})(F^{++})^{N_{3}}(x_{4},\theta_{1},u_{1})$$

$$\times \frac{1}{\Box_{4}^{2}}\delta^{6}(x_{4}-x_{5})\frac{1}{\Box_{4}}\delta^{6}(x_{4}-x_{6})\frac{1}{\Box_{1}}\delta^{6}(x_{1}-x_{4})\frac{1}{\Box_{1}}\delta^{6}(x_{1}-x_{5})\frac{1}{\Box_{1}}\delta^{6}(x_{1}-x_{6}). (5.19)$$

Calculating the momentum integral employing (5.13) and taking the trace over the gauge algebra indices, we obtain

$$\Gamma_{\text{VII, div}}^{\text{FQQ}} = -\frac{4if^2(C_2)^2}{(4\pi)^6 \varepsilon^2} \text{tr} \int d^{14}z \frac{du_1 du_2 du_3(u_1^- u_2^+)}{(u_1^+ u_2^+)(u_2^+ u_3^+)} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{3A,\tau}^+].$$
 (5.20)

The final result for divergent contributions. Summing up the divergent contributions (5.9), (5.14) and (5.20) and singling out the structure we are interested in, we obtain

$$\begin{split} \Gamma^{\text{FQQ}}_{\text{I, div}} + \Gamma^{\text{FQQ}}_{\text{V, div}} + \Gamma^{\text{FQQ}}_{\text{VII, div}} &= -\frac{i \text{f}^2(C_2)^2}{8(2\pi)^6 \, \varepsilon^2} \text{tr} \, \int d^{14}z \, \frac{du_1 du_2 du_3}{(u_1^+ u_2^+)(u_3^+ u_1^+)} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{3A,\tau}^+] \\ &+ \frac{2i \text{f}^2(C_2)^2}{(4\pi)^6 \varepsilon^2} \text{tr} \, \int d^{14}z \, \frac{du_1 du_2}{(u_1^+ u_2^+)^2} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{2A,\tau}^+] \\ &- \frac{4i \text{f}^2(C_2)^2}{(4\pi)^6 \varepsilon^2} \text{tr} \, \int d^{14}z \, \frac{du_1 du_2 du_3(u_1^- u_2^+)}{(u_1^+ u_2^+)(u_2^+ u_3^+)} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{3A,\tau}^+] \ \, (5.21) \end{split}$$

This expression agrees with the power counting analysis performed above. We see that, as expected, the divergent contributions to the effective action are non-local in harmonics, but remain local with respect to the space-time and Grassmann coordinates.

On-shell condition. In Appendix A the expression (5.21) is rewritten in a different form that is local in the harmonic variables. However, the result involves the superfield Q^{-A} defined by the equation $\nabla^{++}Q_A^- = Q_A^+$. Nevertheless, all terms containing Q_A^- vanish on shell. Then it becomes possible to determine the coefficient c_2 in (3.15), and also to fix the last coefficient c_3 in (3.15) using eq. (3.16):

$$c_2 = -\frac{f^2(C_2)^2}{16(2\pi)^6 \varepsilon^2}, \qquad c_3 = \frac{f^2(C_2)^2}{16(2\pi)^6 \varepsilon^2}, \qquad \varepsilon \to 0.$$
 (5.22)

The final expression for the divergent part of the two-loop effective action in the model (2.11) reads

$$\Gamma_{\text{div}}^{(2)} = \frac{f^2(C_2)^2}{8(2\pi)^6 \varepsilon^2} \text{tr} \int d\zeta^{(-4)} \left(F^{++} \widehat{\Box} F^{++} - \frac{i}{2} F^{++} \widehat{\Box} [Q^{+A}, Q_A^+] \right)
+ \frac{1}{2} [Q^{+A}, Q_A^+] \widehat{\Box} [Q^{+B}, Q_B^+] + \text{terms proportional to e.o.m. for } Q^+.$$
(5.23)

The expression (5.23) vanishes for the background superfields satisfying the classical equations of motion (2.14) and so generalizes the result obtained earlier in ref. [40].

6 Summary

In the present paper we have developed a manifestly covariant and $\mathcal{N}=(1,0)$ supersymmetric method for studying the effective action of the six-dimensional $\mathcal{N}=(1,1)$ SYM theory formulated in $\mathcal{N}=(1,0)$ harmonic superspace. In such an approach the classical action (2.11) is that of $\mathcal{N}=(1,0)$ SYM theory minimally interacting with the hypermultiplet in the adjoint representation of the gauge group. The corresponding classical action is invariant under $\mathcal{N}=(1,0)$ supersymmetry and also possesses a hidden on-shell $\mathcal{N}=(0,1)$ supersymmetry. Hence, the action (2.11) really describes the six-dimensional $\mathcal{N}=(1,1)$ SYM theory.

In the previous papers [23,55] we have shown that in the minimal gauge (a supersymmetric version of the Feynman gauge) this theory is finite off shell in the one-loop approximation. The two-loop $\frac{1}{\varepsilon^2}$ divergent contribution in the gauge-multiplet sector has been calculated in ref. [40]. In the present paper we have analyzed the general structure of the two-loop divergences in the theory under consideration and shown that, generically, the corresponding counterterms, being space-time local, can be non-local in harmonics. Extending the result of ref. [40], we have performed the explicit calculation of the leading two-loop divergences in the mixed gauge multiplet-hypermultiplet sector. Note that the calculation of the divergences containing the background hypermultiplet has required considerable efforts with the use of a large arsenal of various harmonic identities.

Throughout the paper we performed the calculation within the background field method in the harmonic superspace for arbitrary background superfields V^{++} and Q^{+A} . Due to the presence of the non-trivial hypermultiplet background the kinetic term for quantum superfields contains the mixed term that is removed by the non-local shift (2.26) of the quantum hypermultiplet. This leads to the appearance of new cubic vertices (2.37) and (2.38) and to the new two-loop diagrams presented in Fig. 3, some of which contribute to the divergent part of the effective action. Also, the two-loop divergences come from the supergraph $\Gamma_{\rm I}$ considered earlier in [40] under the choice $Q^+=0$. In this paper we carried out a direct calculation of the two-loop $\frac{1}{\varepsilon^2}$ divergences and showed that the final result, with contributions from both the gauge multiplet and hypermultiplet, exactly coincides with the one obtained in ref. [40], where the dependence on the hypermultiplet was restored through an indirect analysis.

It is worth noting that, in addition to the leading $\frac{1}{\varepsilon^2}$ divergences, one can also expect the two-loop subleading $\frac{1}{\varepsilon}$ divergences. If the background hypermultiplet is switched off, the expressions for them are proportional to F^{++} and so naturally vanishes on shell due to the equations of motion for the vector multiplet. However, the calculation of the $\frac{1}{\varepsilon}$ divergences is much more technically involved as compared to the $\frac{1}{\varepsilon^2}$ terms (especially in the presence of the background hypermultiplet) and the further work is needed to study the structure of the background-dependent harmonic supergraphs. It would be very interesting (and instructive from the point of view of the harmonic supergraph technique) to derive the total set of the $\frac{1}{\varepsilon}$ divergences by a direct calculation. We are going to study all the relevant aspects in the forthcoming works. However, we wish here to emphasize that all calculations can be carried out using the same techniques. For illustration, in Appendix B we show how the harmonic technique works for calculating the two-loop $\frac{1}{\varepsilon}$ divergences of the supergraph presented in Fig. 3.

As we saw, the one- and two-loop divergences in the theory under consideration are proportional to classical equations of motion. This implies the one- and two-loop on-shell finiteness of the theory under consideration. One can expect that, starting from the three-loop approximation, the structure of divergences will change and the results will not be proportional to the standard classical equations of motion. At present, the off-shell structure of the three- and higher-loop divergences of the effective

action is unknown¹⁰. We plan to address this problem in the future papers.

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A Harmonic locality

The contributions (3.4), (3.7), (3.8) and (3.9) to the two-loop effective action are non-local in the harmonic variables. This does not contradict the non-renormalization theorem which asserts only the locality of the divergent part of effective action in the Grassmann and space-time coordinates. Moreover, all the contributions just mentioned can be related to local expressions such as (3.5), (3.10). Here we are going to demonstrate this.

Let us begin our study with

$$\mathcal{G}_{1}^{(2)} = \operatorname{tr} \int d^{14}z \frac{du_{1}du_{2}}{(u_{1}^{+}u_{2}^{+})^{2}} F_{1,\tau}^{++} F_{2,\tau}^{++}, \qquad (A.1)$$

where $F_{1,\tau}^{++}=F_{\tau}^{++}(z,u_1), \ [F^{++}]=[m^2]$ and τ -representation was introduced in Eq. (2.29). Using the identity

$$\frac{1}{(u_1^+ u_2^+)^2} = D_1^{++} \frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} + \frac{1}{2} (D_1^{--})^2 \delta^{(2,-2)}(u_1, u_2)$$
(A.2)

this expression can be rewritten as

$$\mathcal{G}_{1}^{(2)} = \operatorname{tr} \int d^{14}z du_{1} du_{2} \left(D_{1}^{++} \frac{(u_{1}^{-} u_{2}^{+})}{(u_{1}^{+} u_{2}^{+})^{3}} + \frac{1}{2} (D_{1}^{--})^{2} \delta^{(2,-2)}(u_{1}, u_{2}) \right) F_{1,\tau}^{++} F_{2,\tau}^{++} . \tag{A.3}$$

The first term in the bracket is a total D^{++} harmonic derivative due to the property $\nabla^{++}F^{++} = 0$. In the second term we integrate by parts with respect to the derivative D_1^{--} and then use the harmonic delta function to integrate over u_2 . We obtain

$$\mathcal{G}_{1}^{(2)} = \frac{1}{2} \operatorname{tr} \int d^{14}z \, du \, F_{\tau}^{++} (D^{--})^{2} F_{\tau}^{++} = \operatorname{tr} \int d\zeta^{(-4)} \, F^{++} \, \widehat{\Box} \, F^{++} \,. \tag{A.4}$$

We see that this expression is invariant under the gauge transformation (2.17) (the initial expression which is non-local in the harmonic variables is evidently gauge invariant too).

Now let us consider the expression (3.8),

$$\mathcal{G}_{2}^{(2)} = \operatorname{tr} \int d^{14}z \frac{du_{1}du_{2}}{(u_{1}^{+}u_{2}^{+})^{2}} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{2A,\tau}^{+}]
= \operatorname{tr} \int d^{14}z du_{1}du_{2} \left(D_{1}^{++} \frac{(u_{1}^{-}u_{2}^{+})}{(u_{1}^{+}u_{2}^{+})^{3}} + \frac{1}{2} (D_{1}^{--})^{2} \delta^{(2,-2)}(u_{1}, u_{2}) \right) F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{2A,\tau}^{+}]. \quad (A.5)$$

¹⁰The existence of the proper dimension single-trace operators with the manifest off-shell $\mathcal{N}=(1,0)$ supersymmetry and hidden on-shell $\mathcal{N}=(0,1)$ supersymmetry was shown in [11] (see also [12]). These operators are non-vanishing on shell and so can carry actual divergences.

The first term in the bracket is the total harmonic derivative D^{++} due to the property $\nabla^{++}F^{++} = 0$. In the second term we integrate twice by parts with respect to the harmonic derivative D^{--} ,

$$\mathcal{G}_{2}^{(2)} = \frac{1}{2} \operatorname{tr} \int d^{14}z [Q^{+A}, Q_{A}^{+}] (\nabla^{--})^{2} F^{++} = \operatorname{tr} \int d\zeta^{(-4)} [Q^{+A}, Q_{A}^{+}] \stackrel{\frown}{\Box} F^{++}. \tag{A.6}$$

The expression (3.9) can equivalently be presented in the form

$$\mathcal{G}_{3}^{(2)} = \operatorname{tr} \int d^{14}z \frac{du_{1}du_{2}du_{3}}{(u_{1}^{+}u_{2}^{+})(u_{1}^{+}u_{3}^{+})} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{3A,\tau}^{+}] = \operatorname{tr} \int d^{14}z du \, F^{++}[Q^{-A}, Q_{A}^{-}], \tag{A.7}$$

where the superfield Q_A^- is defined by the equation $\nabla^{++}Q_A^- = Q_A^+$. Its solution in the τ -representation is written as

$$Q_{A,\tau}^{-}(z,u_1) = \int du_2 \frac{Q_{A,\tau}^{+}(z,u_2)}{(u_1^{+}u_2^{+})}.$$
(A.8)

Then, using the identity

$$Q_A^- = \nabla^{--}Q_A^+ - \nabla^{++}\nabla^{--}Q_A^-, \tag{A.9}$$

we rewrite the expression (A.7) in the form

$$\begin{split} \mathcal{G}_{3}^{(2)} &= \operatorname{tr} \, \int d^{14}z \, du \, \Big(F^{++} [\nabla^{--}Q^{+A}, Q_{A}^{-}] - F^{++} [\nabla^{++}\nabla^{--}Q^{-A}, Q_{A}^{-}] \Big) \\ &= -\operatorname{tr} \, \int d^{14}z \, du \, [Q^{+A}, Q_{A}^{-}] \nabla^{--}F^{++} \\ &= -\operatorname{tr} \, \int d^{14}z \, du \, \Big([Q^{+A}, \nabla^{--}Q_{A}^{+}] - [Q^{+A}, \nabla^{++}\nabla^{--}Q_{A}^{-}] \Big) \nabla^{--}F^{++} \\ &= -\operatorname{tr} \, \int d^{14}z \, du \, \Big(\frac{1}{2}\nabla^{--}[Q^{+A}, Q_{A}^{+}] - [Q^{+A}, \nabla^{++}\nabla^{--}Q_{A}^{-}] \Big) \nabla^{--}F^{++} \\ &= \frac{1}{2}\operatorname{tr} \, \int d^{14}z \, du \, F^{++} (\nabla^{--})^{2} [Q^{+A}, Q_{A}^{+}] + \operatorname{tr} \, \int d^{14}z \, du \, [Q^{+A}, \nabla^{++}\nabla^{--}Q_{A}^{-}] \nabla^{--}F^{++} \\ &= \operatorname{tr} \, \int d\zeta^{(-4)} \, F^{++} \, \widehat{\Box} \, [Q^{+A}, Q_{A}^{+}] + \operatorname{tr} \, \int d^{14}z \, du \, [Q^{+A}, \nabla^{++}\nabla^{--}Q_{A}^{-}] \nabla^{--}F^{++} \, . \end{split} \tag{A.10}$$

In the first line we integrated by parts with respect to the harmonic derivatives ∇^{--} and ∇^{++} in the first and second terms, respectively. The first term in the last line of eq. (A.10) coincides with (3.10). The second term vanishes if the background hypermultiplet satisfies the classical equation of motion (2.14) (this is because $\nabla^{++}\nabla^{--}Q_A^-=0$ on shell, see ref. [45] for details).

Let us consider the last term (3.9)

$$\mathcal{G}_{4}^{(2)} = \operatorname{tr} \int d^{14}z \frac{du_{1}du_{2}du_{3} (u_{1}^{-}u_{2}^{+})}{(u_{1}^{+}u_{2}^{+})(u_{2}^{+}u_{3}^{+})} F_{1,\tau}^{++}[Q_{2,\tau}^{+A}, Q_{3A,\tau}^{+}]. \tag{A.11}$$

Using eq. (A.8) and the identity

$$\frac{(u_1^+ u_2^+)}{(u_1^+ u_2^+)} = \frac{1}{2} D_1^{++} \frac{(u_1^+ u_2^+)^2}{(u_1^+ u_2^+)^2} - \frac{1}{2} D_1^{--} \delta^{(0,0)}(u_1, u_2) , \qquad (A.12)$$

we rewrite this expression, modulo a complete harmonic derivative, as

$$\begin{split} \mathcal{G}_{4}^{(2)} &= -\frac{1}{2} \mathrm{tr} \, \int d^{14}z \, du_{1} du_{2} \, D_{1}^{--} \delta^{(0,0)}(u_{1},u_{2}) F_{1,\tau}^{++} [Q_{2,\tau}^{+A},Q_{2A,\tau}^{-}] \\ &= -\frac{1}{4} \mathrm{tr} \, \int d^{14}z \, du_{1} du_{2} \, D_{1}^{--} \delta^{(0,0)}(u_{1},u_{2}) F_{1,\tau}^{++} D_{2}^{--} [Q_{2,\tau}^{+A},Q_{2A,\tau}^{+}] \\ &\quad + \frac{1}{2} \mathrm{tr} \, \int d^{14}z \, du_{1} du_{2} \, D_{1}^{--} \delta^{(0,0)}(u_{1},u_{2}) F_{1,\tau}^{++} D_{2}^{--} [Q_{2,\tau}^{+A},D_{2}^{++} D_{2}^{--} Q_{2A,\tau}^{-}] \\ &= \frac{1}{4} \mathrm{tr} \, \int d^{14}z \, du \, \nabla^{--} F^{++} \nabla^{--} [Q^{+A},Q_{A}^{+}] \\ &\quad - \frac{1}{2} \mathrm{tr} \, \int d^{14}z \, du \, \nabla^{--} F^{++} \nabla^{--} [Q^{+A},\nabla^{++} \nabla^{--} Q_{A}^{-}] \\ &= -\frac{1}{2} \mathrm{tr} \, \int d\zeta^{(-4)} \, F^{++} \, \widehat{\Box} \, [Q^{+A},Q_{A}^{+}] \\ &\quad - \frac{1}{2} \mathrm{tr} \, \int d^{14}z \, du \, \nabla^{--} F^{++} \nabla^{--} [Q^{+A},\nabla^{++} \nabla^{--} Q_{A}^{-}] \, . \end{split} \tag{A.13}$$

Like in the previous case, the second term in the last line of eq.(A.13) vanishes on shell because of the equality $\nabla^{++}\nabla^{--}Q_A^-=0$.

Acting similarly, one can show that the non-local contribution (3.13) coincides with (3.12) up to terms proportional to the background hypermultiplet equation of motion.

Finally, we note that all non-local in the harmonics expressions (3.4), (3.7), (3.8) and (3.9) are gauge invariant. In the λ -representation, this directly follows from the above analysis.

B Calculation of $\frac{1}{\epsilon}$ divergences: an example

In this Appendix we illustrate how the two-loop $\frac{1}{\varepsilon}$ divergences can in principle be calculated in the framework of the harmonic supergraph techniques. As an example, the consideration will be carried out for the supergraph presented in Fig. 3.

It is known that the theory under consideration does not have one-loop divergences off shell at least in the Fermi-Feynman gauge [43], hence no sub-divergences in the two-loop supergraphs can appear. For this reason, the divergent contributions proportional to $\frac{1}{\varepsilon}$ can come out only as the cross terms, with the divergent contributions multiplied by the finite ones. However, all such contributions also vanish on shell. To demonstrate this, we consider, for simplicity, the trivial hypermultiplet background $Q^+=0$. In this case all additional diagrams with "non-local" vertices in Fig. 3, including the diagram $\Gamma_{\rm V}$ which contains the divergent contribution, immediately vanish. Consequently, all possible divergent contributions can come only from the diagram $\Gamma_{\rm I}$.

Let us denote $g^{(1,1)}(z; u_1, u_2)$ the finite contribution of the vector multiplet Green function divided by $(u_1^+u_2^+)$ (5.5) in the $z_2 \to z_1$ limit,

$$\frac{G^{(2,2)}(z_1, u_1|z_2, u_2)}{(u_1^+ u_2^+)} \Big|_{z_2 \to z_1} = \frac{a}{\varepsilon} F_1^{++} (u_1^- u_2^+)^2 \delta^{(1,-1)}(u_1, u_2) + g^{(1,1)}(z; u_1, u_2|V^{++}), \quad \varepsilon \to 0. \quad (B.1)$$

Here, $a = -\frac{4i}{(4\pi)^3}$. The function $g^{(1,1)}(z; u_1, u_2|V^{++})$ is an analytic superfield depending on the background vector multiplet V^{++} . By definition, this function is a gauge invariant polynomial of gauge superfield strength, such that it is local in the z space and can be non-local with respect to harmonic

variables. It contains all the information about the structure of the finite part of the gauge superfield Green function $G^{(2,2)}(z_1,u_1;z_2,u_2)$ in the $z_2 \to z_1$ coincident limit. The explicit form of this function does not matter and will not be discussed in what follows.

As we discussed earlier, the divergent contribution of the diagram $\Gamma_{\rm I}$ arises from the expression (we omitted the gauge group indices and relevant coefficients)

$$\Gamma_{\text{div}}^{(2)}[V^{++}] = f^2 \int d^{14}z \prod_{a=1}^4 u_a \frac{G^{(2,2)}(z, u_1; z, u_2) G^{(2,2)}(z, u_3; z, u_4)}{(u_1^+ u_2^+)(u_2^+ u_3^+)(u_3^+ u_4^+)(u_4^+ u_1^+)} \bigg|_{\text{div}}.$$
(B.2)

Substituting (B.1) in the last expression and integrating with the harmonic delta-functions, we obtain

$$\Gamma_{\text{div}}^{(2)}[V^{++}] = -\frac{a^2 f^2}{\varepsilon^2} \text{tr} \int d\zeta^{(-4)} F^{++} \widehat{\Box} F^{++}
+ \frac{2af^2}{\varepsilon} \text{tr} \int d^{14}z \frac{du_1 du_2 du_3}{(u_1^+ u_2^+)(u_1^+ u_3^+)} F_{1,\tau}^{++} g^{(1,1)}(z; u_2, u_3 | V^{++}).$$
(B.3)

The first term in (B.3) was obtained earlier in [40] by explicit calculation of the effective action, and it corresponds to the highest-order pole in ε . The second term corresponds to the $\frac{1}{\varepsilon}$ divergent contribution. We see that such a contribution also vanishes when the background gauge superfield satisfies the classical equation of motion, $F^{++} = 0$, in agreement with the hidden $\mathcal{N} = (0, 1)$ supersymmetry.

Thus we see that the harmonic supergraph technique successfully works for calculating both $\frac{1}{\varepsilon^2}$ and $\frac{1}{\varepsilon}$ divergences.

References

- [1] Z. Bern, L.J. Dixon and V.A. Smirnov, "Iteration of planar amplitudes in maximally super-symmetric Yang-Mills theory at three loops and beyond", Phys. Rev. D **72** (2005) 085001, arXiv:hep-th/0505205.
- [2] Z. Bern, J.J.M. Carrasco, L.J. Dixon, H. Johansson and R. Roiban, "The Complete Four-Loop Four-Point Amplitude in N=4 Super-Yang-Mills Theory", Phys. Rev. D 82 (2010) 125040, arXiv:1008.3327 [hep-th].
- [3] Z. Bern, J.J.M. Carrasco, L.J. Dixon, H. Johansson and R. Roiban, "Simplifying Multiloop Integrands and Ultraviolet Divergences of Gauge Theory and Gravity Amplitudes", Phys. Rev. D 85 (2012) 105014, arXiv:1201.5366 [hep-th].
- [4] G. Bossard, P.S. Howe and K.S. Stelle, "The Ultra-violet question in maximally supersymmetric field theories," Gen. Rel. Grav. 41 (2009) 919, arXiv:0901.4661 [hep-th].
- [5] G. Bossard, P.S. Howe and K.S. Stelle, "A Note on the UV behaviour of maximally supersymmetric Yang-Mills theories," Phys. Lett. B **682** (2009) 137, arXiv:0908.3883 [hep-th].
- [6] D.I. Kazakov, "Ultraviolet fixed points in gauge and SUSY field theories in extra dimensions," JHEP **0303** (2003) 020, arXiv:hep-th/0209100.
- [7] E.S. Fradkin and A.A. Tseytlin, "Quantum Properties of Higher Dimensional and Dimensionally Reduced Supersymmetric Theories," Nucl. Phys. B **227** (1983) 252.
- [8] N. Marcus and A. Sagnotti, "A Test of Finiteness Predictions for Supersymmetric Theories," Phys. Lett. **135** B (1984) 85.

- [9] N. Marcus and A. Sagnotti, "The Ultraviolet Behavior of N=4 Yang-Mills and the Power Counting of Extended Superspace," Nucl. Phys. B **256** (1985) 77.
- [10] P.S. Howe and K.S. Stelle, "Ultraviolet Divergences in Higher Dimensional Supersymmetric Yang-Mills Theories," Phys. Lett. **137** B (1984) 175.
- [11] G. Bossard, E. Ivanov and A. Smilga, "Ultraviolet behavior of 6D supersymmetric Yang-Mills theories and harmonic superspace", JHEP **1512** (2015) 085, arXiv:1509.08027 [hep-th].
- [12] S. Buyucli and E. Ivanov, "Higher-dimensional invariants in 6D super Yang-Mills theory", JHEP **2107** (2021) 190, arXiv:2105.05899 [hep-th].
- [13] P.S. Howe and K.S. Stelle, "Supersymmetry counterterms revisited," Phys. Lett. B **554** (2003) 190, arXiv:hep-th/0211279.
- [14] R. Kallosh, "The Ultraviolet Finiteness of N=8 Supergravity," JHEP **1012** (2010) 009, arXiv:1009.1135 [hep-th].
- [15] R. Kallosh, " $E_{7(7)}$ Symmetry and Finiteness of N=8 Supergravity," JHEP **1203** (2012) 083, arXiv:1103.4115 [hep-th].
- [16] Z. Bern, J.J. Carrasco, W.M. Chen, A. Edison, H. Johansson, J. Parra-Martinez, R. Roiban and M. Zeng, "Ultraviolet Properties of $\mathcal{N}=8$ Supergravity at Five Loops," Phys. Rev. D **98** (2018) no.8, 086021, arXiv:1804.09311 [hep-th].
- [17] M.F. Sohnius and P.C. West, "Conformal Invariance in N=4 Supersymmetric Yang-Mills Theory," Phys. Lett. B **100** (1981) 245.
- [18] M.T. Grisaru and W. Siegel, "Supergraphity. 2. Manifestly Covariant Rules and Higher Loop Finiteness," Nucl. Phys. B **201** (1982) 292; [Erratum: Nucl. Phys. B **206** (1982) 496].
- [19] S. Mandelstam, "Light Cone Superspace and the Ultraviolet Finiteness of the N=4 Model," Nucl. Phys. B **213** (1983) 149.
- [20] L. Brink, O. Lindgren and B.E.W. Nilsson, "N=4 Yang-Mills Theory on the Light Cone," Nucl. Phys. B **212** (1983) 401.
- [21] P.S. Howe, K.S. Stelle and P.K. Townsend, "Miraculous Ultraviolet Cancellations in Supersymmetry Made Manifest," Nucl. Phys. B **236** (1984) 125.
- [22] L.V. Bork, D.I. Kazakov, M.V. Kompaniets, D.M. Tolkachev and D.E. Vlasenko, "Divergences in maximal supersymmetric Yang-Mills theories in diverse dimensions," JHEP 1511 (2015) 059, arXiv:1508.05570 [hep-th].
- [23] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "One-loop divergences in 6D, $\mathcal{N} = (1,0)$ SYM theory," JHEP **1701** (2017) 128, arXiv:1612.03190 [hep-th].
- [24] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "Supergraph analysis of the one-loop divergences in 6D, $\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$ gauge theories," Nucl. Phys. B **921** (2017) 127, arXiv:1704.02530 [hep-th].
- [25] Z. Bern, J. S. Rozowsky and B. Yan, "Two loop four gluon amplitudes in N=4 superYang-Mills," Phys. Lett. B **401** (1997), 273, arXiv:hep-ph/9702424.
- [26] Z. Bern, L. J. Dixon, D. C. Dunbar, M. Perelstein and J. S. Rozowsky, "On the relationship between Yang-Mills theory and gravity and its implication for ultraviolet divergences," Nucl. Phys. B 530 (1998), 401, arXiv:hep-th/9802162.

- [27] Z. Bern, J. J. Carrasco, L. J. Dixon, M. R. Douglas, M. von Hippel and H. Johansson, "D=5 maximally supersymmetric Yang-Mills theory diverges at six loops," Phys. Rev. D 87 (2013) 025018, arXiv:1210.7709 [hep-th].
- [28] J. J. M. Carrasco, A. Edison and H. Johansson, "Maximal Super-Yang-Mills at Six Loops via Novel Integrand Bootstrap," arXiv:2112.05178 [hep-th].
- [29] S.J. Gates, M.T. Grisaru, M. Rocek and W. Siegel, "Superspace Or One Thousand and One Lessons in Supersymmetry," Front. Phys. **58** (1983) 1.
- [30] P.C. West, "Introduction to supersymmetry and supergravity," Singapore, Singapore: World Scientific (1990) 425 p.
- [31] I.L. Buchbinder and S.M. Kuzenko, "Ideas and methods of supersymmetry and supergravity: Or a walk through superspace," Bristol, UK: IOP (1998) 656 p.
- [32] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, "Unconstrained N=2 matter, Yang-Mills and supergravity theories in harmonic superspace", Class. Quant. Grav. 1 (1984) 469 [Corrigendum ibid. 2 (1985) 127].
- [33] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, "Harmonic Superspace: Key To N=2 Supersymmetry Theories," JETP Lett. **40** (1984) 912 [Pisma Zh. Eksp. Teor. Fiz. **40** (1984) 155].
- [34] A.S. Galperin, E.A. Ivanov, V.I. Ogievetsky and E.S. Sokatchev, "Harmonic superspace", Cambridge, UK: Univ. Pr. (2001) 306 p.
- [35] E.I. Buchbinder, B.A. Ovrut, I.L. Buchbinder, E.A. Ivanov and S.M. Kuzenko, "Low-energy effective action in N = 2 supersymmetric field theories", Phys. Part. Nucl. **32** (2001) 641 [Fiz. Elem. Chast. Atom. Yadra **32** (2001) 1222].
- [36] I.L. Buchbinder, S.M. Kuzenko and B.A. Ovrut, "On the D = 4, N=2 nonrenormalization theorem," Phys. Lett. B **433** (1998) 335, arXiv:hep-th/9710142.
- [37] I.L. Buchbinder, N.G. Pletnev and K.V. Stepanyantz, "Manifestly N=2 supersymmetric regularization for N=2 supersymmetric field theories," Phys. Lett. B **751** (2015) 434, arXiv:1509.08055 [hep-th].
- [38] P.S. Howe, K.S. Stelle and P.C. West, "N=1, d = 6 harmonic superspace", Class. Quant. Grav. **2** (1985) 815.
- [39] B.M. Zupnik, "Six-dimensional Supergauge Theories in the Harmonic Superspace," Sov. J. Nucl. Phys. 44 (1986) 512 [Yad. Fiz. 44 (1986) 794].
- [40] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "On the two-loop divergences in 6D, N=(1,1) SYM theory," Phys. Lett. B 820 (2021) 136516, arXiv:2104.14284 [hep-th].
- [41] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "One-loop divergences in the $6D, \mathcal{N} = (1,0)$ abelian gauge theory," Phys. Lett. B **763** (2016) 375, arXiv:1609.00975 [hep-th].
- [42] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "On the two-loop divergences of the 2-point hypermultiplet supergraphs for 6D, $\mathcal{N}=(1,1)$ SYM theory," Phys. Lett. B 778 (2018) 252, arXiv:1711.11514 [hep-th].
- [43] I.L. Buchbinder, E.A. Ivanov and B.S. Merzlikin, "Leading low-energy effective action in 6D, $\mathcal{N} = (1,1)$ SYM theory," JHEP **1809** (2018) 039, arXiv:1711.03302 [hep-th].

- [44] I.L. Buchbinder, E. Ivanov, B. Merzlikin and K. Stepanyantz, "Harmonic Superspace Approach to the Effective Action in Six-Dimensional Supersymmetric Gauge Theories," Symmetry 11 (2019) no.1, 68, arXiv:1812.02681 [hep-th].
- [45] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "Gauge dependence of the one-loop divergences in 6D, $\mathcal{N}=(1,0)$ abelian theory," Nucl. Phys. B **936** (2018) 638, arXiv:1808.08446 [hep-th].
- [46] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "On gauge dependence of the one-loop divergences in 6D, $\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$ SYM theories," Phys. Lett. B **798** (2019) 134957, arXiv:1907.12302 [hep-th].
- [47] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "Supergraph calculation of one-loop divergences in higher-derivative 6D SYM theory," JHEP **08** (2020) 169, arXiv:2004.12657 [hep-th].
- [48] I.L. Buchbinder, E.A. Ivanov, B.S. Merzlikin and K.V. Stepanyantz, "The renormalization structure of 6D, $\mathcal{N} = (1,0)$ supersymmetric higher-derivative gauge theory," Nucl. Phys. B **961** (2020) 115249, arXiv:2007.02843 [hep-th].
- [49] E.A. Ivanov, A.V. Smilga and B.M. Zupnik, "Renormalizable supersymmetric gauge theory in six dimensions," Nucl. Phys. B **726** (2005) 131, arXiv:hep-th/0505082.
- [50] P.K. Townsend and G. Sierra, "Chiral Anomalies and Constraints on the Gauge Group in Higher Dimensional Supersymmetric Yang-Mills Theories," Nucl. Phys. B **222** (1983) 493.
- [51] A.V. Smilga, "Chiral anomalies in higher-derivative supersymmetric 6D theories," Phys. Lett. B **647** (2007) 298, arXiv:hep-th/0606139.
- [52] S.M. Kuzenko, J. Novak and I.B. Samsonov, "The anomalous current multiplet in 6D minimal supersymmetry," JHEP **1602** (2016) 132, arXiv:1511.06582 [hep-th].
- [53] S.M. Kuzenko, J. Novak and I.B. Samsonov, "Chiral anomalies in six dimensions from harmonic superspace," JHEP **1711** (2017) 145, arXiv:1708.08238 [hep-th].
- [54] I.L. Buchbinder, E.I. Buchbinder, S.M. Kuzenko and B.A. Ovrut, "The background field method for $\mathcal{N}=2$ superYang-Mills theories in harmonic superspace", Phys. Lett. B **417** (1998) 61, arXiv:hep-th/9704214.
- [55] I.L. Buchbinder, E.A. Ivanov and N.G. Pletnev, "Superfield approach to the construction of effective action in quantum field theory with extended supersymmetry", Phys. Part. Nucl. 47 (2016) no.3, 291 [Fiz. Elem. Chast. Atom. Yadra 47 (2016) No.3, 541-698].
- [56] S.M. Kuzenko, "Self-dual effective action of N = 4 SYM revisited," JHEP 0503 (2005) 008, arXiv:hep-th/0410128.
- [57] I.L. Buchbinder, A.S. Budekhina, E.A. Ivanov, B.S. Merzlikin, K.V. Stepanyantz, "On two-loop divergences in 6D, $\mathcal{N}=(1,1)$ supergauge theory ", Physics of Particles and Nuclei Letters, 19 (2022) 666.