Chapter 1 Exercises

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1 Combinators

Determine if each of the follow are combinators or not.

1. $\lambda x.xxx$ — This is a combinator because the only variable to appear in the body, x, is bound as it also appears in the head.

2. $\lambda xy.xz$ — This is not a combinator because z is an unbound variable.

3. $\lambda xyz.xy(zx)$ — This is a combinator because this simplifies to $\lambda xyz.zxy$ and all of the body variables are bound in the head.

4. $\lambda xyz.xy(zyx)$ — This is a combinator because this simplifies to $\lambda xyz.zyxy$ and all of the variables in the body are bound in the head.

5. $\lambda xy.xy(zxy)$ — This is not a combinator because this simplifies to $\lambda xy.zxyy$ and z is an unbound variable as it is headless.

2 Normal form or diverge?

Determine if each of the following can be reduced to a normal form or if they diverge.

1. $\lambda x.xxx$ — This is already in normal form and thus it is not divergent.

2. $(\lambda z.zz)(\lambda y.yy)$ — This is divergent. We can say that we will bind z to $(\lambda y.yy)$ which then evaluates to $(\lambda y.yy)(\lambda y.yy)$. As $(\lambda y.yy)$ is alpha equivalent to $(\lambda z.zz)$ we can say that this diverges.

3. $(\lambda x.xxx)z$ — This can be reduced to normal form. We can bind x to z and then get zzz which is not divergent.

3 Beta reduce

Evaluate (that is, beta reduce) each of the following expressions to normal form.

1. $(\lambda abc.cba)zz(\lambda wv.w)$ — This evaluates relatively easily. Here are the bindings $a \to z$, $b \to z$, and $c \to (\lambda wv.w)$. We can do the first reduction like this: $(\lambda z.zz(\lambda wv.w))$. This reduces to $(\lambda wv.w)(\lambda wv.w)$.