9 Continuum Mechanics

9.1 Example: Motion of a String

We have an elastic string lying along the x-axis. Consider when we displace the string in the y direction by $y(x)$. If we slice the string into infinitesimally small segments of mass m, length l, and tension T, balancing forces says (using small angle approximation)

$$\begin{aligned} m\ddot{y}_n &= -\frac{T}{l}(y_n - y_{n-1}) + \frac{T}{l}(y_{n+1} - y_n) \\ \ddot{y}_n &= \frac{T}{ml}(y_{n+1} - 2y_n + y_{n-1}) \end{aligned} \tag{1}$$