The "Expected Profit" Strategy in Poker

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Sports Betting

Suppose you are betting that the Rams will beat the Patriots in the Super Bowl. Suppose the Rams win with probability p. Suppose you bet \$$X$ to win \$$Y$ (for instance, you bet \$100 to profit \$130). Then your expected profit is

$$\mathbb{E}[\text{profit}] = pY + (1-p)(-X)$$

because with probability p you profit \$$Y$ and with probability $1-p$ you lose \$$X$. You would like to have a positive expected profit, because this indicates that in the long run you are expected to profit from making the bet over and over. Now, we see that

$$\mathbb{E}[\text{profit}] = pY + (1-p)(-X) > 0$$

if and only if

$$\boxed{p > \frac{X}{X+Y}}$$

We call the term $\frac{X}{X+Y}$ the *pot odds*, and come to the conclusion that it is a good idea to make the bet in the long run if the above inequality holds (i.e., if the probability of winning the bet exceeds the pot odds), and a bad idea to make the bet if it doesn't hold.

The main problem with this strategy is that it is difficult to obtain the probability p that the Rams win, if it even exists. We may resort to statistical methods (or to our intuition) to estimate p, but to obtain p with certainty is impossible. But if for some reason you feel deeply that you have obtained a value for p, I encourage you to consider this strategy.

In poker, however, there are many instances in which we can find p.

Poker

We call $\frac{X}{X+Y}$ the pot odds because we can apply this strategy to poker. Suppose you are playing poker and you have to call \$$X$ to stay in the hand, and the total pot size is \$$Y$. So, you have to call \$$X$ to profit \$$Y$ if you win. Then we find ourselves in a similar situation as the Super Bowl example - you bet \$$X$ to profit \$$Y$. Now, if you know the probability p that you win the hand, then you can employ the previous strategy to ensure profit in the long run.

Fortunately, in poker there are instances where we can easily determine p. For instance, suppose you are playing Texas Holdem and have an A♡7♡ of Hearts in your hand, and the table cards are Q♡6♠3♡2♣. So, only the river card (one more table card) remains. Since you have two hearts and the Ace of hearts in your hand, a flush (the nut-flush) virtually guarantees that you win the hand. Now, there are 13 hearts in the deck and 4 have been dealt, so 9 remain in the deck. We call the cards we need to win (these 9 hearts) *outs*. You have seen 6 cards (2 in your hand, 4 on the table), so 46 cards remain unkown to you, so the probability of drawing a heart on the last card is 9/46. Since 46 is so close to 50, at the poker table we can approximate this probability by calling it 9/50, and doubling it gives us 18/100, which gives us our probability of winning on the river: $p = .18$. The fast way to obtain this .18 is by taking the number of outs (9) and doubling it (18). So, if we have to call \$$X$ with a pot size of \$$Y$, we should make the call if and only if

$$p > \frac{X}{X+Y}$$

We can perform similar calculations to find p in many other instances. As we saw in the example, if only the river card remains undrawn,

$$p = \frac{2 * \binom{\text{number}}{\text{of outs}}}{100}$$

Poker Strategy, Succintly

If you can determine the probability that you win a hand and need to call a certain amount to stay in the hand, make the call if

$$\boxed{\begin{matrix}\text{probability}\\ \text{you win}\end{matrix} \quad > \quad \frac{\text{call amount}}{\text{call amount} + \text{pot size}}}$$

and fold if not.