

Lab: Kelly Betting

1. A smart -EV Bet

Kelly (1956) extended his analysis to horses

m horses $1, \dots, m$
true (known) win probabilities p_1, \dots, p_m
decimal odds $\alpha_1, \dots, \alpha_m$
payroll fractions $f = (f_1, \dots, f_m)$ to bet on each horse.
make N sequential bets, solve for f .

Use Lagrange Multiplier/KKT conditions to maximize expected log bankroll after N bets, yielding the Kelly allocations f via this algorithm:

(a) Permute indices so that $p(s)\alpha_s \geq p(s+1)\alpha_{s+1}$

(b) Set b equal to the minimum positive value of

$$\frac{1 - p_t}{1 - \sigma_t} \quad \text{where } p_t = \sum_1^t p(s), \quad \sigma_t = \sum_1^t \frac{1}{\alpha_s}$$

(c) Set $f(s) = p(s) - b/\alpha_s$ or zero, whichever is larger. (The $f(s)$ will sum to $1 - b$.)

Now, Consider a horse Race with $m=3$ horses. Find an example of true horse win probabilities (p_1, p_2, p_3) and decimal odds (d_1, d_2, d_3) such that it is Kelly optimal to bet on a $-EV$ horse.

The Kelly bet allocations (f_1, f_2, f_3) according to the above algorithm must consist of $f_i > 0$ for some horse i , where betting on horse i is $-EV$: $p_i d_i - 1 < 0$.

Then, $M=100$ times, simulate $N=1000$ sequential bets on this horse Race and Record your bankroll at the end using allocations (f_1, f_2, f_3) according to ① Kelly and ② separately letting $f_i = 0$ (where i is the $-EV$ horse). Which yields more wealth on average?

Why is it possible that making a $-EV$ bet is smart?

If the game is profitable on the whole, you should put money into the game.

If you only bet on the $+EV$ horse you can lose more than if you hedge.