Kelly Criterion for 2021 Elite 8

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0
5	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0
6	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0
7	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0
8	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1

p = win probability, from 538 "true probs"

mL = moneyline odds, from dRaftKings

Implied.p = the win probability that is implied from the moneyline odds

Ex $mL = -360 \Rightarrow \text{Implied.p} = \frac{360}{360+100} = .7826$

$mL = +260 \Rightarrow \text{Implied.p} = \frac{100}{260+100} = .2778$

$$\text{plusEV} = \begin{cases} \text{true if} & p > \text{implied.p} \\ \text{false if} & p \leq \text{implied.p} \end{cases}$$

"positive expected value bet"

- One betting strategy is to only make +EV bets. But this is not necessarily Kelly optimal... see the end of these notes

α_s the odds paid on the occurrence of the s'th transmitted symbol, i.e., α_s is the number of dollars returned for a one-dollar bet (including that one dollar). $s \in \{1, 2\}$

- Need to convert moneyline odds to α.

Ex $mL = -360 \Rightarrow$ bet \$360 to profit \$100

$\Rightarrow$ bet \$1 to profit $\frac{100}{360}$

$\Rightarrow \alpha = 1 + \frac{100}{360} = 1.2778$

$mL = +260 \Rightarrow$ bet \$100 to profit 260

$\Rightarrow$ bet \$1 to profit $\frac{260}{100}$

$\Rightarrow \alpha = 1 + \frac{260}{100} = 3.6$

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0
5	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0
6	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0
7	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0
8	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1

Kelly Step 1

For each game,

(a) Permute indices so that $p(s)\alpha_s \geq p(s+1)\alpha_{s+1}$

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000

4 games

For each game,

b = the fraction Not bet on this game.

Kelly step 2

(b) Set b equal to the minimum positive value of

$$F_t = \frac{1-p_t}{1-\sigma_t} \quad \text{where} \quad p_t = \sum_1^t p(s), \quad \sigma_t = \sum_1^t \frac{1}{\alpha_s}$$

$t \in \{0,1\}$ is the index that minimizes F_t. $F_0 = 1$.

- $t=0$ if do not bet on this game $\iff p(1)\alpha_1 < 1$ (both bets are negative EV!)
- $t=1$ if bet on the team with the higher $p(s)\alpha_s$ value $\iff p(1)\alpha_1 > 1$
- $t \neq 2$ since can't bet on team with lowest $p(s)\alpha_s$ value

Consequence for 2-team Kelly, bet on a team $\Rightarrow$ plus EV bet!

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha	F_0	F_1	t	b
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056	1	1.1256200	0	1.0000000
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200	NA	NA	NA	1.0000000
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721	1	0.9165150	1	0.9165150
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350	NA	NA	NA	1.0000000
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500	1	1.0103231	0	1.0000000
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092	NA	NA	NA	1.0000000
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500	1	0.8844545	1	0.8844545
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000	NA	NA	NA	1.0000000

$$\begin{cases} F_0 = 1 \\ F_1 = \dfrac{1-p(1)}{1-1/\alpha(1)} = \dfrac{\alpha(1)-\alpha(1)p(1)}{\alpha(1)-1} \end{cases}$$

so to bet on team 1, we need $F_1 < F_0 = 1$, so need $\alpha(1)p(1) > 1$.

Kelly Step 3

(c) Set $a(s) = p(s) - b/\alpha_s$ or zero, whichever is larger. (The $a(s)$ will sum to $1 - b$.)

Here: $a = 1-b$, since 2 outcomes per game

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha	b	a	a_
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056	1.0000000	0.0000000	0.00000000
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200	1.0000000	0.0000000	0.00000000
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721	0.9165150	0.0834850	0.04174250
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350	1.0000000	0.0000000	0.00000000
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500	1.0000000	0.0000000	0.00000000
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092	1.0000000	0.0000000	0.00000000
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500	0.8844545	0.1155455	0.05777273
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000	1.0000000	0.0000000	0.00000000

Step 4: Make Bets, and see how we did!

we only make +EV bets! This is because it's 2-team Kelly (see later)

$B = \$1000$, total bet size

	team	opp	teamWon	p	alpha	ml	implied.p	plusEV	p.times.alpha	a_	bet	profit
1	2 Houston	12 Oregon St	1	0.7553	1.277778	-360	0.7826087	0	0.9651056	0.00000000	0.00000	0.00000
2	12 Oregon St	2 Houston	0	0.2447	3.600000	260	0.2777778	0	0.8809200	0.00000000	0.00000	0.00000
3	1 Baylor	3 Arkansas	1	0.7737	1.327869	-305	0.7530864	1	1.0273721	0.04174250	41.74250	13.68607
4	3 Arkansas	1 Baylor	0	0.2263	3.450000	245	0.2898551	0	0.7807350	0.00000000	0.00000	0.00000
5	6 USC	1 Gonzaga	0	0.2274	4.250000	325	0.2352941	0	0.9664500	0.00000000	0.00000	0.00000
6	1 Gonzaga	6 USC	1	0.7726	1.229885	-435	0.8130841	0	0.9502092	0.00000000	0.00000	0.00000
7	11 UCLA	1 Michigan	1	0.3514	3.750000	275	0.2666667	1	1.3177500	0.05777273	57.77273	158.87500
8	1 Michigan	11 UCLA	0	0.6486	1.289855	-345	0.7752809	0	0.8366000	0.00000000	0.00000	0.00000

Total profit from Elite 8: $172.56

Cool! other Rounds didn't do as well, but still made money!

Question from Adi (case: 3 teams, $(P_1+P_2<1, P_3>0)$)

One of the results in the paper is that there exist horses with true probabilities and odds that make bets on the horse negative EV. Yet Kelly betting on the horse suggests wagering some amount of your bankroll anyway.

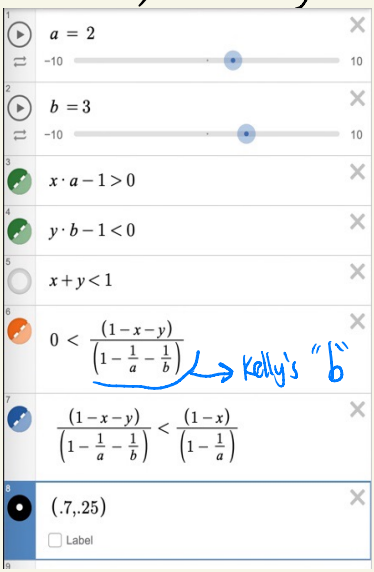
$$\begin{cases} P_1+P_2<1 \\ P_1, P_2 \geq 0 \end{cases}$$ probabilities

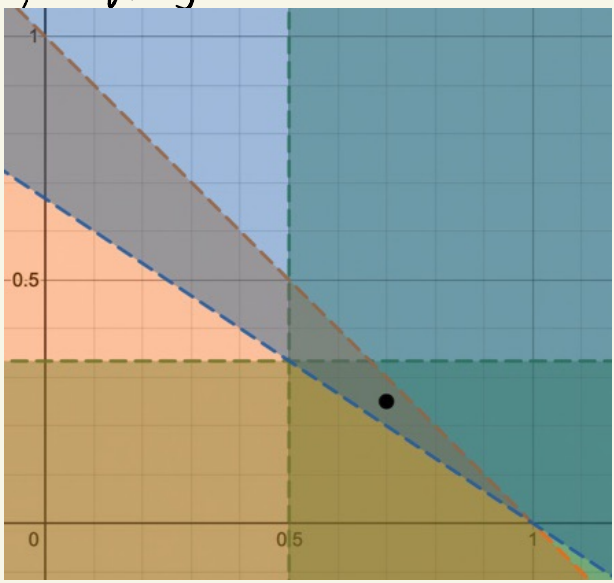
$$\begin{cases} P_1\alpha_1 - 1 > 0 \\ P_2\alpha_2 - 1 < 0 \end{cases}$$ option 1 is $+EV \Rightarrow$ we will bet on some team; option 2 is $-EV$

$$\begin{cases} 0 < \dfrac{1-P_1-P_2}{1-\frac{1}{\alpha_1}-\frac{1}{\alpha_2}} < \dfrac{1-P_1}{1-\frac{1}{\alpha_1}} \\ \text{Equivalently, } 0 < F_2 < F_1 \end{cases}$$

make a nonzero bet on team 2!

$\alpha_1 \mapsto a$, $\alpha_2 \mapsto b$, $P \mapsto x$, $q \mapsto y$





Check $a_2 = \max\left(P_2 - \frac{b}{\alpha_2}, 0\right) = 0.1$, so we make a $-EV$ bet as desired!