How to Choose a Wife: A Generalization of the Secretary Problem for Top-k Selection

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July 2025

The problem. There are n applicants (potential spouses). You will interview them in a uniformly random order (think of an interview as a date or series of dates). After each interview, you immediately and permanently accept or reject the applicant. You may select only one applicant (monogamous marriage). Your goal is to maximize the probability of choosing one of the top k candidates.

Strategy. Let's use an *optimal stopping strategy.* The strategy begins with an observation phase: interview and reject the first r candidates. Then, select the first subsequent candidate who you like more than every previous candidate. You win if that chosen candidate is among the top k.

Results. The formula for the success probability $P(n, k, r)$ that you end up with a top-k applicant out of n total applicants and r applicants in the observation phase is

$$P(n, k, r) = \frac{r}{n} \sum_{t=r+1}^{n} \frac{1}{t-1} \left(\sum_{j=0}^{k-1} \frac{\binom{n-t}{j}}{\binom{n-1}{j}} \right), \tag{0.1}$$

which we derive in Appendix A.

The author is nearly 28 years old, and it would be nice if he finds a wife by 34 (his mother would certainly be happy). Supposing he has the bandwidth to date, say, 3 new girls per month, let's round to a total of $n = 200$. In Figure 1 we visualize success probability $P(n = 200, k, r)$ as a function of k (color) and r (x-axis), and the optimal r values are noted by $\times$ symbols.

The solution to the classical marriage problem[1] (k=1) is to use $1/e \approx 37\%$ of the applicants for the observation period ($r = n/e$), and the optimal success probability is $P = 1/e$. In our example, $r = n/e \approx 73$. These values align with the yellow curve in Figure 1.

An observation period of $r \approx 30$ gives a $P \approx 80\%$ chance of selecting a top 5% girl ($k/n = 10/200$). An observation period of $r \approx 15$ gives a $P \approx 90\%$ chance of selecting a top 15% girl ($k/n = 30/200$). In real life, these success probabilities are in some sense underestimates because you could fall in love during the observation period or return to an ex. To me, somewhere in-between—say, $r = 22$—seems like a good choice.

[1] https://en.wikipedia.org/wiki/Secretary_problem

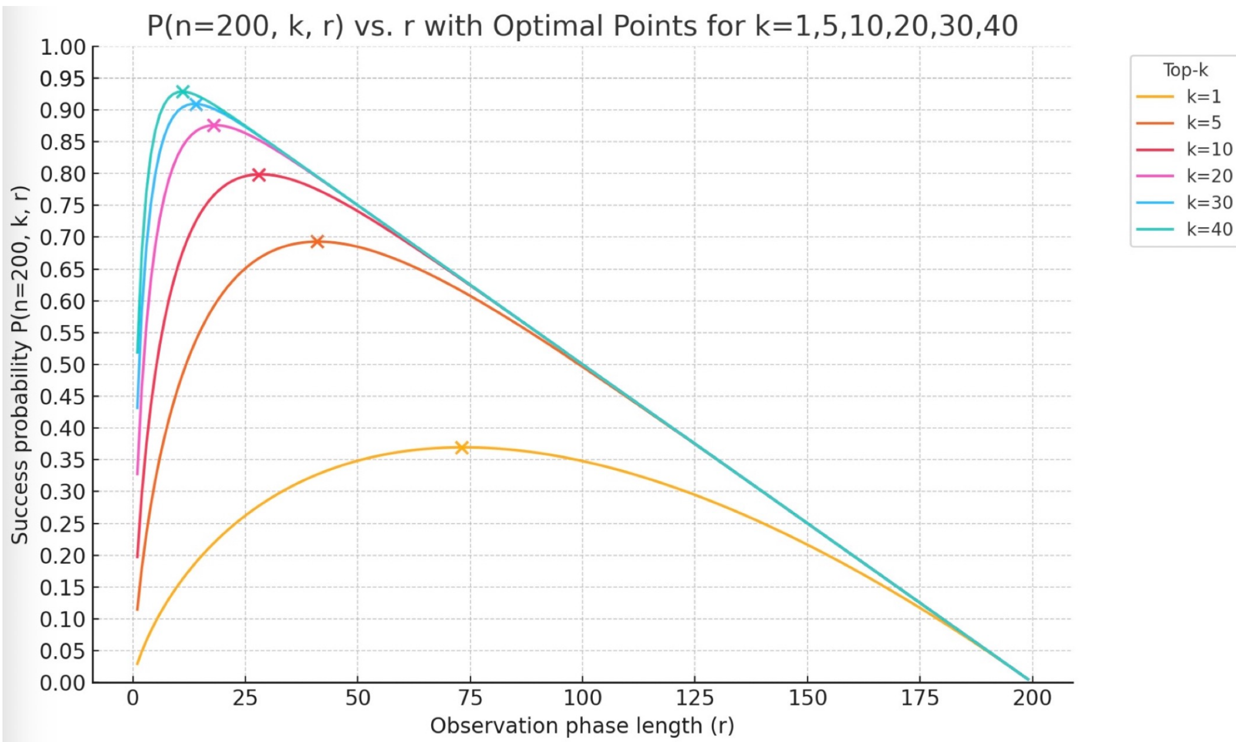

Figure 1: Success probability $P(n = 200, k, r)$ (y-axis) as a function of k (color) and r (x-axis). Optimal r values are denoted by $\times$ symbols.

Appendix

A Deriving Success Probability

Denote the true ranking of the n applicants by $1, ..., n$ where 1 is the best. Then,

$$
\begin{aligned}
P(n,k,r) &= \mathbb{P}(\text{the strategy selects an applicant in the top } k) \\
&= \sum_{j=1}^{k} \mathbb{P}(\text{the strategy selects applicant of rank } j) \\
&= \sum_{j=1}^{k} \sum_{t=r+1}^{n} \mathbb{P}(\text{the strategy stops at timestep } t \text{ AND candidate } t \text{ has rank } j) \\
&= \sum_{j=1}^{k} \sum_{t=r+1}^{n} \mathbb{P}(A \cap B \cap C) \\
&= \sum_{j=1}^{k} \sum_{t=r+1}^{n} \mathbb{P}(C \mid A \cap B) \cdot \mathbb{P}(B \mid A) \cdot \mathbb{P}(A)
\end{aligned}
$$

where

- A: Candidate t has true rank j,
- B: No one better than the t^{th} candidate appears at times $1, \ldots, t-1$,
- C: The local best in times $1, \ldots, t-1$ falls in the observation phase $1, \ldots, r$.

Well,

$$\mathbb{P}(A) = \frac{1}{n}, \quad \mathbb{P}(C \mid A \cap B) = \frac{r}{t-1}, \quad \mathbb{P}(B \mid A) = \frac{\binom{n-t}{j-1}}{\binom{n-1}{j-1}}.$$

Therefore,

$$
\begin{aligned}
P(n,k,r) &= \sum_{j=1}^{k} \sum_{t=r+1}^{n} \left(\frac{r}{t-1} \cdot \frac{1}{n} \cdot \frac{\binom{n-t}{j-1}}{\binom{n-1}{j-1}} \right) \\
&= \frac{r}{n} \sum_{t=r+1}^{n} \frac{1}{t-1} \left(\sum_{j=0}^{k-1} \frac{\binom{n-t}{j}}{\binom{n-1}{j}} \right).
\end{aligned}
$$

B Continuous approximation (large n)

Let $\alpha = \frac{r}{n}$ and $w = \frac{t}{n}$. We have

$$\frac{\binom{n-t}{j}}{\binom{n-1}{j}} \approx (1-w)^j \quad \text{as } n \to \infty \tag{B.1}$$

and

$$\sum_{j=0}^{k-1}(1-w)^j = \frac{1-(1-w)^k}{w}, \tag{B.2}$$

so

$$P(k,\alpha) \approx \alpha \int_{\alpha}^{1} \frac{1-(1-w)^k}{w^2}\, dw. \tag{B.3}$$

This will be easier to evaluate numerically for large n.