Lab: Kelly Betting

1. A Smart −EV Bet

Kelly (1956) extended his analysis to horses

$$
\begin{cases}
\text{m horses } 1, \ldots, m \\
\text{true (known) win probabilities } P_1, \ldots, P_m \\
\text{decimal odds } \alpha_1, \ldots, \alpha_m \\
\text{payroll fractions } f = (f_1, \ldots, f_m) \text{ to bet on each horse.} \\
\text{make N sequential bets, solve for } f.
\end{cases}
$$

He used **Lagrange Multipliers / KKT** conditions to maximize expected log bankroll after N bets, yielding the kelly allocations f via this algorithm:

(a) Permute indices so that $p(s)\alpha_s \geqq p(s+1)\alpha_{s+1}$

(b) Set b equal to the minimum positive value of

$$
\frac{1 - p_t}{1 - \sigma_t} \quad \text{where} \quad p_t = \sum_1^t p(s), \quad \sigma_t = \sum_1^t \frac{1}{\alpha_s}
$$

(c) Set $f(s) = p(s) - b/\alpha_s$ or zero, whichever is larger. (The $f(s)$ will sum to $1 - b$.)

Consider a horse race with $m = 3$ horses.
Find an example of true horse win probabilities
(P_1, P_2, P_3) and decimal odds (d_1, d_2, d_3) such that
it is Kelly optimal to bet on a $-EV$ horse.
The Kelly bet allocations (f_1, f_2, f_3) according to the
above algorithm must consist of $f_i > 0$ for some
horse i, where betting on horse i is $-EV$: $P_i d_i - 1 < 0$.

Then, $M = 100$ times, simulate $N = 1000$ bets on this
horse race and record your bankroll at
the end using allocations (f_1, f_2, f_3) according to
①Kelly and ②separately letting $f_i = 0$ (where i is the
$-EV$ horse). Which yields more wealth on average?

Why is it possible that making a $-EV$ bet is smart?

If the game is profitable on the whole, you
should put money into the game.
If you only bet on the $+EV$ horse you can
lose more than if you hedge.