Simulation: Win Probability in Simplified Football

* XGBoost → win probability estimates

$$\widehat{WP}_{Go} = .63, \quad \widehat{WP}_{FG} = .60, \quad \widehat{WP}_{Punt} = .57$$

People say: therefore you should Go.

But what if these values are not right?
What if the model sucks?

* People saw a dataset of $\approx$ 200,000 1st down plays in the last 20 years

and $\geq$ 500,000 plays altogether

so they think Big Data $\Rightarrow$ Good Model.

* i index of i^{th} play

$$y_i = \begin{cases} 1 & \text{if team with possession on play } i \text{ wins that game} \\ 0 & \text{if loses} \end{cases}$$

What do you notice about this outcome variable?

- noisy
- Extreme Autocorrelation

* Every play i from the same game shares the exact same value of y_i or ($1-y_i$ if other team)

→ $\boxed{\text{there is only one independent draw of the outcome win/loss for each game}}$

* there are about ≈ 4000 games in the last 15 years, so the effective sample size of our model is $\approx 4{,}000$ not $\approx 500{,}000$.

→ SMALL data Regime.

yardline	point spread	...	score diff	outcome win/loss	
70	3		0	1	200 plays game 1
60	3		0	1	
52	3		0	1	
		⋮			
40	-3		-7	0	
					200 plays game 2

* XGBoost $\rightarrow$ win probability estimates

$$\widehat{WP}_{Go} = .63, \quad \widehat{WP}_{FG} = .60, \quad \widehat{WP}_{Punt} = .57$$

* If we have a good WP model, which would arise from XGBoost fit on a dataset with large number of Rows, then

prediction interval

confidence interval $\hat{I}(x) = [\widehat{WP}_L, \widehat{WP}_u]$

$$\widehat{WP}_{go} = .63 \qquad \hat{I}_{go} = [.625, .635]$$

What is a confidence interval?

Imagine we had 100 different training datasets (X, y) each with the same X, but y each time is a new draw from our model $y_i \sim \text{Bernoulli}(WP(x_i))$.

Then a 95% CI on $\hat{y}_i = \widehat{WP}_i(x_i)$ is an interval which contains 95% of the time the WP estimate fit on the dataset.

$$(X, y^{(1)}) \quad (X, y^{(2)}) \quad \cdots \quad (X, y^{(100)})$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad\qquad \downarrow$$
$$\widehat{WP}^{(1)} \quad \widehat{WP}^{(2)} \qquad\qquad \widehat{WP}^{(100)}$$

at game-state x, 95 of these 100 $\widehat{WP}(x)$ estimates must lie in the 95% confidence interval.

if $\widehat{WP}^{(1)}(x) \geq \widehat{WP}^{(2)}(x) \geq \cdots \geq \widehat{WP}^{(100)}(x)$

then 95% CI would be $\left[\widehat{WP}^{(97)}(x), \widehat{WP}^{(3)}(x)\right]$.

What is a prediction interval?

A 95% PI says the true (unseen) outcome y^* lies in the PI 95% of the time.

Out-of-sample dataset $\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$ $\begin{pmatrix} y_1^* \\ \vdots \\ y_m^* \end{pmatrix}$

prediction intervals $\begin{pmatrix} PI_1 \\ \vdots \\ PI_m \end{pmatrix}$

then $y_i^* \in PI_i$ in .95 of the rows.

* If we have a good WP model, which would arise from XGBoost fit on a dataset with large number of datapoints, then

confidence interval $\hat{I}(x) = [\widehat{WP}_L, \widehat{WP}_u]$

$\widehat{WP}_{go} = .63 \qquad \hat{I}_{go} = [.625, .635]$

* If we have a bad WP model, which would arise from XGBoost fit on a dataset with a small number of datapoints, then

$\widehat{WP}_{go} = .63 \qquad \hat{I}_{go} = [.54, .76]$

yardline
game sec, Rem
point spread
score diff

How can we get confidence intervals on our win probability estimates?

→ Obtaining confidence or prediction intervals for general blackbox machine learning models like XGBoost, Random Forests, or Neural Nets is a fundamental open problem in machine learning today...

→ for some special cases you can do ok.

Process

- Conjectured a way to get CI
- Created a simplified version of football in which the win probability is known and can be explicitly calculated.
- Then we generated a fake historical dataset of football plays which has the same autocorrelated win/loss outcome vector as our real dataset.
- Then we fit XGBoost on this fake historical dataset to estimate WP and get CI
- Then, because this is Simplified Football in which true WP is known, we can simply check if our CI worked.

Simplified Football

- begins at midfield
- each play, the ball moves left or right by 1 yardline with equal probability
- if ball reaches left endzone, team 1 scores TD +1 point
- if ball reaches right endzone, team 2 scores TD −1 point
- ball resets to midfield after each TD
- after N plays, game ends
- if tied after N plays, flip coin to determine winner.

How do we generate a fake historical dataset of simplified football plays?

index: n^{th} play of g^{th} game

Outcome play: $\varepsilon_{gn} \overset{iid}{\sim} \pm 1$

game starts at midfield: $X_{g0} = \frac{L}{2}$, L = length of field

game starts tied: $S_{g0} = 0$

field position at start of play n+1:

$$X_{g,n+1} = \begin{cases} X_{gn} + \xi_{gn} & \text{if prev play not TD} \\ L/2 & \text{if prev play was TD} \end{cases}$$

not a TD $\rightarrow$ $0 < X_{gn} + \xi_{gn} < L$

Score differential at start of play n+1:

$$S_{g,n+1} = \begin{cases} S_{gn}+1 & \text{if } X_{gn}+\xi_{gn} = 0 \\ S_{gn}-1 & \text{if } X_{gn}+\xi_{gn} = L \\ S_{gn} & \text{else} \end{cases}$$

Response column:

identically equal to

$$y_{gn} \equiv y_{g,N+1} = \begin{cases} 1 & \text{if } S_{g,N+1} > 0 \\ 0 & \text{if } S_{g,N+1} < 0 \\ \text{Bernoulli}(\tfrac{1}{2}) & \text{if } S_{g,N+1} = 0 \end{cases}$$

autocorrelation

Fake Historical Dataset

$$\boxed{\mathcal{D} = \left\{ (\underbrace{n}_{\text{time (game sec. Rem)}}, \underbrace{X_{gn}}_{\text{field position (yardline)}}, \underbrace{S_{gn}}_{\text{score diff}}, \underbrace{y_{gn}}_{\text{win/loss outcome}}) : \begin{array}{l} g = 1, \dots, G \\ n = 1, \dots, N \end{array} \right\}}$$

autocorrelation

$G \approx 4000$

How do we explicitly calculate WP?

true win probability

$$\boxed{WP(\underbrace{n}_{\text{time}}, \underbrace{x}_{\text{field pos}}, \underbrace{s}_{\text{score diff}}) = \mathbb{P}\left(S_{g,N+1} > 0 \,\middle|\, X_{gn} = x, S_{gn} = s \right)}$$

How to evaluate $WP(n,x,s)$?

$$WP(N+1, x, s) = \begin{cases} 1 & \text{if } s > 0 \\ 1/2 & \text{if } s = 0 \\ 0 & \text{if } s < 0 \end{cases}$$

dynamic programming

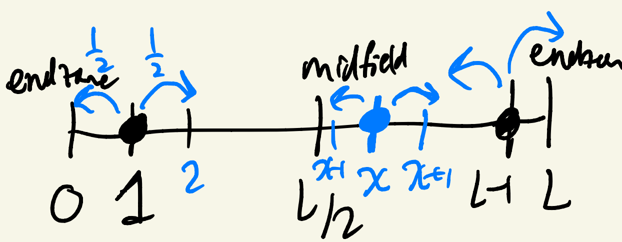
Recursion

write $WP(n-1, x, s)$ in terms of $WP(n, x, s)$.

$$WP(n-1, x, s) = \begin{cases} \frac{1}{2}\cdot WP(n, x=2, s) + \frac{1}{2}\cdot WP(n, \frac{L}{2}, s+1) & \text{if } x=1. \\ \frac{1}{2}\cdot WP(n, x=\frac{L}{2}, s-1) + \frac{1}{2}\cdot WP(n, L-2, s) & \text{if } x=L-1. \\ \frac{1}{2}\cdot WP(n, x+1, s) + \frac{1}{2}\cdot WP(n, x-1, s) & \text{else.} \end{cases}$$

- WP of last play (known)
- WP of 2nd to last play known in terms of WP of last play

...



Repeat this logic all the way to the 1st play.

$WP(n, x, s)$ is Known.

Fake historical dataset $\mathcal{D} = \left\{ (n, X_{gn}, S_{gn}, y_{gn}) : \begin{matrix} g=1,\ldots,G \\ n=1,\ldots,N \end{matrix} \right\}$

$\widehat{WP} = \text{XGBoost}(\mathcal{D})$

$\widehat{CI} = \text{some\_method}(\mathcal{D})$

We can evaluate how good $\widehat{WP}$ and $\widehat{CI}$ are because $\mathbf{WP}(n, x, s)$ is Known!

Tomorrow: $\widehat{CI}$ for $\widehat{WP}$