Moneylines and Win Probability

Ryan Brill

What is a moneyline bet?

Suppose the Eagles are playing the Patriots on Sunday night. If you go online to place a sports bet, there's a good chance you'll run into a bet that looks like this:

Philadelphia Eagles +200 New England Patriots -220

This type of bet is called a **moneyline** bet. In fact, these were the opening moneylines for the Superbowl LII in Minneapolis (in 2018). The team whose moneyline is negative is the favorite to win the game (the favorite is more likely to win the game), and the team whose moneyline is positive is the underdog. Here's how moneyline bets work: since the Patriots are the favorite, if the Patriots win the game, then you win \$100 for every \$220 you bet on the Patriots. Since the Eagles are the underdog, if the Eagles win the game, then you win \$200 for every \$100 you bet on the Eagles. We will refer to this example throughout the article.

Finding the implied win probability

Suppose that the favorite wins the game with some probability p, so $0.5 \le p \le 1$. Then the underdog wins the game with probability 1-p. Let M_f denote the moneyline of the favorite, so in our example, $M_f = 220$; and let M_u denote the moneyline of the underdog, so in our example, $M_u = 200$. Let the moneyline M_u be the average of M_f and M_u , so in our example $M_u = 210$. Note that in general, M_u is an integer multiple of 10 and M_u > 100. We'd like to find the **implied win probability**, the probability that the favorite wins the game, given the moneyline – i.e., an expression for p using M_u .

If you bet \$B on the favorite, then your expected net return is

$$\mathbb{E}[R_f] = p \cdot \$B(\frac{\$100}{\$M}) + (1-p) \cdot (-\$B)$$

Similarly, if you bet \$B on the underdog, then your expected net return is

$$\mathbb{E}[R_u] = p \cdot (-\$B) + (1-p) \cdot \$B(\frac{\$M}{\$100})$$

If $\mathbb{E}[R_f] > 0$, then everyone who could calculate $\mathbb{E}[R_f]$ would bet on the favorite, which would expose the sportsbook to a lot of risk, meaning the sportsbook would lose money if the favorite wins. If $\mathbb{E}[R_f] < 0$, then

$$\mathbb{E}[R_f] = p \cdot \$B(\frac{\$100}{\$M}) + (1-p) \cdot (-\$B) < 0$$

Rearranging this inequality yields

$$\rho < \frac{M}{M + 100}$$

which implies

$$\mathbb{E}[R_{u}] = p \cdot (-\$B) + (1-p) \cdot (\$B)(\frac{\$M}{\$100})$$

$$> -\left(\frac{M}{M+100}\right)(\$B) + (1-\left(\frac{M}{M+100}\right))(\$B)(\frac{M}{100})$$

$$= -\left(\frac{M}{M+100}\right)(\$B) + \left(\frac{100}{M+100}\right)(\$B)(\frac{M}{100})$$

$$= -\left(\frac{M}{M+100}\right)(\$B) + \left(\frac{M}{M+100}\right)(\$B)$$

$$= 0$$

i.e. $\mathbb{E}[R_u] > 0$. In this case, everyone who could calculate $\mathbb{E}[R_u]$ would bet on the underdog, which would expose the sportsbook to a lot of risk. The sportsbook is not exposed to this extra risk if $\mathbb{E}[R_f] = 0$ (in which case $\mathbb{E}[R_u] = 0$ too). Solving $\mathbb{E}[R_f] = 0$ yields

$$p = \frac{M}{M + 100}$$
 and $1 - p = \frac{100}{M + 100}$

so we have found the desired implied win probability.

Assumptions

The primary assumption of this derivation is setting $\mathbb{E}[R_f] = 0$. In English, this assumption says that the expected value of betting on the favorite is 0. Intuitively, this makes sense - you should not expect to make money in the long run simply by betting on the favorite. In future work, it would be interesting to see whether this holds in practice.

Conclusion

So, the next time you are at a Las Vegas sportsbook, take a look at some of the moneylines and find an implied win probability.