A ~ N(m, v) where m and v are known. Two-tailed test that A is significantly nonzero.

If $A \sim N(m, v)$, and you want to perform a **two-tailed test** of the null hypothesis $H_0 : A = 0$ versus the alternative $H_1 : A \neq 0$, here's how to proceed:

Test Setup

- Null hypothesis: $\quad H_0 : m = 0$
- Alternative hypothesis: $\quad H_1 : m \neq 0$
- Known variance: $\quad \mathrm{Var}(A) = v$
- A is normally distributed.

Test Statistic

The standardized test statistic is:

$$Z = \frac{m}{\sqrt{v}} \sim N(0, 1) \text{ under } H_0$$

P-Value (Two-Tailed)

$$p = 2 \cdot \Phi\left(-\left|\frac{m}{\sqrt{v}}\right|\right)$$

where Φ is the standard normal CDF.

The setup:

You have a **single known quantity** $A \sim N(m, v)$, and you know both the mean m and variance v. But you want to test if A is **significantly different from 0**—that is, whether the value of A is statistically inconsistent with 0, given the uncertainty from the distribution.

Why the Z-test is appropriate:

1. **Known distribution:**
   Since $A \sim N(m, v)$, the distribution of A is fully specified under the null hypothesis $H_0 : m = 0$. Under H_0, $A \sim N(0, v)$.
2. **Standardization:**
   To evaluate how extreme $A = m$ is under the null, we standardize:
   $$Z = \frac{A - 0}{\sqrt{v}} = \frac{m}{\sqrt{v}}$$
   This follows a standard normal distribution under H_0.
3. **Two-tailed nature:**
   You want to know if A is *significantly nonzero*—that is, either too positive or too negative to plausibly come from $N(0, v)$. Hence, a two-tailed test is appropriate.
4. **Decision framework:**
   Using the standard normal distribution, you can compute how likely it is to observe a value as extreme as m, assuming $A \sim N(0, v)$. If it's too unlikely (e.g., $p < 0.05$), you reject the idea that A came from $N(0, v)$, i.e., reject H_0.

In summary:

You're not trying to estimate parameters from data—you already know m and v. Instead, you're assessing whether a known value $A = m$ is consistent with being drawn from a normal distribution centered at 0. That makes the Z-test using $Z = m/\sqrt{v}$ the natural and correct approach.

Would you like to see a Bayesian perspective or gec tric intuition for this test?