CSE881 HW3

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Sep 25th, 2016

Problem 1

(a)

$$
\begin{aligned}
d(\boldsymbol{x},\boldsymbol{y}) &= 1 - c(\boldsymbol{x},\boldsymbol{y}) \\
c(\boldsymbol{x},\boldsymbol{y}) &= 1 - d(\boldsymbol{x},\boldsymbol{y}) \\
cos(\boldsymbol{x},\boldsymbol{y}) &= 1 - d(\boldsymbol{x},\boldsymbol{y}) \\
cos\frac{\pi}{4} &\le cos(s,y) \le cos0 \\
1 - cos0 &\le d(\boldsymbol{x},\boldsymbol{y}) \le cos1 - \frac{\pi}{4} \\
0 &\le d(\boldsymbol{x},\boldsymbol{y}) \le 1
\end{aligned}
$$

(b)

Based on (a), Yes, it satisfies the positivity preperty

(c)

$$
\begin{aligned}
d(\boldsymbol{x},\boldsymbol{y}) &= 1 - cos(\boldsymbol{x},\boldsymbol{y}) \\
&= 1 - cos(\boldsymbol{y},\boldsymbol{x}) \\
&= d(\boldsymbol{y},\boldsymbol{x})
\end{aligned}
$$

Yes, it satisfies the symmetry preperty

(d)

No, for example:

$$
\begin{aligned}
\boldsymbol{x} &= (3,4) \\
\boldsymbol{y} &= (5,12) \\
\boldsymbol{z} &= (7,24) \\
d(\boldsymbol{x},\boldsymbol{y}) &= 1 - \frac{15+48}{5*13} = \frac{2}{65} \\
d(\boldsymbol{y},\boldsymbol{z}) &= 1 - \frac{35+288}{13*25} = \frac{2}{325} \\
d(\boldsymbol{x},\boldsymbol{z}) &= 1 - \frac{21+96}{5*25} = \frac{8}{125} \\
d(\boldsymbol{x},\boldsymbol{y}) + d(\boldsymbol{y},\boldsymbol{z}) &= \frac{12}{325} < \frac{8}{125} = d(\boldsymbol{x},\boldsymbol{z})
\end{aligned}
$$

Problem 2

(a)
Let $\boldsymbol{U}(\boldsymbol{q}, \epsilon)$ denote a circle neighbourhood of point $\boldsymbol{q}$ within a radius of ϵ;

$$\forall \boldsymbol{p} \in \boldsymbol{S}$$

$$\begin{aligned}
d(\boldsymbol{p}, \boldsymbol{y}) + d(\boldsymbol{p}, \boldsymbol{q}) &\geq d(\boldsymbol{q}, \boldsymbol{y}) \\
d(\boldsymbol{p}, \boldsymbol{q}) &\geq d(\boldsymbol{q}, \boldsymbol{y}) - d(\boldsymbol{p}, \boldsymbol{y}) \\
d(\boldsymbol{q}, \boldsymbol{y}) + d(\boldsymbol{p}, \boldsymbol{q}) &\geq d(\boldsymbol{p}, \boldsymbol{y}) \\
d(\boldsymbol{p}, \boldsymbol{q}) &\geq d(\boldsymbol{p}, \boldsymbol{y}) - d(\boldsymbol{q}, \boldsymbol{y}) \\
if\ min[d(\boldsymbol{q}, \boldsymbol{y}) - d(\boldsymbol{p}, \boldsymbol{y}), d(\boldsymbol{p}, \boldsymbol{y} - d(\boldsymbol{q}, \boldsymbol{y})] &> \epsilon \\
d(\boldsymbol{p}, \boldsymbol{q}) &> \epsilon \\
\boldsymbol{p} &\notin \boldsymbol{U}(\boldsymbol{q}, \epsilon) \\
No\ more\ calculation\ needed.
\end{aligned}$$

$$\begin{aligned}
d(\boldsymbol{p}, \boldsymbol{q}) &\leq d(\boldsymbol{p}, \boldsymbol{y}) + d(\boldsymbol{q}, \boldsymbol{y}) \\
if\ d(\boldsymbol{p}, \boldsymbol{y}) + d(\boldsymbol{q}, \boldsymbol{y}) &< \epsilon \\
d(\boldsymbol{p}, \boldsymbol{q}) &< \epsilon \\
\boldsymbol{p} &\in \boldsymbol{U}(\boldsymbol{q}, \epsilon) \\
No\ more\ calculation\ needed.
\end{aligned}$$

(b)
Let $\boldsymbol{U}(\boldsymbol{x}, \beta)$ denote a circle neighbourhood of point $\boldsymbol{x}$ within a radius of β;

$$
\begin{aligned}
\forall \boldsymbol{x_i} &\in \boldsymbol{S}; \\
we\ have\ \boldsymbol{x_i^*} &\in \boldsymbol{\Psi}\ (i \in \mathbb{N}^*) \\
d(\boldsymbol{x_i}, \boldsymbol{x_i^*}) &\leq \epsilon \\
\forall i, j &\in \mathbb{N}^*
\end{aligned}
$$

$$
\begin{aligned}
d(\boldsymbol{x_i}, \boldsymbol{x_j}) &\leq d(\boldsymbol{x_i}, \boldsymbol{x_i^*}) + d(\boldsymbol{x_i^*}, \boldsymbol{x_j}) \\
&\leq d(\boldsymbol{x_i}, \boldsymbol{x_i^*}) + (d(\boldsymbol{x_i^*}, \boldsymbol{x_j^*}) + d(\boldsymbol{x_j}, \boldsymbol{x_j^*}) \\
&= (d(\boldsymbol{x_i}, \boldsymbol{x_i^*}) + d(\boldsymbol{x_j}, \boldsymbol{x_j^*})) + d(\boldsymbol{x_i^*}, \boldsymbol{x_j^*}) \\
&\leq 2\epsilon + d(\boldsymbol{x_i^*}, \boldsymbol{x_j^*}) \\
if\ 2\epsilon + d(\boldsymbol{x_i^*}, \boldsymbol{x_j^*}) &< \beta \\
d(\boldsymbol{x_i}, \boldsymbol{x_j}) &< \beta \\
\boldsymbol{x_i} &\in \boldsymbol{U}(\boldsymbol{x_j}, \beta)
\end{aligned}
$$

$No\ more\ calculation\ needed.$

$$
\begin{aligned}
d(\boldsymbol{x_i^*}, \boldsymbol{x_j^*}) &\leq d(\boldsymbol{x_i}, \boldsymbol{x_i^*}) + d(\boldsymbol{x_i}, \boldsymbol{x_j^*}) \\
&\leq d(\boldsymbol{x_i}, \boldsymbol{x_i^*}) + (d(\boldsymbol{x_i}, \boldsymbol{x_j}) + d(\boldsymbol{x_j}, \boldsymbol{x_j^*}) \\
&= (d(\boldsymbol{x_i}, \boldsymbol{x_i^*}) + d(\boldsymbol{x_j}, \boldsymbol{x_j^*})) + d(\boldsymbol{x_i}, \boldsymbol{x_j}) \\
&\leq 2\epsilon + d(\boldsymbol{x_i}, \boldsymbol{x_j}) \\
d(\boldsymbol{x_i}, \boldsymbol{x_j}) &\geq d(\boldsymbol{x_i^*}, \boldsymbol{x_j^*}) - 2\epsilon \\
if\ d(\boldsymbol{x_i^*}, \boldsymbol{x_j^*}) - 2\epsilon &> \beta \\
d(\boldsymbol{x_i}, \boldsymbol{x_j}) &> \beta \\
\boldsymbol{x_i} &\notin \boldsymbol{U}(\boldsymbol{x_j}, \beta)
\end{aligned}
$$

$No\ more\ calculation\ needed.$

Problem 3

(a)

$$
\begin{aligned}
Var(\boldsymbol{x}) &= 0.07263963 \\
Var(\boldsymbol{y}) &= 0.1110305
\end{aligned}
$$

The variance on dimension $\boldsymbol{y}$ is larger.The median of this dimension is 0.5472

For the first part:

$$var(0.58530.22380.50600.7513) = 0.04854933$$
$$var(0.13860.14930.25430.2575) = 0.004198389$$

The variance on dimension $\boldsymbol{x}$ is larger. The median is 0.5060.

For the second part:

$$var(0.11900.25510.49840.9597) = 0.1364748$$
$$var(0.69910.84070.89090.9593) = 0.01215053$$

The variance on dimension $\boldsymbol{x}$ is larger. The median is 0.2551.

For (0.5853,0.1386),(0.7513 ,0.2575):

$$var(0.5853, 0.7513) = 0.013778$$
$$var(0.1386, 0.2575) = 0.007068605$$

The variance on dimension $\boldsymbol{x}$ is larger. The median is 0.5853.

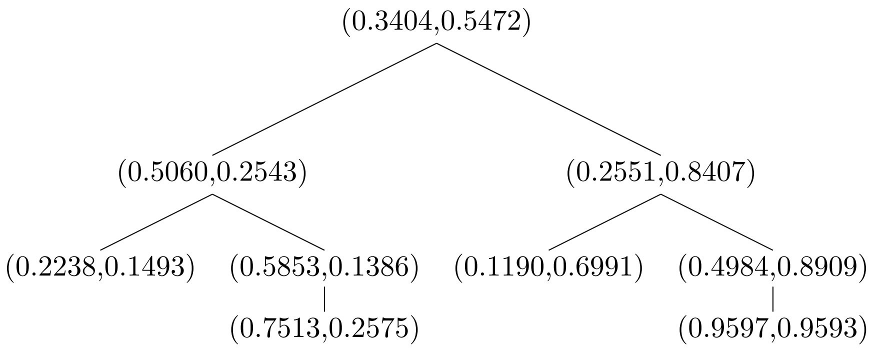
For (0.4984,0.8909),(0.9597,0.9593)

$$var(0.4984, 0.9597) = 0.1063988$$
$$var(0.8909, 0.9593) = 0.00233928$$

The variance on dimension $\boldsymbol{x}$ is larger. The median is 0.4984

The ordered date set:

	$[x]$	$[y]$
[1,]	0.2238	0.1493
[2,]	0.5060	0.2543
[3,]	0.5853	0.1386
[4,]	0.7513	0.2575
[5,]	0.3404	0.5472
[6,]	0.1190	0.6991
[7,]	0.2551	0.8407
[8,]	0.4984	0.8909
[9,]	0.9597	0.9593



(b)

(c)

$$
\begin{gathered}
(0.3404, 0.5472) \\
(0.2551, 0.8407) \\
(0.2238, 0.1493) \\
(0.5853, 0.1386) \\
(0.4984, 0.8909)
\end{gathered}
$$

Problem 4

(a)

$$
\begin{array}{c}
\begin{array}{ccccc} & d_1 & d_2 & d_3 & d_4 \end{array} \\
\begin{array}{c} d_1 \\ d_2 \\ d_3 \\ d_4 \end{array}
\begin{pmatrix}
1 & \frac{1}{3} & \frac{2}{7} & \frac{3}{8} \\
\frac{1}{3} & 1 & \frac{3}{7} & \frac{5}{7} \\
\frac{2}{7} & \frac{3}{7} & 1 & \frac{1}{7} \\
\frac{3}{8} & \frac{5}{7} & \frac{1}{7} & 1
\end{pmatrix}
\end{array}
$$

(b)

Calculation for this question are shown at the end of the work.

$$
\begin{array}{c}
\begin{array}{ccccccc} & h_1 & h_2 & h_3 & h_4 & h_5 & h_6 \end{array} \\
\begin{array}{c} d_1 \\ d_2 \\ d_3 \\ d_4 \end{array}
\begin{pmatrix}
10 & 3 & 2 & 3 & 9 & 10 \\
10 & 3 & 4 & 6 & 6 & 10 \\
3 & 3 & 4 & 4 & 4 & 4 \\
10 & 7 & 2 & 6 & 6 & 10
\end{pmatrix}
\end{array}
$$

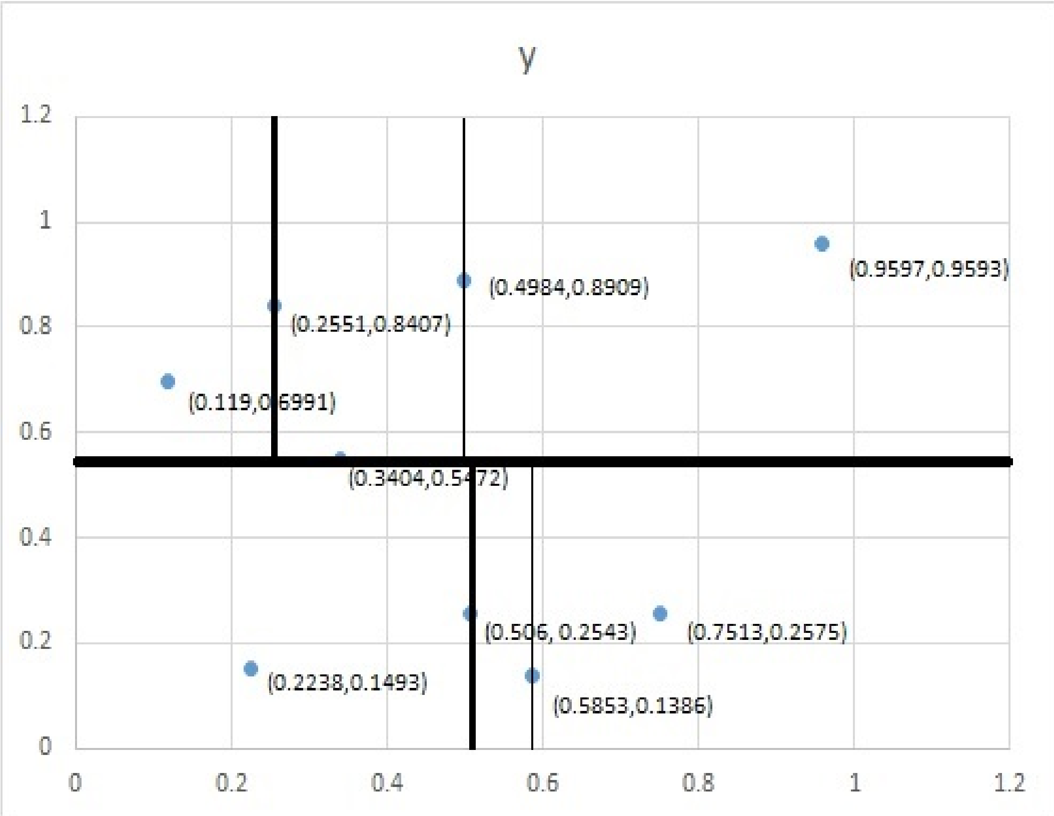


Figure 1: Problem 3 (b)

(c)

$$\begin{array}{c|cccc} & d_1 & d_2 & d_3 & d_4 \\ \hline d_1 & 1 & \frac{1}{2} & \frac{1}{6} & \frac{1}{2} \\ d_2 & \frac{1}{2} & 1 & \frac{1}{3} & \frac{2}{3} \\ d_3 & \frac{1}{6} & \frac{1}{3} & 1 & 0 \\ d_4 & \frac{1}{2} & \frac{2}{3} & 0 & 1 \end{array}$$

(d) Yes, it's $p[h(d_2) = h(d_4)]$

(e) Yes, it's $p[h(d_3) = h(d_4)]$

Calculation for 3(b)

$$\begin{pmatrix} \mathbf{[10]} & \mathbf{[3]} & \mathbf{[8]} & \mathbf{[9]} & \mathbf{[6]} & \mathbf{[5]} & \mathbf{[4]} & \mathbf{[2]} & \mathbf{[1]} & \mathbf{[7]} \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$h(D_1) = (10, 10, 3, 10)$$

$$\begin{pmatrix} \mathbf{[3]} & \mathbf{[9]} & \mathbf{[7]} & \mathbf{[5]} & \mathbf{[4]} & \mathbf{[2]} & \mathbf{[6]} & \mathbf{[8]} & \mathbf{[1]} & \mathbf{[10]} \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$h(D_2) = (3, 3, 3, 7)$$

$$\begin{pmatrix} \mathbf{[4]} & \mathbf{[2]} & \mathbf{[3]} & \mathbf{[1]} & \mathbf{[7]} & \mathbf{[5]} & \mathbf{[6]} & \mathbf{[9]} & \mathbf{[8]} & \mathbf{[10]} \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$h(D_3) = (2, 4, 4, 2)$$

$$\begin{pmatrix} \mathbf{[6]} & \mathbf{[4]} & \mathbf{[3]} & \mathbf{[10]} & \mathbf{[7]} & \mathbf{[8]} & \mathbf{[5]} & \mathbf{[9]} & \mathbf{[1]} & \mathbf{[2]} \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$h(D_4) = (3, 6, 4, 6)$$

$$\begin{pmatrix} \mathbf{[9]} & \mathbf{[6]} & \mathbf{[8]} & \mathbf{[5]} & \mathbf{[1]} & \mathbf{[4]} & \mathbf{[7]} & \mathbf{[2]} & \mathbf{[3]} & \mathbf{[10]} \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$h(D_5) = (9, 6, 4, 60)$$

$$\begin{pmatrix} \mathbf{[10]} & \mathbf{[1]} & \mathbf{[4]} & \mathbf{[6]} & \mathbf{[9]} & \mathbf{[2]} & \mathbf{[7]} & \mathbf{[3]} & \mathbf{[8]} & \mathbf{[5]} \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$h(D_6) = (10, 10, 4, 10)$$