CSE881 HW9

Nan Cao, A52871775

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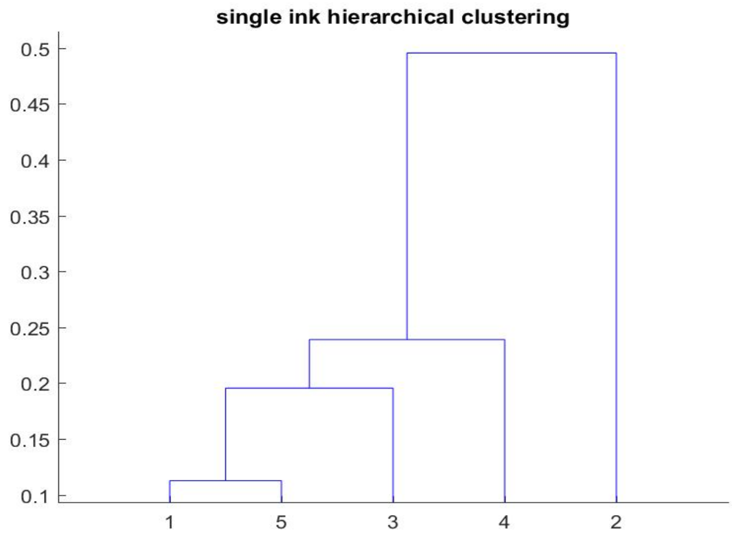
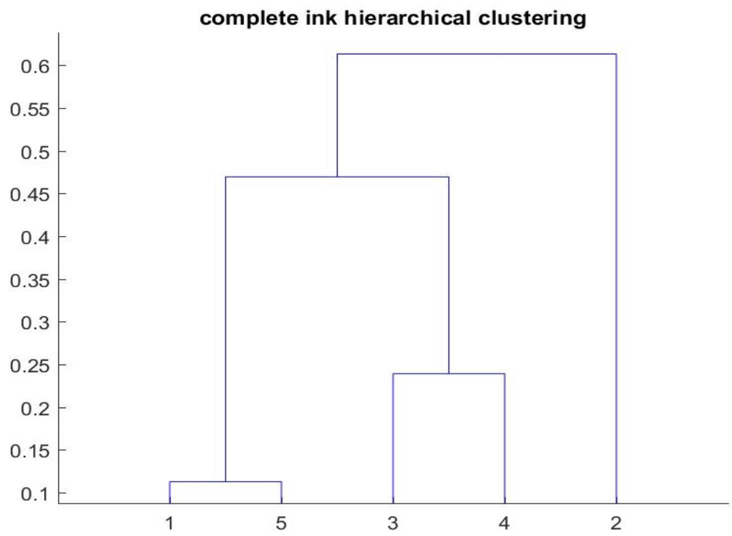
Problem 1





Problem 2

(a)
a,b,c,d,e,f,g,h,i,j,k;
m,o,p,q,r,s,t,u,v,w,x
(b)
l,n,y
(c)
z
(d)
2 clusters will be ontained.

Problem 3

(a)

$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\mathbf{L} = \mathbf{D} - \mathbf{W} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

(b)

$$
\lambda = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 & 0 & 0.2679 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 1.5858 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 3.7321 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 4.4142
\end{bmatrix}
$$

$$
\mathbf{V} = \begin{bmatrix}
0.3780 & -0.4440 & -0.2808 & 0.7071 & 0.0020 & 0.2299 & -0.1683 \\
0.3780 & -0.4440 & -0.2808 & -0.7071 & -0.0020 & 0.2299 & -0.1683 \\
0.3780 & -0.3251 & 0.1645 & -0.0000 & 0.0000 & -0.6280 & 0.5745 \\
0.3780 & -0.0000 & 0.7941 & -0.0000 & 0.0000 & 0.0000 & -0.4760 \\
0.3780 & 0.4440 & -0.2808 & 0.0020 & -0.7071 & -0.2299 & -0.1683 \\
0.3780 & 0.3251 & 0.1645 & 0.0000 & -0.0000 & 0.6280 & 0.5745 \\
0.3780 & 0.4440 & -0.2808 & -0.0020 & 0.7071 & -0.2299 & -0.1683
\end{bmatrix}
$$

$$\lambda = (0, 0.2679, 1.5858, 3, 3, 3.7321, 4.4142)^T$$

The smallest three eigenvalues:

$$\lambda_{(1)} = 0, \lambda_{(2)} = 1.5858, \lambda_{(3)} = 3$$

(c)

$$
\begin{aligned}
\mathbf{e}_{(1)} =& (0.3780, 0.3780, 0.3780, 0.3780, 0.3780, 0.3780, 0.3780)^T \\
\mathbf{e}_{(2)} =& (-0.4440, -0.4440, -0.3251, -0.0000, 0.4440, 0.3251, 0.4440)^T \\
\mathbf{e}_{(3)} =& (-0.2808, -0.2808, 0.1645, 0.7941, -0.2808, 0.1645, -0.2808)^T
\end{aligned}
$$

(d)

EigVec	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	$\mathbf{e}_4$	$\mathbf{e}_5$	$\mathbf{e}_6$	$\mathbf{e}_7$
Cluster	1	1	1	2	3	3	3

$\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}$; $\{\mathbf{e}_4\}$; $\{\mathbf{e}_5,\mathbf{e}_6,\mathbf{e}_7\}$;

(e)

$$
\begin{aligned}
Ncut(V_1, V_2, V_3) =& \frac{Cut(V_1, V - V_1)}{d(V_1)} + \frac{Cut(V_2, V - V_2)}{d(V_2)} + \frac{Cut(V_3, V - V_3)}{d(V_3)} \\
=& \frac{2}{7} + 1 + \frac{2}{7} \\
=& \frac{11}{7}
\end{aligned}
$$

(f)

$$
\begin{aligned}
Ncut(V_1, V_2, V_3) =& \frac{Cut(V_1, V - V_1)}{d(V_1)} + \frac{Cut(V_2, V - V_2)}{d(V_2)} + \frac{Cut(V_3, V - V_3)}{d(V_3)} \\
=& \frac{2}{4} + \frac{4}{8} + \frac{2}{4} \\
=& \frac{3}{2}
\end{aligned}
$$

It's larger than the solution found in (d).