CSE881 HW3

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Problem 1

(a)

(i)

Please see the codes in the last part of this pdf.

(ii)

Please see the codes in the last part of this pdf.

(iii)

Use the codes in "Q1.m", we can mathematically find the following combination of non-linearly independent column pairs.

```
(\ 4,\ 35);\ (\ 4,\ 36);\ (\ 4,\ 37);\ (\ 22,\ 35);\ (\ 22,\ 36);\ (\ 22,\ 37);\ (\ 30,\ 35);\ (\ 30,\ 36);\ (\ 30,\ 37);\ (\ 31,\ 35);\ (\ 31,\ 36);\ (\ 31,\ 37);\ (\ 32,\ 35);\ (\ 32,\ 36);\ (\ 32,\ 37);\ (\ 33,\ 35);\ (\ 33,\ 36);\ (\ 33,\ 37);\ (\ 34,\ 35);\ (\ 34,\ 36);\ (\ 34,\ 37);\ (\ 35,\ 37);\ (\ 35,\ 38);\ (\ 35,\ 39);\ (\ 35,\ 40);\ (\ 35,\ 41);\ (\ 35,\ 42);\ (\ 35,\ 48);\ (\ 36,\ 37);\ (\ 36,\ 38);\ (\ 36,\ 39);\ (\ 36,\ 40);\ (\ 36,\ 41);\ (\ 36,\ 42);\ (\ 36,\ 47);\ (\ 36,\ 48);\ (\ 37,\ 38);\ (\ 37,\ 39);\ (\ 37,\ 40);\ (\ 37,\ 41);\ (\ 37,\ 42);\ (\ 37,\ 47);\ (\ 37,\ 48).
```

Only group 4 that contains both of the columns in a pair.

Following are the names of columns:

- 30. weekday is monday: Was the article published on a Monday?
- 31. weekday is tuesday: Was the article published on a Tuesday?
- 32. weekday is wednesday: Was the article published on a Wednesday?
- 33. weekday is thursday: Was the article published on a Thursday?
- 34. weekday is friday: Was the article published on a Friday?
- 35. weekday is saturday: Was the article published on a Saturday?
- 36. weekday is sunday: Was the article published on a Sunday?
- 37. is weekend: Was the article published on the weekend?

Mathematically, all of following are correct answers:

```
( 30, 35); ( 30, 36); ( 30, 37); ( 31, 35); ( 31, 36); ( 31, 37); ( 32, 35); ( 32, 36); ( 32, 37); ( 33, 35); ( 33, 36); ( 33, 37); ( 34, 35); ( 34, 36); ( 34, 37); ( 35, 37); ( 36, 37)
```

And I picked (36,37) as my answer. Firstly, it's mathematically correct. Secondly, the remaining variables show us an aesthetic feeling of mathematics.

Please see the codes in the last part of this pdf.

(b)

The correlation of MLR is -0.0339

The correlation of lasso is -0.229

- (c) THe correlation of kernel is -0.082
- (d) all 3 correlations (MLR lasso kernel) remain the same.

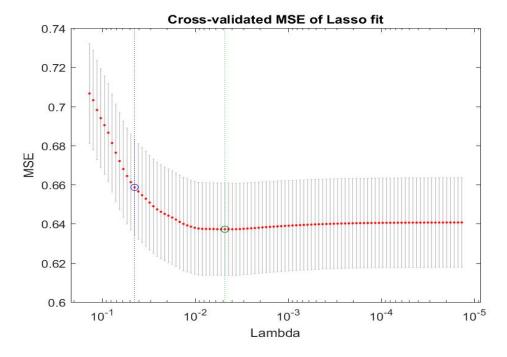


Figure 1: Problem 2b-lasso

Problem 2

(a)

An outlier may have an unusual X or Y. An outlier is an influential point that have great effects on the slope of a regression model. If there are more than 1 outlier, their effect may be quite small, if the effect can be canceled by other's. For example:

$$\begin{array}{lll} x1\!=\!1,2,3,4,5,6 & y1\!=\!2.1,3.9,6.1,8.1,10.2,12.3 \\ x2\!=\!1,2,3,4,5,6,7 & y2\!=\!2.1,3.9,6.1,8.1,10.2,12.3,30 \\ x3\!=\!1,2,3,4,5,6,7 & y3\!=\!2.1,3.9,6.1,8.1,10.2,12.3,3 \end{array}$$

(b)

$$min \sum_{i} a_{i}(y_{i} - w^{T}x_{i})^{2} + \lambda ||w||$$

$$= min(\sqrt{a_{i}}y_{i} - w^{T}\sqrt{a_{i}}x_{i}) + \lambda ||w||$$
so let X^{*} denote $(\sqrt{a_{1}}x_{1}, \sqrt{a_{2}}x_{2}, ..., \sqrt{a_{n}}x_{n})$

$$let Y^{*}$$
 denote $(\sqrt{a_{1}}y_{1}, \sqrt{a_{2}}y_{2}, ..., \sqrt{a_{n}}y_{n})$

$$w = [X^{*T}X^{*} + \lambda I]^{-1}X^{*T}y^{*}$$

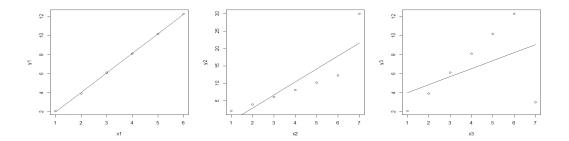


Figure 2: Problem 3c-2

Problem 3

$$\begin{split} E(Sample) &= -\frac{1}{2}log_2\frac{1}{2} - \frac{1}{2}log_2\frac{1}{2} \\ &= 1 \\ E(A) &= \frac{1}{2}(-\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3}) \\ &= 0.9183 \\ E(B) &= \frac{1}{3}(-\frac{1}{2}log_2\frac{1}{2} - \frac{1}{2}log_2\frac{1}{2}) + \frac{1}{12}(-\frac{5}{5}log_5\frac{5}{5} - \frac{0}{5}log_2\frac{0}{0}) + \frac{7}{12}(-\frac{3}{7}log_2\frac{3}{7} - \frac{4}{7}log_2\frac{4}{7}) \\ &= 0.9081 \end{split}$$

B is better because 1-E(B) is bigger.

Codes

1 % NAN CAO CSE881 HW4 2 % set dir in nan's win lap 3 **cd** C:\Users\nan66\Dropbox\CSE881\HW4\; %set dir in nan's linux lap 5 % cd /home/nan/Dropbox/CSE881/HW4; 6 *787.77*% 7 %(a)%%%%% 8 ONP=csvread('OnlineNewsPopularity.csv',1,0); Pre=ONP(:,1:58);%col 1~58 as Predict Variable Tar=ONP(:,59); %col 59 as the Response/Target Variable 11 Tar = log(Tar);12 Tar=Tar(:);13 $\operatorname{Tar}(\operatorname{Tar} = \mathbf{Inf}) = 0;$ rPre=rank(Pre); %rank of Predict Matrix 15 a=0;%set 3 initial variables16 17 b=0;18 c = 0;

```
19 for i = 1:57
20 for j=(i+1):58 % try all the possible combination of 2 columns
21 A=Pre;
22 A(:,j) = []; %remove the one with greater column number first
23 A(:, i) = [];
24 rA=rank(A);
25 if rA==56;
26 \quad a = a + 1; \\ \textit{\%use a to calculate the number of the possible ij}
27
   b(a)=i; %save possible i
   c(a)=j; %save possible j
29
   \mathbf{end}
30 end
31
   end
   d=[b;c]'; %show all the possible combination of removing
32
33 d
34 %Remove column 36(is it Su) and column 37(is it Weekend);
35 Pre1=Pre;
36 \text{ Rm} = [36, 37];
37 Pre1=Pre;
38 Pre1(:,Rm) = [];
39 rank(Pre1)
40 %%%%%
41 %(b)
42 %%%%%
43 Len=length(Tar);
44 TrainPre=Pre1(1:2000,:);
45 TrainTar=Tar(1:2000);
46 TestPre=Pre1(2001:Len,:);
   TestTar=Tar(2001:Len);
47
48
   sTrainPre=zscore (TrainPre); %standardize
   sTestPre=zscore (TestPre);
49
   % TrainTar=zscore (TrainTar);%standardize
   % TestTar=zscore(TestTar);
   TrainPre1 = [TrainPre ones(2000,1)]; % add a column of 1s
53
   sTrainPre1 = [sTrainPre ones(2000,1)];
   TestPre1 = [TestPre ones(Len-2000,1)];
54
55
   sTestPre1 = [sTestPre ones(Len-2000,1)];
56 %MLR
57 wMLR=regress (TrainTar, sTrainPre1);
58 \quad \% lasso
   [w, stats]=lasso(sTrainPre1, TrainTar, 'Alpha', 1, 'CV', 10);
  Figure1=lassoPlot(w, stats, 'PlotType', 'CV');
   saveas(Figure1, 'Q1b', 'jpeg');
62
   wbest=w(:, stats.Index1SE');
63
64 %cal top 10
65 [a1,b1]=sort(abs(wMLR), 'descend');
66 a1(1:10)
67 b1(1:10)
68
  [a2,b2]=sort(abs(wbest), 'descend');
69 a2(1:10)
70 b2(1:10)
   %cal predicted values
72 MLRPreVal=sTestPre1*wMLR;
```

```
73 CorMLR=corr (MLRPreVal, TestTar)
74 LasPreVal=sTestPre1*wbest;
75 CorLas=corr (LasPreVal, TestTar)
76 %%%%
77 \%(c)
78 %%%%
79 Len=length(Tar);
80 TrainPre=Pre1(1:2000,:);
81 TrainTar=Tar(1:2000);
    TestPre=Pre1(2001:Len,:);
   TestTar=Tar(2001:Len);
   lambda = 0.001
84
    % calculate \ alpha \ for \ function \ 'apply Kernel'
85
    dist1 = pdist(sTrainPre); % calculate distance between every pair of points
    dist1 = squareform(dist1); % convert to square matrix
87
88 K = \exp(-(1e-7) * dist1.^2); % mu: kernel parameter (specified by user)
   alpha = inv(K + lambda*eye(2000)) * TrainTar;
    pred = applyKernel(sTrainPre, TestPre, \ alpha, 1e-7);
    CorKer=corr (pred, TestTar)
92
93
94
95
96
97
   98 \%(d)
99 %%%%
100 Pre2=Pre1;
101 \operatorname{Pre2}(:,18:29) = \log(\operatorname{Pre2}(:,18:29));
102 Pre2 (Pre2 - Inf) = 0;
103 Len=length(Tar);
104
   TrainPre=Pre2(1:2000,:);
105
    TrainTar=Tar(1:2000);
106
    TestPre=Pre2(2001:Len,:);
   TestTar=Tar(2001:Len);
107
   sTrainPre=zscore (TrainPre); %standardize
108
109 sTestPre=zscore(TestPre);
110 TrainPre1 = [TrainPre ones(2000,1)]; % add a column of 1s
   sTrainPre1 = [sTrainPre ones(2000,1)];
111
112 TestPre1 = [sTestPre ones(Len-2000,1)];
113 sTestPre1 = [sTestPre ones(Len-2000,1)];
115 %MLR
116 wMLR=regress (TrainTar, sTrainPre1);
117 \quad \% lasso
118 [w, stats] = lasso(sTrainPre1, TrainTar, 'Alpha', 1, 'CV', 10);
119 Figure1=lassoPlot(w, stats, 'PlotType', 'CV');
120 saveas (Figure1, 'Qld', 'jpeg');
121
   wbest=w(:, stats.Index1SE');
122
123 % % cal top 10
124 % [a1,b1] = sort(abs(wMLR), 'descend');
125 % a1(1:10)
126 % b1(1:10)
```

```
127 \% [a2, b2]=sort(abs(wbest), 'descend');
128 % a2(1:10)
129 % b2(1:10)
130~\% cal~predicted~values
131 \quad \text{MLRPreVal} \!\!=\!\! \text{sTestPre1} \!*\! \text{wMLR};
132 CorMLR2=corr (MLRPreVal, TestTar)
133 \quad Las PreVal = s Test Pre1*wbest;
134 CorLas2=corr(LasPreVal, TestTar)
135 %%%%
136 \%(c)
137 %%%%
138 Len=length(Tar);
139 TrainPre=Pre1(1:2000,:);
140 TrainTar=Tar(1:2000);
141 \quad \texttt{TestPre=Pre1} \left( \, 2\,0\,0\,1 \colon \texttt{Len} \,\, , \colon \right) \,\, ;
142 \operatorname{TestTar}=\operatorname{Tar}(2001:\operatorname{Len});
143 lambda=0.1;
144 \quad \% calculate \quad alpha \quad for \quad function \quad `apply Kernel'
145 dist1 = pdist(sTrainPre); % calculate distance between every pair of points
146 \operatorname{dist1} = \operatorname{squareform}(\operatorname{dist1}); \% \ convert \ to \ square \ matrix
147 K = \exp(-(1e-7) * dist1.^2); % mu: kernel parameter (specified by user)
148 alpha = inv(K + lambda*eye(2000)) * TrainTar;
149 pred = applyKernel(sTrainPre, TestPre, alpha, 1e-7);
150 CorKer2=corr (pred, TestTar)
```