Nonlinear Optimization
Homework #7
(Due on May 20, 2020)

1) Find and classify all the critical (first order necessary condition) points of the function

$$f(x) = -x_1^2 - 4x_2^2 - 16x_3^2$$

subject to $h(x) = 0$ where $h(x)$ is given by: (a) $h(x) = x_1 - 1$, (b) $h(x) = x_1x_2 - 1$.

2) Find the value of $a \in \mathbb{R}$ such that the point $(1, 1)^T$ is a solution of the problem:

$$\begin{array}{ll} \text{Minimize} & (x-a)^2 + (y-1)^2 \\ \text{subject to:} & x^2 + y^2 = 2. \end{array}$$

3) Solve the following problem:

$$\begin{array}{lc} \text{Optimize} & x + y \\ \text{subject to:} & x + y + z = 1 \\ & x^2 + 2y^2 + z^2 = 1. \end{array}$$

Find all the critical points and discuss their regularity condition. Which of them are global extreme points? Which of them are local extreme points?

4) Consider the problem of finding the point on the parabola $5y = (1-x)^2$ that is closest to $(1, 2)^T$ in the Euclidean norm. Find all the first order necessary condition points, and discuss the regularity condition of each one of them. Which of these points are solutions? By directly substituting the constraint into the objective function and eliminating the variable x, we obtain an unconstrained optimization problem. Show that the solutions of this problem cannot be solutions of the original problem.