Bidding for the Antamina Mine: Valuation and Incentives in a Real Options Context

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This chapter studies the bidding for a copper mine that was offered for sale by the Peruvian government as part of the country's privatization program. The mine itself had a valuable real option component, in the form of the right to develop the mine after completing exploration, which we analyze using Monte Carlo methods. A novel aspect of the transaction was the type of bid requested by the Peruvian government, essentially asking bidders to state both the *premium* that they would pay and *exercise price* (pledged development expenditure) they would set for this real option. This structure gave rise to incentives which affected the amount that firms would offer, their preferences between bidding premium and exercise price, the identity of bidders, the likelihood of ultimate development, and the likelihood of *ex post* renegotiation of the contract.

1. Introduction

Since the publication of the seminal work on real options, it has been recognized that real option analysis is a valuable analytical tool that allows managers to quantify the value of flexibility. While the rate of adoption of real option analysis has been uneven across industries, it has been embraced most enthusiastically in the natural resources industry, where academic research first proposed the application of these techniques. This chapter details the bidding for a classic real option in a natural resource industry: the right to develop a copper deposit.

In 1996, the Peruvian government privatized a number of state-owned assets. One of the first to be privatized was Antamina, a polymetallic ore deposit about 500 kilometers north of Lima, Peru. The winning bidder would have two years to explore the property before deciding whether or not to develop the site. This ability to wait-and-see gave rise to a classic development option, which bidders would need to incorporate into their valuation. The first section of the paper discusses the practical issues involved in carrying out a real options analysis, in the tradition of the work of Paddock, Siegel and Smith (1988) and others.

What is unique about bidding for Antamina, however, is not the valuation of the project *per se*, but rather the real option *embedded in the bidding rules* set up by the Peruvian government, and the resultant implications for the behavior of the bidders and the ultimate developer of the deposit. In effect, the auction rules had firms submit both the option premium and the exercise price of the real option to develop the property. By allowing bidders to set both elements of the bid, the government may have substantially reduced the problem of winners' curse and motivated bidders to propose very high investment levels in the plant;

however, it simultaneously increased the likelihood that the ultimate winner would walk away from the project. Furthermore, the rules created an important time-inconsistency problem, giving incentives for re-negotiation after initial exploration of the property was completed.

The next section provides background data on the Antamina mine and the privatization process set up by the Peruvian government. The third section presents the valuation of the mine, using now-standard real option valuation techniques. The fourth section revisits the bidding rules put in place by the Peruvian government and the incentives brought about by these rules. The last section is a concluding postscript.

2. Background on the Antamina Mine[1]

Beginning in the early 1990's, the Peruvian government sought to return many of its state-owned companies to private ownership. The government planned privatizations that would raise $5.75 billion in cash and investment commitments during 1996-1999. One of the larger entities to be sold was Centromin, Peru's largest state-owned mining company, which controlled seven mines, a metallurgical complex, four hydroelectric plants, a railway system, port facilities, and numerous undeveloped natural resource deposits. The firm had been owned by U.S.-based Cerro de Pasco Corporation until 1974, when it was nationalized by the Peruvian government. After two decades of state-ownership, the government attempted to sell the entire company in 1994 and 1995 but without success. In 1996, the plan was to sell

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[1] The material in this section is excerpted with modifications from the Harvard Business School case study "Bidding for Antamina," No. 297-054, prepared by Peter Tufano and Alberto Moel. Copyright © 1997 by the President and Fellows of Harvard College; all rights reserved.

Centromin's eleven properties piecemeal, with the first two parcels being a gold mine and the Antamina mine.

The Antamina mine, located 482 kilometers north of Lima, Peru, was a rich polymetallic copper and zinc deposit. Based on engineering reports made public by the government as part of the auction, the property had proven and probable ore reserves estimated at 127 million metric tons of ore containing 1.7% copper and 0.8% zinc. Centromin's management had publicly stated that these figures vastly underestimated the likely reserves of the property. While part of the ore body had been well-studied by geologists, there had been virtually no geologic study of a large portion of the deposit. As a result, there was large uncertainty about the size of the reserves.

From a practical perspective, none of the potential bidders could carry out anything but limited geological work on the property before the bids were due, and therefore they had to rely on their analysis of surveys produced by Cerro de Pasco and Centromin. In addition, the government was not willing to represent or guarantee the amount of reserves. However, based on previous experience, some industry experts estimated that the actual reserves might range from 100 to 175 million metric tons.

A feasibility study, which would largely consist of additional geological exploration through drilling, could more precisely establish the amount and quality of ore in the Antamina property. Such exploration, which would cost approximately $24 million, was expected to take about 2 years and would be completed before mine construction began. Once additional geological work had been completed, the developer of the property would be "more confident" of the expected reserves, i.e., within ±20%.

Were the subsequent geological work to suggest the mine was economically exploitable, the property would need to be developed to extract the copper and other metals. Antamina was located in a remote mountainous region in Peru at 4000 meters above sea level, and 200 km from the ocean. Developing this site would require the construction of roads, mining rigs, crushing plants and other ancillary facilities, and the purchase and transport of heavy mining equipment, and was expected to last three years. Based on available estimates, the capital expenditures to develop the mine (excluding the feasibility study) ranged from $581 million to $622 million (in 1996 U.S. dollars) and were a function of the amount of ore found.

Table 1 shows estimates of the mine's life, yearly production, capital expenditures, and operating costs for three development scenarios, corresponding to three potential outcomes for the quantity of ore in the deposit. These estimates were derived from discussions with industry experts. The *expected* scenario was thought to represent the mean or median of the distribution of possible ore reserves, the *low* scenario represented 1 to 1.5 standard deviations below the mean (corresponding to the 100 million metric tons estimate), and the *high* scenario (175 million metric tons) represented 1 to 1.5 standard deviations above the mean.

3. Valuation of the Antamina Investment Opportunity

In a "typical" *all-cash* auction, interested firms are asked to bid the amount of cash they are willing to pay up-front for the right to develop the mineral resource. Some of the first applications on real options, such as Paddock, Siegel and Smith (1988) have dealt with this type of problem. In this section, we apply classic real option valuation methodology to value the investment opportunity, as if bidders were asked to submit standard all-cash bids.

The economic value of the Antamina mine is the value of the discounted stream of expected future sales of copper and zinc, less any development and extraction costs. It is well-known that the standard valuation technique of risk-adjusted discounted cash flow (DCF) analysis fails to capture all sources of value associated with this type of investment, in that it assumes that the decision to invest is *irreversible* and *inflexible,* i.e., the investment cash flows are committed and fixed for the life of the project. A main contribution of real options analysis is to incorporate managerial flexibility inherent in the project in its valuation. Added flexibility value, overlooked in DCF analysis, comes from managerial decisions that can take advantage of mineral price movements: operating flexibility and investment timing flexibility.

Operating flexibility includes various options to vary operating parameters, including shutting down and reopening, expanding and contracting operations, abandoning operations completely, optimizing cutoff grades, and varying production rates as prices fluctuate.[2] Investment timing flexibility is the ability to delay and optimally time the start of new property development.[3] Of course, both types of managerial flexibility can be present jointly. For example, Paddock, Siegel, and Smith (1988) value the option to delay exploration and/or

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[2] Models of the operating options to shut down, reopen and abandon a mineral resource project were developed by Brennan and Schwartz (1985 a,b). Applications include those by Palm, Pearson, and Read (1986), who value the option to shut down and reopen low cost and high cost copper mines, Cavender (1992) who values the option to shut down and reopen a small open-pit gold mine, and Mardones (1993) who analyzes flexibility to adjust cutoff grade and to stockpile work in process.

[3] The value of the option to delay development of an oil field for a finite time has been studied by Brennan and Schwartz (1985 a,b), McDonald and Siegal (1986), Paddock, Siegal and Smith (1988), Trigeorgis (1990), Bjerksund and Ekern (1990), Laughton (1991) and Laughton and Jacoby (1991).

extraction of undeveloped offshore petroleum leases for up to 5 years, and then the option to shutdown, reopen, or abandon once the field has been developed.[4]

In this section, we value the *right to develop* the Antamina mineral reserves, focusing on the investment options available to the winning bidder. We value the winning bidder's right to develop the property as a call option, treating the developed mine as the underlying asset and the development expenses as the exercise price of this option. Following the extant literature, we consider the developed mine as a levered claim on a traded underlying asset, where the leverage comes from operating leverage and the traded underlying is a set of forward contracts on copper and zinc. By mapping the underlying asset (the developed mine) into traded assets (metals prices), we can use the risk-neutral valuation methodology developed by Cox and Ross (1976) and Harrison and Kreps (1979).

To value Antamina, we use a Monte Carlo simulation model, implemented within the popular spreadsheet program, Excel. This decision was made for three reasons. First, the European style of this option makes Monte Carlo a feasible approach. Second, the analysis is a practical approach for classroom or practitioner use, as its transparency allows users to see the assumptions used and the workings of the valuation model. Finally, it allows varying both the optimal amount of investment and the mine life based on the amount of ore found. In particular, it allows simulating a series of possible price paths for copper and zinc prices which can then be used as inputs to a DCF model to establish the distribution of values for the underlying asset (the developed mine). The model then values the real option to develop the mine after the two-year

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[4] Trigeorgis (1990) values the option to defer, expand, or cancel a non-fuel mineral project. Bjerksund and Ekern (1990)

feasibility study is completed and the related ore uncertainty is resolved. We make a number of simplifying assumptions:

1. *We focus on the investment timing option, ignoring any operating options.* In particular, we assume that once development of the project has begun, any options to delay or speed up development, temporarily shut down, or reopen, change the production profile (e.g., speed up or slow down production) or the ore grade, or abandon the project can be ignored.

2. *The probability distribution of ore quantity can be adequately characterized by three discrete outcomes.*   In particular, we adopt the three possible quantity outcomes described in Table 1, each with a known probability of occurring.[5]

3. *The risk-free discount rate is known and deterministic.*   We discount the mine revenues (derived from forward prices or simulated at a risk-neutral drift rate) at a constant risk-free rate, as in Brennan and Schwartz (1985a,b). They argue that if the mine owner can enter into forward contracts to sell the mine output in the future at currently agreed-upon prices, the price risk would be eliminated and the relevant discount rate would be the risk-free rate. In Brennan and Schwartz (1985b), they also hypothesize known and certain costs, discounted at the risk-free rate as well.[6] We follow their treatment. Other uncertain variables, such as exchange rates and inflation rates, are also assumed to be known and deterministic.[7]

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carry out a similar analysis, except the option to delay is perpetual.

[5] We estimate that the *low* and *high* case have a 0.2 probability of occurring, while the *expected* outcome has a probability of 0.6. In reality, the quantity would be continuous random variable (while the investment programs would be lumpy.) We use a discrete version of the distribution to simplify the computational burden of the Monte Carlo model.

[6] The assumption that costs can be discounted at the risk-free rate is appropriate if costs are riskless in the sense that they have no systematic risk.

[7] The extension of commodity price modeling to include stochastic interest rates is discussed in Schwartz (1997).

4. Project estimates gathered from industry sources are representative. To arrive at the information needed to value the mine given in Table 1, we interviewed industry experts and consultants. Although the estimated parameters are not necessarily those used by bidders for Antamina, we have been told that they are representative of those used. In addition to the assumptions made in Table 1, we assume working capital of 25% of net revenue (gross revenue less treatment charges), 5-year straight-line depreciation starting in 2001, a tax rate of 30%, and a long-term annual inflation rate of 3.5% for all costs.[8]

5. Copper and zinc prices follow a joint diffusion process. Following the work of Gibson and Schwartz (1990) and Schwartz (1997), we assume that the prices of copper and zinc and their convenience yields follow a joint diffusion process. In particular, the copper and zinc prices follow geometric Brownian motion, while their convenience yields are mean-reverting. The prices of each commodity are negatively correlated with that commodity's convenience yield, while copper and zinc returns are positively correlated with each other. More formally, the dynamics of prices and convenience yields are given by the following:[9]

$$
\begin{aligned}
\frac{dP_c}{P_c} &= (r - \delta_c)\, dt + \sigma_c\, dz_c \\
\frac{dP_z}{P_z} &= (r - \delta_z)\, dt + \sigma_z\, dz_z \\
d\delta_c &= k_c(\alpha_c - \delta_c)\, dt + \sigma_{\delta c}\, dz_{\delta c} \\
d\delta_z &= k_z(\alpha_z - \delta_z)\, dt + \sigma_{\delta z}\, dz_{\delta z}
\end{aligned}
\tag{1}
$$

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[8] The project value will be sensitive to these assumptions. For example, inflation affects the operating costs, investment expenditures, and closure costs of the project; higher inflation will decrease the value of the project.

[9] We make the additional simplifying assumption that the market price of convenience yield risk λ is zero. Schwartz (1997) finds that λ for copper was not significantly different from zero in the period 1988-1995. Given our assumption that the mine can be expressed as a claim on the copper, its λ would also be zero. While this assumption simplifies our

where P's are prices, r is the risk-free rate, δ's are convenience yields, and the subscripts c and z correspond to the copper and zinc processes, respectively. The $\boldsymbol{a}'s$ are the long-term mean convenience yields, while the k's are the mean-reversion coefficients dictating the speed with which the convenience yields approach their long-term means. The dz's are correlated increments to standard Brownian processes, with correlations among their noise terms:

$$
\begin{aligned}
dz_c \, dz_z &= \boldsymbol{r}_{cz} \, dt \\
dz_c \, dz_{\boldsymbol{dc}} &= \boldsymbol{r}_{c,\boldsymbol{dc}} \, dt \\
dz_z \, dz_{\boldsymbol{dz}} &= \boldsymbol{r}_{z,\boldsymbol{dz}} \, dt
\end{aligned}
\tag{2}
$$

where the ρ's denote the corresponding correlation coefficients between the relevant Brownian motions. The first equation relates the prices of copper and zinc, while the second and third equations relate the commodity prices to their respective convenience yields.

To obtain estimates for the relevant parameters, we used copper and zinc spot and futures market data from the London Metals Exchange for the period September 1991 to July 1996. We used the spot and 3-month futures prices of zinc and copper to determine the convenience yield, as given by Gibson and Schwartz (1990), equation (9):

$$
\boldsymbol{d} = r' - 4\ln\left(\frac{F(S,3)}{S}\right)
\tag{3}
$$

where r' is the three-month risk-free rate, S is the spot price, and $F(S,3)$ is the 3-month copper or zinc futures price. To determine the mean-reversion parameters k and $\boldsymbol{a}$, we ran the regression[10]

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analysis, we recognize its limitations. In particular, a portion of the project (the cost structure) is not a tradable asset, and therefore may have a market price of risk.

[10] The methodology used is described in Gibson and Schwartz (1990), and in Dixit and Pindyck (1994).

$$\boldsymbol{d}_t - \boldsymbol{d}_{t-1} = a + b\boldsymbol{d}_{t-1} + \boldsymbol{e}_t \tag{4}$$

for both zinc and copper convenience yields, and calculated

$$\boldsymbol{a} = -\frac{\hat{a}}{\hat{b}}$$

$$k = -\ln(1+\hat{b}) \tag{5}$$

$$\hat{\boldsymbol{s}} = \hat{\boldsymbol{s}}_e \sqrt{\frac{\ln(1+\hat{b})}{(1+\hat{b})^2 - 1}}$$

where $\hat{\boldsymbol{s}}_e$ is the standard error of the regression. Table 2 shows the mean-reversion

parameters k and $\boldsymbol{a}$, the volatilities (calculated over a 90-day window), and the relevant

correlations.

We forecast the copper and zinc prices and convenience yields monthly for the first two

years, using a discrete-time version for the diffusion processes of equation (1), and the relevant

parameters from Table 2. The initial spot prices and convenience yields (at the time of the

bidding in mid-1996) were \$1.00/pound and 20% for copper, and \$0.55/pound and -5% for

zinc, respectively. Under the assumption that a forward market extends indefinitely into the

future and that the convenience yields are mean reverting, the forward price of copper (or zinc)

at time t after year 2 can be approximated by Schwartz (1997), equations (18) and (20):

$$F_1(P,\boldsymbol{d},t) = P \exp\left[-\boldsymbol{d}\frac{1-e^{-kt}}{k} + A(t)\right] \tag{6}$$

$A(t)$ is given by:

$$A(t) = \left(r - \boldsymbol{a} + \frac{1}{2}\frac{\boldsymbol{s}_d^2}{k^2} - \frac{\boldsymbol{s}_d\boldsymbol{s}\boldsymbol{r}_d}{k}\right)t + \frac{1}{4}\boldsymbol{s}_d^2\frac{1-e^{-2kt}}{k^3} + \left(\boldsymbol{a}k + \boldsymbol{s}_d\boldsymbol{s}\boldsymbol{r}_d - \frac{\boldsymbol{s}_d^2}{k}\right)\frac{1-e^{-kt}}{k^2} \tag{7}$$

where σ is the commodity price volatility, $\boldsymbol{d}$ is the (stochastic) convenience yield, α is the long-

term mean convenience yield, k is the convenience yield mean reversion coefficient, $\boldsymbol{r}_d$ is the

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correlation of the convenience yield with the commodity price, and s_d is the convenience yield volatility.[11]

Our Monte Carlo simulation generates 10,000 price and convenience yield paths from year 0 to year 2 using monthly time steps. For each run, the ending metal prices and convenience yields at year 2 are used as inputs into Equations (6) and (7) to generate the forward prices for copper and zinc in future periods. These forward prices are then used to derive the cash flows in three DCF scenarios, which correspond to the high, medium (expected), and low ore outcomes. As a base case, the top graph in Figure 1 shows the distribution of values for the Antamina mine if the mine *had to be developed* in year 2 (i.e., there was no option to delay development, and a standard DCF model applied). The mean NPV of this "real forward contract", i.e., of a commitment to develop at the end of two years, is $454 million, with a wide distribution as shown in the figure.

This distribution (in the top panel of Figure 1) shows that in about four out of ten outcomes the value of developing the mine at year two is negative. The essence of the *real option* is that at this go/no-go point, managers could optimally decide not to proceed after learning the outcome of the exploration by year if the economic value of developing the mine

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[11] Under the assumption that the term structure of convenience yields at the end of year two does *not* display mean-reversion, the forward curve is given by the usual expression

$$F_1(P, d, T) = P \exp[(r - d)T], \tag{6'}$$

where the convenience yield (d) is assumed constant. Schwartz (1997, esp. fig.7), shows that the incorporation of stochastic convenience yields into the calculation of the term structure has a large effect on the shape of these curves, and the effect differs across different time periods. While the spot prices of the metals are correlated in our analysis, our calculation of the forward curves does not explicitly take metal price correlation into account in the construction of the term structure of the forward curves.

turns out to be negative.[12] By walking away from the project these negative outcomes can be truncated, as shown in the bottom graph of Figure 1.

As shown in Table 3, the decision not to develop would be made in 23% of the high-ore scenarios, 39% of the medium ore scenarios, and 53% of the low-ore scenarios. The mine would not be developed when metals prices are low, convenience yields are large (i.e., when the forward curves are shallow or downward sloping) or when the quantity of ore is low. Under these circumstances, the value of a developed mine is not high enough to justify the expenditure of the development charges.

By being able to abandon the project at year 2, management can eliminate all negative-NPV outcomes. In that case, the "expanded" mean NPV of the mine would rise to $704 million. The difference between the two of $250 million, is the value of the development timing (or abandonment) option at year 2. This amounts to 55% of the standard DCF valuation, which is on the high end of the findings of Davis (1996).

As with any valuation model, the outputs are only as valid as the parameters used as inputs to the model. Some executives in the mining industry suggested that these valuations might be "too high," but without access to their private assumptions we cannot ascertain whether this difference is due to our parameter inputs or other factors. Certainly, the choice of the diffusion process is an important modeling decision. Schwartz (1997) points out that the mean-reverting characteristics of metal prices have a first-order impact on valuation and the decision of when to exercise real options. Our application confirms his point. If we calculate

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[12] In reality, the managers could postpone this decision by accepting the penalty, but we do not model this choice.

the mine value ignoring mean reversion after year 2 (i.e., if the forward curve is given by equation given in footnote 10), the base-case NPV of the "real forward contract" is $1.3 billion, while the development timing option increases it to $1.7 billion, for a $400 million value for the option to defer development. These values are 2.5 to 3 times those resulting from the analysis using Schwartz's (1997) mean-reverting specification for the dynamics of metals' prices. Were we to ignore all volatility in both the price and convenience yields of these metals, the value of the project would be about a quarter: $112 million for the commitment, and about $158 million with the option to develop.[13] While it is obvious that the level of volatility affects the valuation of the real option, the seemingly-technical choice of modeling the diffusion process of commodity prices may have as large an impact on project valuation.

The historical data used to generate the convenience yield process parameters can also have a material impact on the valuation. In the past, copper and zinc convenience yields have been highly unstable. For example, the 3-month convenience yield of zinc oscillated between - 10% and 60% in the period 1991-1996, with periods of relative stability followed by short bursts of high volatility (i.e. the distribution of convenience yields is "long-tailed" and skewed.) For this reason, the choice of the window for estimation of the convenience yield parameters will influence these parameters, and the resulting mine valuation.

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[13] Even if there is no commodity price volatility, the option value is still positive since there is ore quantity uncertainty captured by the model.

4. The Incentive Impacts of Real-Option Bids

The above valuation of the real option to develop the Antamina mine, while important to potential bidders, is a well-studied problem. The role of information and bidding structure on bid prices has also been extensively studied—at least under all-cash bidding rules.[14] However, the bidding rules established by the Peruvian government—and the incentives they created—are more novel than those previously studied. The auction rules themselves recognized and embodied the development timing option, asking bidders to specify both the premium and the exercise price of the option to develop the property. These rules created strong, and arguably perverse, incentives for potential bidders, leading them to promise to invest large sums in developing the property while increasing the likelihood that they would ultimately choose to walk away from the project.

4.1 The rules of the game:. Bidders for Antamina were asked to submit two figures: an *initial payment* and a *pledged investment commitment*:

- The *initial payment*, was the amount of cash to be paid immediately, and had to exceed $17.5 million.

- For the next two years, the winning bidder could explore the property and, under the auction rules, was obligated to spend at least $13.5 million on exploration. As exploration was projected to cost $24 million, this constraint was not binding in a real sense, except if initial exploration were to rule out all subsequent development.

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[14] For an analysis of the role of private and public information on bids for oil and gas lease auctions using all-cash bids, see Porter (1995). The number and size of the bids received can also be influenced by elements of the bidding process itself, such as the auction mechanism, the speed of the auction process, and the number or order of assets to be auctioned. For a review of this literature, see Lopez-de-Silanes (1996).

- At the end of two years, the winner had to either:

  (1) walk away from the project, give up rights to the property, and owe the government no additional funds; or

  (2) develop the mine. If the winning bidder decided to proceed, and its investment by the end of year five fell short of the amount it promised to invest up front (the pledged *investment commitment*), it would owe the Peruvian government a penalty equal to 30% of this shortfall, to be paid at the end of year 5. The minimum pledged investment commitment was $135 million. The penalty was presumably put in place to prevent firms from making "false promises" regarding their intended investment plans.

- The winning bidder would be the firm proposing the largest total "bid." To establish the value of the "bid," the authorities would add the initial payment and 30% of the investment commitment, giving greater reward to cash payments over promises of future investment.

The actions and decisions of the winning bidder are summarized in the time line of Figure 2.

4.2 *Analyzing the government's bidding rules.*

To place these bidding rules in perspective, it is useful to contrast this auction mechanism to a more traditional one in which firms make *all-cash* bids, proposing only how much cash they would pay today for the right to develop in the future.

Under the all-cash bid, the expected value for each bidder, conditional on winning, is:

$$E[PV(Max(S - X, 0))] - B \qquad (8)$$

where the actual investment cost (X) is the exercise price of the real option to develop the property, the cash bid (B) is the premium paid for the option, and the underlying asset is the value of the developed reserves (S). $PV(\bullet)$ is the present value operator, and $E[\bullet]$ represents the expectation operator.

By contrast, under the rules established to sell Antamina, firms bid both the initial payment B_1 and the promised investment commitment B_2. The expected value now is:

$$E[PV(Max(S - X - PEN, 0))] - B_1 \qquad (9)$$

where *PEN* is the penalty, given by:

$$PEN = 0.3Max(B_2 - X, 0) \qquad (10)$$

Since bidders would always bid B_2 at least equal to X,[15] this simplifies to:

$$E[PV(Max(S - 0.7X - 0.3B_2, 0))] - B_1 \qquad (11)$$

Comparing this actual bidding structure to the above all-cash scheme, the difference is that the Antamina rules have firms propose not only the option premium B_1, but also a portion $(0.3B_2)$ of the total exercise price, $0.7X+0.3B_2$. Thus, the government has bidders propose the two option terms simultaneously.

4.3 *Incentives for bidding.*

In analyzing this auction, one can ask how the all-cash and Antamina bidding rules compare along a variety of dimensions: the bids submitted by the firms, the likely identity of bidders, the winner's subsequent decision regarding development of the property, and the possibility of renegotiating the contract.

In setting bids under all-cash, first-price bid rules, bidders would be unwilling to offer a cash bid that would make their expected return negative, and their reservation price would equal

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[15] If X is the amount a bidder would spend to develop, conditional on proceeding, then it can make an investment commitment promise of at least X without incurring any risk of penalties. By bidding B_2 less than X, the bidder would

the first term in equation (8) above. They face the classic winners' curse problem, in that the winning bidder confronts the distinct possibility of having made the most over-optimistic valuation, and hence *ex-post* overpaying for the property.[16]

The Antamina bidding rules create a number of subtle complications, both with respect to a bidder's choice between the cash and investment commitment portions of their bid, and with respect to their subsequent decisions. In setting the bid, a firm must figure out how to allocate its bid between the initial payment B_1 and the investment commitment B_2. Consider the relative costs and benefits of adding another dollar to each of these portions of the bid. By bidding another dollar in the initial cash payment, the bidder gets an additional dollar in "bid points" under the government's weighting scheme, but must also spend a dollar (conditional on winning.) By bidding another dollar in investment commitment, the bidder gets only $0.30 in "bid points," but must pay the present value of $0.30, conditional on deciding to go forward at year 2. If we compare the bid points received to the expected cost, conditional on winning the bid, there is an incentive for firms to increase their pledged investment commitment rather than the initial cash payment, as shown in Table 4.

Quite simply, it is "cheaper" to bid future promises—on which one can renege—than to bid cash. At an extreme, if the initial payment were zero (and exploration expenses were also zero), a firm could bid an extraordinarily high investment commitment B_2 and still be assured of not losing money or suffering from the winner's curse. This is because a call option cannot have

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not reduce its penalty, but would reduce the likelihood of winning the auction. Thus $B_2 = X$ dominates $B_2 < X$, so that the penalty can be expressed as 0.3 $(B_2 - X)$.

negative value, and with zero premium paid, there is no exercise price that could make buying the option worth less than zero. In this extreme case, the bidder would only develop if ore prices were very high and/or there was a large ore deposit; otherwise it could walk away without spending any money on the project. It couldn't overbid because it could walk away after exploration but before spending any money on the project.

Actually, as the government's rules insisted that bidders invest at least $17.5 as initial payment plus another $13.5 million in exploration, there could be some winner's curse. For example, if a firm bids the minimum initial cash amount and spends $24 million on exploration, one can calculate the highest exercise price (investment commitment) that it could bid and still break even (its reservation investment commitment). Figure 3 plots the outcome to a bidder as a function of the pledged investment commitment bid.[17] Given the low option premium (initial payment and exploration expenditures), bidders could bid up to about $18 billion in investment commitment before the value of the project turned negative. The notion that a firm might bid such a "high" investment commitment may seem puzzling at first. However, once expressed in option terms, it is less difficult to grasp: Suppose that you could buy a two-year call option on a stock that is currently priced at $10, but which is quite volatile ($s = 50\%$).[18] The call-writer asks you, "How high an exercise price would you accept for paying a penny premium?" In

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[16] The first mention of winner's curse in the academic literature was in the context of bidding for oil and gas drilling rights by Capen, Clapp and Campbell in 1971.

[17] This calculation assumes that the bidder makes the minimal initial payment of $17.5 million and a $24 million exploration expenditure. We assume that once development is begun, the bidder will complete the project. In reality, the bidder could abandon the project after partial development or may under-invest (and pay a penalty if its development expenditures were less than the promised investment commitment.) These features would increase the value of the project because the holder would only exercise them when it is optimal to do so.

[18] The risk-free rate is assumed to be 5% (continuously compounded) in this example.

principle, you would be willing to set the strike price as high as $80, or eight times the current

stock price in return for paying a penny for the option.

The bidding rules also affect the outcome in another way, by giving a comparative

advantage to smaller, less-capitalized companies. Ultimately, only three firms submitted bids for

Antamina. Two were among the largest and best capitalized in the industry. The smallest of the

three, a joint venture of Rio Algom and Inmet, had a combined market capitalization of only

$1.5 billion, or 10% that of the largest bidder and 33% that of the next bidder. Under an all-

cash auction, the joint venture of the smaller mining companies would likely be unable to raise

sufficient funds to match the bids of the larger firms.[19] However, under the Peruvian bidding

rules, smaller bidders could make future *promises* to develop the mine *without having to raise*

much money today. This type of bidding structure could therefore encourage smaller players

to bid aggressively, given their comparative advantage in bidding future promises.

4.4. Incentives for subsequent investment.

Of course, the level of investment commitment—and hence the penalty to be paid if the

property is developed—will affect the subsequent development decision. Unlike an all-cash

bid, in which the cash payment becomes an irrelevant sunk cost, under the Antamina rules, the

investment commitment is not a sunk cost. The property will be developed subsequently only if

it is economically profitable to do so, which from Equation (11) will occur when the value of net

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[19] In this instance, a $1+ billion cash bid by RioAlgom/Inmet might not be feasible, as it would represent two-thirds of
their market capitalization.

operating profits less 0.7 times the actual investment less 0.3 times the pledged investment commitment is greater than zero, or if:

$$S - 0.7X - 0.3B_2 > 0 \qquad\qquad (12)$$

As the pledged investment commitment (B_2) rises, it becomes harder to clear this hurdle, and the probability that the property will be developed falls. Figure 3 also plots (on the right scale) the probability that the mine would be developed, conditional on the winning bid pledging the investment commitment shown along the horizontal axis.[20] As the investment commitment (exercise price) increases, the cost of developing the mine and paying the penalty increases. As a result, the call option to develop is more likely to remain unexercised at maturity. Each point represents the percentage of trials for which the firm would optimally choose not to proceed, conditional on the stated level of investment commitment. Only when the investment commitment rises above $18 billion, does the probability that the project will be developed fall to 0%. Thus, competition among bidders would tend to reduce the probability that the mine would be developed, unlike the case of an all-cash bid.

In addition, the bidding procedure instituted by the Peruvian government might induce firms to make marginal investments that are uneconomic, rather than pay the penalty. Suppose the management of the winning bidder finds it optimal to proceed, despite the firm having bid a large investment commitment and owing a penalty to the Peruvian government. In particular, suppose it bid an investment commitment of $900 million while the necessary investment to actually develop Antamina was $600 million. If the firm invested only $600 million, it would

owe the Peruvian government a penalty of $90 million, or 30% of $300 million. Alternatively, suppose it could spend another $300 million on the project, buying additional trucks, helicopters, or jets. Preferably, these "extra" investments would have some fungible value beyond that delivered to the Antamina project. As long as any losses from purchasing these assets were less than $90 million, it would be optimal for the firm to purchase these extra assets and claim them as capital investments for Antamina to avoid the penalty; at some later date it could redeploy them to other uses to recoup their value. Given this disincentive, the government might need to closely monitor the type of investments made by the winning bidder. A related problem is that the winning bidder might try to use inflated transfer prices to increase its recorded investment in the project. For example, the winning bidder could hire an affiliated firm to carry out some of the development and pay it above-market rates to reduce the penalty owed to the government.[21]

4.5 Incentives for renegotiation.

The Antamina rules also seem to encourage *ex-post* re-negotiation that could render the penalty moot. At the end of two years, suppose the winning bidder concluded that the project was not economically feasible if it must pay the penalty, but that it would be feasible without the penalty. Thus, it could either walk away from the project or appeal to the government to

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[20] Technically, this represents the *risk-neutral* probability that the mine would be developed, rather than the *objective* probability of its exercise.

[21] Whenever contractual payoffs are determined by accounting figures, there is a temptation for firms to manipulate these reports to their advantage. There are many examples, ranging from accelerating or delaying large write-offs to

forgive the penalty. In response, the government could either accept this proposal, or call the firm's bluff, reacquire the property and offer it again for sale. At that time, however, the firm that just completed the exploration would be most informed potential bidder, and others might be reluctant to bid against it. If the information already collected by the winning bidder were revealed to the market, the government could not recover the penalty because other bidders would also know the project is not feasible if they have to make the original investment commitment. Thus, the penalty may be unenforceable in practice even when the pure development option (without the penalty) has positive intrinsic value.

This time-inconsistency problem plagued another recent auction: the Federal Communication Commission's spectrum auction of wireless phone licenses.[22] The FCC auctioned off these spectrum rights in 1996, with 89 companies bidding $10.2 billion for 493 licenses. Bidders with less than $40 million in annual revenues received attractive financing terms from the government, with no interest payable for six years. For a variety of reasons, the wireless business has failed to be as attractive as initially anticipated, and bidders requested the government to restructure their obligations, or else the winning bidders would default. While one solution would be to re-offer the licenses for sale, these winning bidders could seek protection in bankruptcy courts, and it could be costly and time-consuming for the FCC to gain legal access to the assets. In essence, firms bid future promises, like the investment

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maximize managers' bonuses (Healy (1985)) or under-reporting earnings to avoid making payments to income bondholders (Tufano (1997)).

[22] The problem has been covered widely in the press. For example, see Bryan Gruley and Quentin Hardy, "Wireless Bidders Ask to Restructure Debt," *Wall Street Journal*, June 26, 1997, page A3. For an academic discussion of the auction rules, see McMillan (1994).

commitments in the Antamina example. When things didn't work out, they optimally could chose not to exercise the real option to proceed with development, and at that point the original contracts ceased to be binding.

4.6 Does this structure meet the government's stated objectives?

Starting in the mid-1990s, the right-of-center administration of President Alberto Fujimori sought to undo decades of state intervention and to return many of its state-owned firms to private ownership. The objectives of this large-scale privatization program were many, including: (a) the development of projects that would generate local employment, taxable revenue, and hard currency for the country, and (b) cash proceeds for the government from the initial sale. Officials in the privatization program publically emphasized the former goal over the latter. One stated: "At the moment the treasury is not in particular need of money, but what's imperative is that we allow private companies to carry out what state entities have been doing."[23] Speaking directly about the Antamina privatization, one official said: "We obviously want to sell at an interesting price, but the principal objective is to maintain and develop the sector by attracting quality companies."[24]

Based on the above analysis, we doubt whether the proposed rules met the government's stated objectives. As discussed above, the rules encouraged weaker firms to bid large investment commitments, reducing the likelihood of actual development. The incentive to bid more in investment commitment over cash payments also meant that the government would

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[23] Saul Hudson, "Interview—Peru upbeat on energy and mining," *Reuters Financial Service*, June 14, 1996.

receive less money up-front. Further, given the incentives for potential re-negotiation of any penalties due, their future payment is suspect. Finally, the resulting disincentives seem to require substantial *ex post* monitoring of the type of investments made by the winning bidder. By setting up a scheme that not only delivers little money up-front but also reduces the likelihood of development later on, the government seems unlikely to meet its stated goals.[25]

4.7 Alternative bidding rules.

To assess alternative bidding rules, one must have a clear specification of the government's objective function. In much of auction theory, the seller is assumed to maximize expected revenue. For example, the classic literature on auctions, going back to Vickrey (1961), analyzed alternative auction mechanisms on the basis of their expected revenue.[26] In Antamina's case, the government's objective is not simply to maximize expected revenue, and the option character of the bid further complicates the task of comparing alternative bidding rules.

Consider the stylized example given earlier, where the owner of a share offers it up for "sale" via an auction where bidders can bid any pair of option premia and exercise prices. The share-owner is offering to write a call option on the share rather than selling it outright, and gives potential bidders the right to select any contract type they prefer. If the seller's goal were to maximize its expected revenue, it would select the bid for which the premium received exceeded

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[24] Saul Hudson, "Forty Firms Qualify for Peru Copper Prospect Sale," *Reuters Financial Service*, June 21, 1996.
[25] The rules may be consistent with an unstated goal of favorable public relations. A large investment commitment could be viewed positively as a signal of the government's privatization offerings.

the value of the call delivered, or $P - c(X)$, where P is the premium offered and $c(X)$ is the value of the call with exercise price X. There are multiple combinations of these parameters that make the seller indifferent. These are diagrammed in Figure 4, which shows the set of bids which have zero net expected value to both buyer and seller. While these various bids have the same value, they present the bidder with different risk exposuress this is shown on the bottom panel, which plots the profit/loss to the seller at maturity for two proposed contracts. Given the sellers' risk preferences, we could select from among these equal-value choices.

For Antamina, the government has stated a preference for development as a primary goal and high cash proceeds as a secondary goal. The probability of development (exercise) is inversely related to the exercise price, while cash proceeds are directly related to the option premium. Thus, while points A and B on the figure have equal value, based on these preferences the option represented by A dominates B, in that it is more likely to be exercised as well as deliver a higher up-front cash price. Taken to an extreme, this suggests that the government's interests are best served by reducing the exercise price and eliminating the penalty. Thus, it appears that an all-cash auction would seem better suited to meeting the government's objectives.

5. Postscript and Conclusion

The Peruvian government three bids for Antamina and announced the winner on July 12, 1996. The largest firm that submitted a bid, RTZ-CRA Ltd., offered $17.5 million as initial

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[26] See Riley (1989) for an overview of this literature.

payment and pledged a $900 million investment commitment. Noranda, the second largest firm, also bid $17.5 million upfront but promised a more generous, $1,900 million investment commitment. The winner was the small Rio Algom/Inmet partnership, which bid $20 million up front and a pledged investment commitment of $2,500 million.[27]

At a press conference later on the same day, Rio Algom Ltd. and Inmet Mining Corp. said that they intended to spend about *$1 billion* to develop their new Peruvian copper purchase. "If it does not prove viable, we just lose our up-front investment," stated a Rio Algom spokesman.[28] Clearly, the joint venture partners were well aware of the options embedded in the bidding rules, signaling that they did not plan to spend the full investment commitment of $2.5 billion, but rather a more reasonable $1 billion (plus penalty) were they to proceed. They apparently wanted to convey this understanding to security analysts and make sure they were not perceived as having "overpaid" for the mine. The winning bidder clearly understood the option-nature of the offering, and followed a bidding strategy consistent with what should have been predicted.[29]

The power of real option analysis—in conjunction with simple game theory—is that it promises to help us better understand not only how things are valued but also how managers will behave when faced with flexibility and choices. In this case, a real options perspective can help us understand how firms would bid on a large capital investment project, as well as to predict what types of problems might crop up in the future.

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[27] See Sally Bowen, "Canadians in $2.5 billion Peruvian Copper Plan," *Financial Times*, July 16, 1996, p. 33.
[28] "Canadian Group Hopes to Double Antamina Reserves," *Reuters Financial Service*, July 12, 1996.

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[29] In the MBA classroom, students played the role of bidders for Antamina. Those who appreciated the nature of the real option apparently bid differently from those who did not, with the former offering lower initial payments and investment commitments over $15 billion.

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Table 1
Summary of Information Used to Value Antamina

Estimates of mine life, yearly production, capital expenditures, and operating costs for three development scenarios corresponding to the quantity of ore in the deposit. The expected scenario represents the mean or median of the distribution of possible ore content, the low scenario represents 1 to 1.5 standard deviations below the mean, and the high scenario represents 1 to 1.5 standard deviations above the mean.

Scenario		Low Case	Expected	High Case
Reserves	(million metric tons)	100	127	175
Mine life	(years)	12	14	18
Copper production	(million lbs/year)	313	339	365
Zinc production	(million lbs/year)	155	168	181
Operating costs[a]	(millions of 1996 U.S. $)	131	138	145
Copper treatment charge[b]	(1996 U.S. dollars/lb)	0.28	0.28	0.28
Zinc treatment charge[b]	(1996 U.S. dollars/lb)	0.22	0.22	0.22
Feasibility study[c]				
1996	(millions of 1996 U.S. $)	6	6	6
1997	(millions of 1996 U.S. $)	18	18	18
Capital expenditure[c]				
1998	(millions of 1996 U.S. $)	54	55	55
1999	(millions of 1996 U.S. $)	246	255	264
2000	(millions of 1996 U.S. $)	281	292	303
Per year after 2000[c]	(millions of 1996 U.S. $)	8.7	9.0	9.3
Closure costs[c]	(millions of 1996 U.S. $)	45	45	45

Notes

[a] While the operating costs are expressed in U.S. dollars, less than half would be set in Peruvian Sols. These costs are expressed in real (1996) amounts, and exclude the treatment charges paid to a smelter.

[b] The treatment charges were paid by the mining company to smelters (typically in Japan) and would be set and paid in U.S. dollars. These treatment charges are set by contract with the smelter, and tend to adjust with the relative demand for smelting capacity which tends to increase with the price of copper. From the smelter, the mining company received revenue equal to the amount of metal produced times the U.S. dollar market price of copper (or zinc) minus the treatment charge.

[c] The feasibility study, the initial and continuing capital expenditures, and the closure costs (environmental clean-up and reclamation) are expressed in real (1996) amounts. These expenses would primarily be set and paid in U.S. dollars.

Table 2
Price Process Parameter Estimates

Daily data from the London Metals Exchange for the period September 1991 to July 1996 was used to determine the metal price process parameters used in text equations (1) and (2). Panel A shows the zinc and copper 3-month futures convenience yield mean reversion parameters k and α, estimated from the data using equations (3), (4), and (5). Panel B shows the copper and zinc annualized return volatilities calculated on the basis of the prior 90-day price changes. A year is assumed to have 250 trading days. Panel B also shows the annualized convenience yield volatilities, obtained from the third equation in (5). Panel C shows the return correlations (ρ) for copper and zinc and their convenience yields. All correlations are for daily data for the period September 1991 - July 1996 and are significant to 1%.

Panel A: Convenience yield mean reversion parameters (k and a)

k_c	k_z	a_c	a_z
0.8	0.6	6.0%	-0.2%

Panel B: Copper and zinc volatilities (s)

s_c	s_z	s_{dc}	s_{dz}
22%	18%	25%	23%

Panel C: Copper and zinc correlations (r)

	P_c	P_z	d_c	d_z
P_c	1.00			
P_z	0.44	1.00		
d_c	0.57		1.00	
d_z		0.73		1.00

Table 3
Characteristics of the Antamina Mine Project,
Conditioned on Whether It is Developed After Year 2

Mean copper and zinc prices and convenience yields, conditioned on whether the project is developed at the end of year 2. The means are shown for the High, Expected, and Low ore outcomes, and for all outcomes combined. We also show the percent of trials in which the mine was developed for all outcomes. T-tests of the differences between developed and not-developed outcomes show that all means of prices and convenience yields are significantly different from one another, with p-values exceeding .01 in all cases.

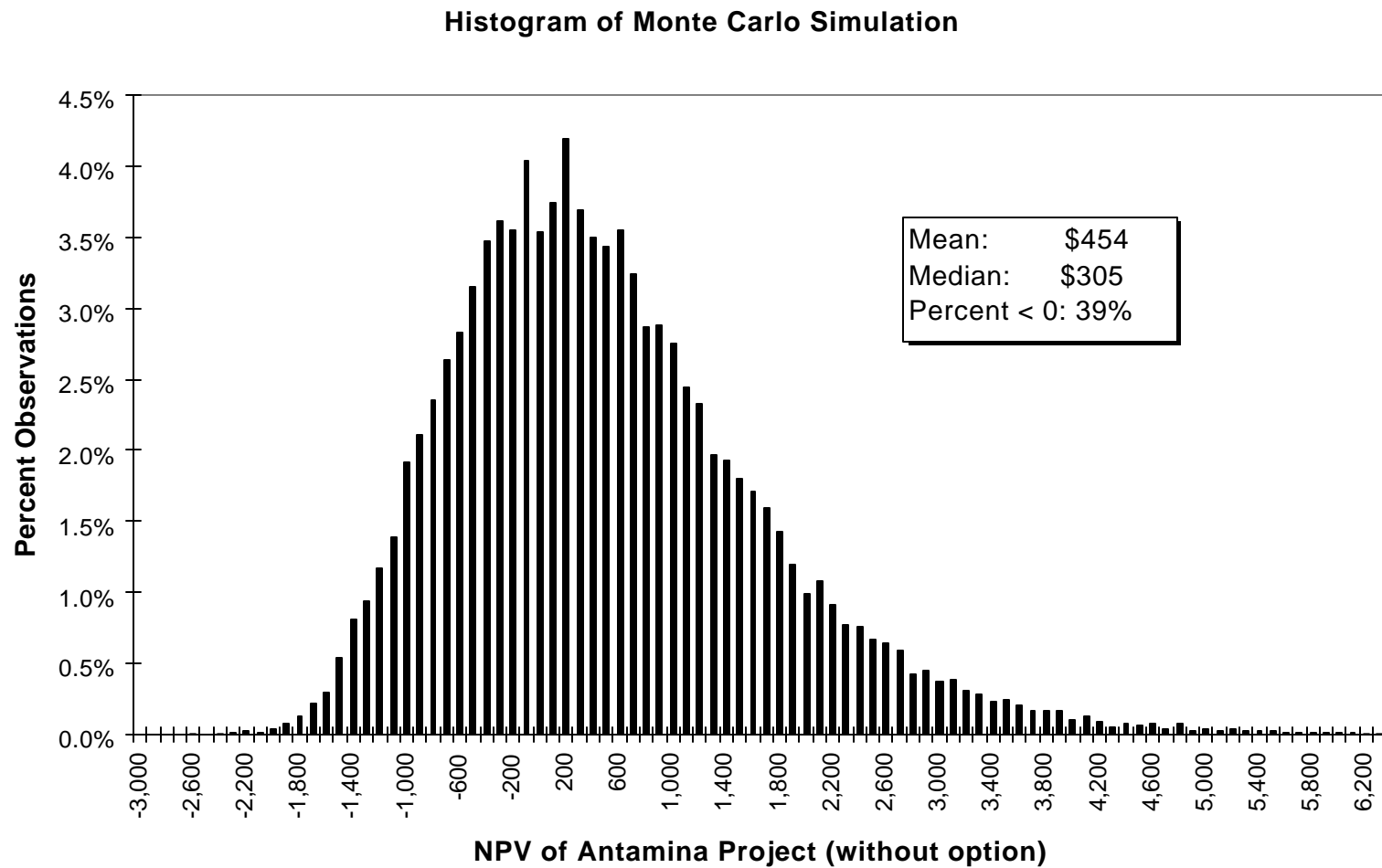|	**Project Developed**	**Project Not Developed**
High ore:		
Copper price/convenience yield	$1.04/5.5%	$0.63/8.9%
Zinc Price/ convenience yield	$0.64/-0.5%	$0.47/0.5%
Percent of trials	77%	23%
Expected ore:		
Copper price/convenience yield	$1.07/5.4%	$0.67/7.9%
Zinc Price/ convenience yield	$0.65/-0.6%	$0.50/0.4%
Percent of trials	61%	39%
Low ore:		
Copper price/convenience yield	$1.10/5.3%	$0.70/7.4%
Zinc Price/ convenience yield	$0.70/-0.7%	$0.52/0.3%
Percent of trials	47%	53%
All outcomes:		
Copper price/convenience yield	$1.07/5.4%	$0.68/7.8%
Zinc Price/ convenience yield	$0.65/-0.6%	$0.50/0.3%
Percent of trials	61%	39%

Table 4
Comparison of Increasing Initial Cash Payment Vs. Pledged Investment Commitment Under the Antamina Bidding Rules

The table below shows the number of "bid points" a firm would receive by adding an additional dollar to either the initial cash payment or the pledged investment commitment, under the rules set by the Peruvian government. A "bid point" is a measure used to judge alternative bids by the Peruvian government.

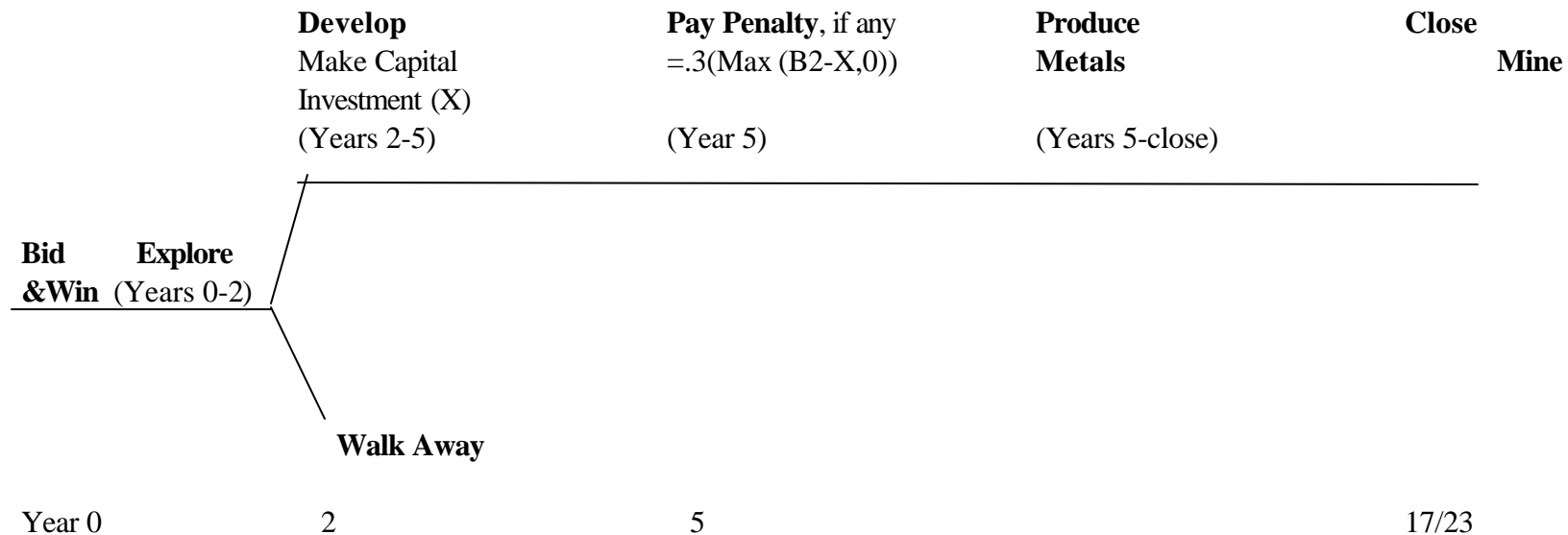|	**Bidder Bids Another $1 On The Initial Cash Payment**	**Bidder Bids Another $1 On Investment Commitment**
Number of *bid points* received	$1.00	$0.30
Expected cost, conditional on winning the bid	$1.00 Must pay this immediately, regardless of ultimate decision to develop	$E[PV(\$0.30)] < \$0.30$ The penalty is not paid until later, and only if the project is ultimately developed
Ratio of *expected cost* to *bid points*	1	Less than 1

Figure 1
Distribution of Values for Antamina if Winning Bidder is Committed to Develop (top panel) or has an Option to Develop (bottom panel).

Histogram of Monte Carlo Simulation
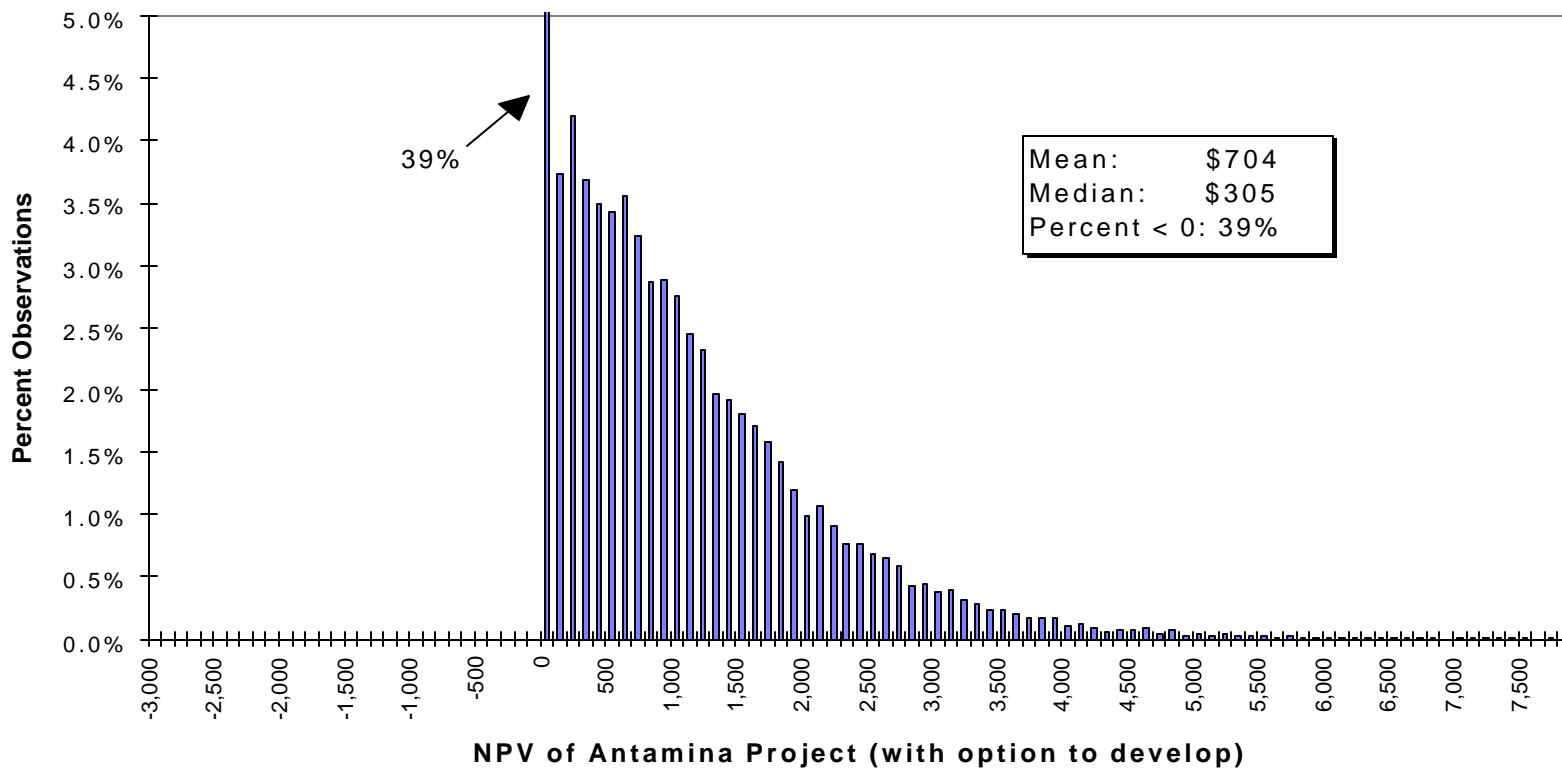
Note: The top panel shows the results of a 10,000 run Monte Carlo simulation for the Antamina mine if the mine had to be developed at the end of year 2. The mean NPV is $454 million. The project has a negative NPV in 39% of the trials. The bottom panel shows the effect of the option to abandon development on the NPV of the project. In this case, negative-NPV outcomes are not developed, resulting in an expanded mean NPV of $704 million.

Figure 2
Time Line of Managerial Decisions in Antamina Auction.

	Develop	**Pay Penalty**, if any	**Produce**	**Close**
	Make Capital	=.3(Max (B2-X,0))	**Metals**	**Mine**
	Investment (X)			
	(Years 2-5)	(Year 5)	(Years 5-close)	

Bid   Explore
&Win (Years 0-2)

Walk Away

Year 0	2	5	17/23

This figure shows the set of decisions facing a firm bidding on Antamina, focusing on the decision whether to develop the mine at the end of exploration (year two). B_1 represents the initial cash payment made at year zero, B_2 the pledged investment commitment, X represents the actual amount spent on development in years two through five (exercise price) if the firm proceeds. The Penalty is paid in year five, after development is completed. Closure takes place when the reserves are depleted, which is predicted to occur between years 17 and 23, depending on the ore found.
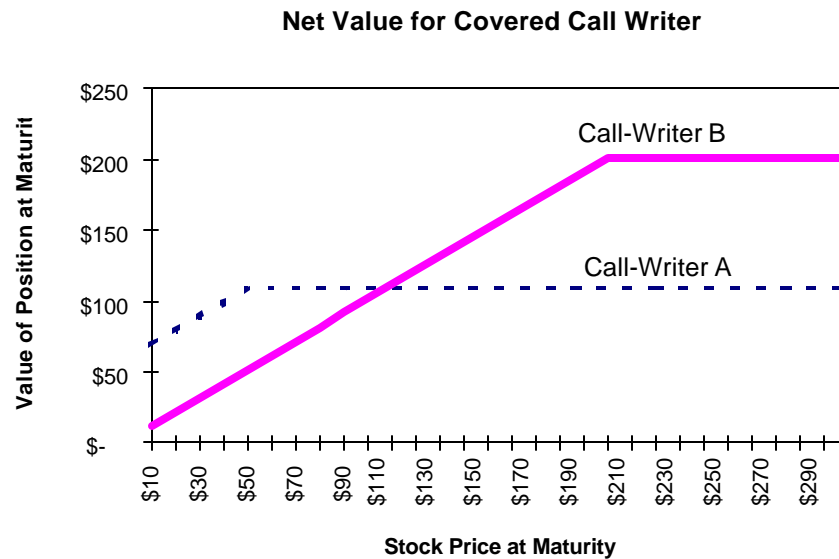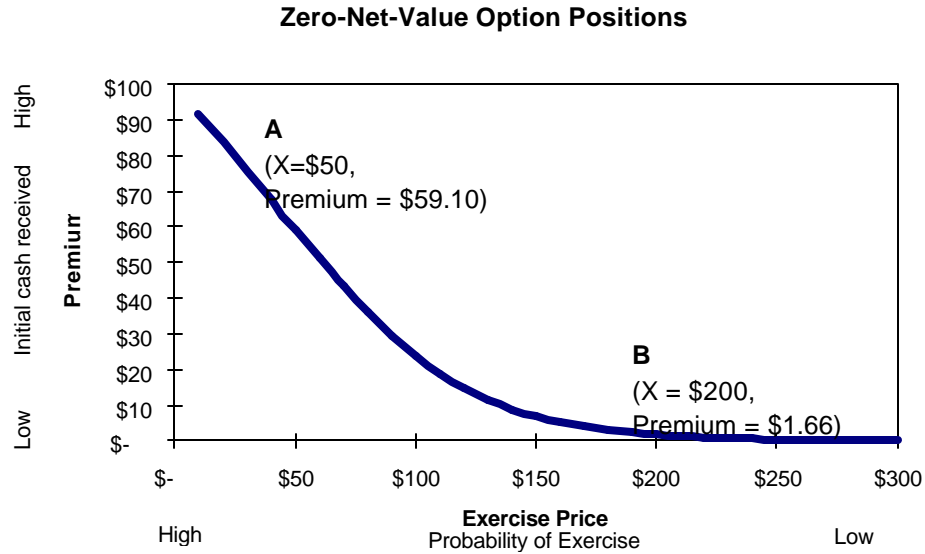
Figure 3
Net Present Value to Winning Bidder Under Antamina Auction, and Probability that the Project Would be Developed vs. Size of Pledged Investment Commitment.
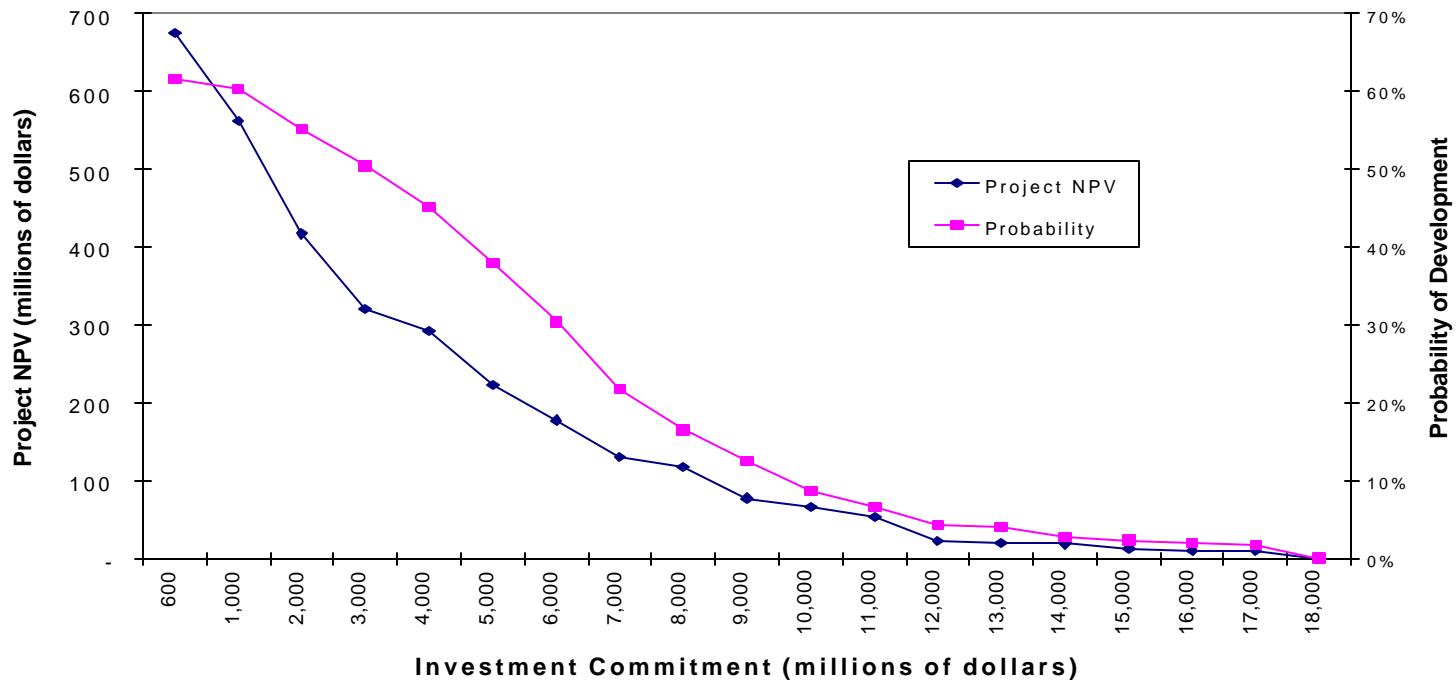
The left-hand scale shows the net present value of the Antamina mine as a function of the pledged investment commitment. The right-hand scale shows the probability of mine development as a function of the investment commitment. The curves intersect the x-axis (the NPV of the project turns negative) at about $18 billion (with a probability of development of 0%). Each point on the graph represents the mean value of a 1,000 run Monte Carlo simulation.

Figure 4
Tradeoff Between the Premium and Exercise Price of a Call Option.

Zero-Net-Value Option Positions

Net Value for Covered Call Writer

The top figure shows the combination of call option exercise prices and premia for which the value of a two-year European-style call option (struck at the given exercise price) less the premium paid equals zero, i.e., the set of fairly-priced calls. (Assumes current stock price of $100, stock return volatility of 25%, and a risk-free rate of 10%.) The lower figure shows the exposure at maturity of two covered-call writers, each of whom wrote a fairly-priced option at exercise prices A and B from the top panel.