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We greatly appreciate the thorough comments from the editor and referees, and have made changes as requested. Below we detail our response to the referees' reports.

Ben Snowball and Sheehan Olver

Response to Referee 1

- *The authors spend an inadaquate amount of time discussing other approaches for solving this problem and comparing their methodology (e.g., closest point method, radial basis functions). What is the complexity convergence behavior of other approaches that can handle spherical caps?*

  We have added in a discussion of the closest point method. We have chosen to omit discussion on radial basis functions as we are not aware of references that consider their usage on spherical caps.

- *Definition 2: It should be mentioned somewhere that OPs should defined by the condition that they are orthogonal to all lower degree polynomials.*

  Added a comment before Definition 2 to explain this, as well as an extra line in Definition 2.

- *Above eq 7: These are not Jacobi matrices in the classical sense, but they are block Jacobi matrices.*

  Addressed.

- *Beginning of Section 2.3: What is the "triangle case"?*

  Explained via a comment in the relevant place.

- *Remark at the bottom of page 7: I do not know what it means for z to be rotionally invariant. But the z coordinate is invariant under any rotation involving only the θ variable.*

  Addressed.

- *Definition 6: Since $\tilde{a} \geq 2$, it only needs to be stated that $\tilde{a}$ is an integer.*

  Fixed.

- *Figure 1: Extra the in the the*

  Fixed.

- *All figures: They should be larger with larger font sizes.*

  Fixed.

- *About Section 4.3: Is it possible to leverage the DCT to speed up the expansion of functions in the basis?*

  Added a remark to state its an open problem.

- *Figure 4: 'N is the the degree' is ambiguous. How does N relate to the number of unknowns?*

  This is mentioned in the body of the text, but now explained in the caption.

Response to Referee 2

- *One shortcoming of this paper is that it has no discussion of the condition number of the linear systems to be solved. I suspect that the condition numbers can get large, but if so, can the matrices be preconditioned in a similar way the Olver-Townsend ultraspherical spectral method (so that it has bounded condition number)?*

  Added small subsection and plots to show the condition numbers of each m-block of Laplacian operator, along with a preconditioned Laplacian (where the preconditioner P is the inverse of the diagonal of the Laplacian). While the Laplacian does not have bounded condition number, the trivial preconditioning keeps it bounded.

- *If the condition numbers do not get large, can you prove it, indicate why, or give numerical evidence?*

  The evidence we can provide is that which addresses the first question.

- *The paper only includes numerical experiments involving zero Dirichlet boundary conditions. Please could the authors clarify in the manuscript if there are any roadblocks to boundary conditions beyond the zero Dirichlet ones demonstrated in Section 5?*

  Added comment a small subsection to Section 5 to address this.

- *Is the code used to produce the numerical experiments going to be made publicly available? For example in the ApproxFun Julia package?*

  The package that makes up the framework for expanding functions, obtaining the operator matrices etc. is purely experimental at this stage, but is publicly available. This fact and where it can be found has been added to the paper.

Other Changes

- Typo has been corrected in one equation in Section 2.3 (missing z from $Q(x, y, z)$).

- Proof for quadrature rule has been fixed - reference to $\boldsymbol{s}_l$ removed; $(1-x^2)^{-1/2}$ weight needed adding to the integrals over dx; clarified the maths that shows the symmetry that is referenced.