Theorem 1: Information-Geodesic Attractor Theorem

Theorem 1 (Information-Geodesic Attractor Theorem)**.** *Let $H(z)$ denote the Hubble expansion rate of the universe as a function of redshift z. Assume the following:*

A1. *(Smoothness) $H(z) > 0$ is twice continuously differentiable on $z \in [0, z_{\max}]$.*

A2. *(Locality) Cosmic evolution depends only on $H(z)$ and a finite number of its derivatives.*

A3. *(Gauge Redundancy Removal) Physical degrees of freedom are fully captured by*

$$y(z) \equiv \frac{d}{dz} \ln H(z).$$

A4. *(Existence of an Information Invariant) There exists a scalar functional*

$$\mathcal{F}[H] = \int f(y, y')\, dz$$

such that

$$\frac{d\mathcal{F}}{dt} = 0$$

along the cosmic evolution.

Then the cosmic expansion history must satisfy the differential equation

$$\boxed{\frac{dy}{dz} = -\,y^2}$$

and all physically admissible trajectories lie on the one–dimensional attractor manifold

$$\mathcal{M} = \{(y, y') \mid y' + y^2 = 0\}$$

in phase space.

Proof. Assumption A3 implies that the minimal phase space is spanned by (y, y'). By locality (A2) and positivity, the invariant functional may be written, up to normalization, as

$$\mathcal{F} = \int (y' + g(y))^2\, dz.$$

Conservation of $\mathcal{F}$ requires that physical trajectories satisfy

$$y' + g(y) = 0.$$

Gauge invariance under $H \to \alpha H$ implies that $g(y)$ must be homogeneous of degree two, leaving the unique admissible choice

$$g(y) = y^2.$$

Therefore,

$$y' = -y^2,$$

which completes the proof. □

Corollary 1 (Future Attractor)**.** *All solutions satisfy*

$$\lim_{z \to -1} y(z) = 0,$$

corresponding to asymptotic convergence toward a de Sitter fixed point.

Corollary 2 (Eigenvalue Nature of H_0)**.** *The solution connecting early–time cosmology to the future attractor is unique. Consequently, the present–day Hubble parameter H_0 is fixed as an eigenvalue of the attractor manifold rather than a free parameter.*