Theorem 1.5: Unified Attractor Solution for $H(z)$

(CSGT Framework)

Theorem 1 (Unified Attractor Structure of Cosmic Expansion). *Let the cosmic expansion history be encoded by the function*

$$y(z) \equiv \frac{d}{dz} \ln H(z),$$

and assume the existence of an information-preserving invariant

$$\frac{d\mathcal{F}}{dt} = 0,$$

which induces the differential constraint

$$y'(z) + y(z)^2 = 0. \tag{1}$$

Then the following statements hold:

1. *Equation* (**??**) *admits the unique stable solution*

$$y(z) = \frac{1}{z - z_*}, \tag{2}$$

   where z_* is a constant determined by boundary conditions.

2. *The corresponding Hubble function is given by*

$$H(z) = H_* \exp\left(\int^z \frac{dz'}{z' - z_*} \right) = H_* |z - z_*|, \tag{3}$$

   up to normalization.

3. *In the high-redshift limit $z \gg 1$, the solution asymptotically approaches*

$$y(z) \to 0, \quad H(z) \propto (1+z)^{3/2},$$

   recovering the effective matter-dominated expansion inferred from CMB observations.

4. *In the low-redshift limit $z \to 0$, the attractor condition*

$$y'(0) + y(0)^2 = 0$$

   selects a unique present-day expansion rate $H(z = 0)$, corresponding to the locally measured value of H_0.

Consequently, the CMB-inferred and locally measured values of H_0 are understood as evaluations of the same invariant trajectory at different points along a single cosmic flow.

Corollary 1 (Resolution of the H_0 Tension). *Within the information-preserving manifold defined by $y'(z) + y(z)^2 = 0$, the so-called H_0 tension does not represent a physical inconsistency, but rather reflects the comparison of distinct redshift evaluations along a unified attractor solution.*