Problem Set 2

Question 1.1)

We can draw the recursion tree as :

$$T(n) = 2T\left(\tfrac{2}{3}n\right) + n^2$$
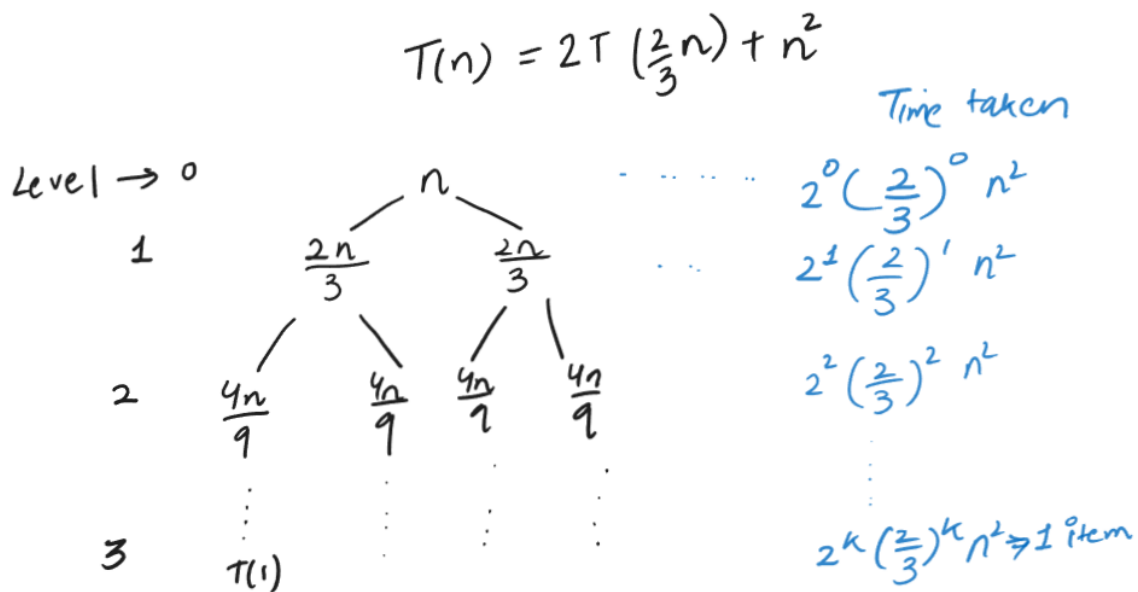
When we have reduce the tree down to individual elements, we have k levels in the tree.

A singular element can be represented as : $\left(\tfrac{2}{3}\right)^k n = 1$

Which means $k = log_{3/2}(n)$

The total cost of all the levels can be represented as :

$$T(n) = \sum_{i=0}^{i=k} 2^i \left(\tfrac{2}{3}\right)^i n^2 \Rightarrow n^2 \sum_{i=0}^{i=k}\left(\tfrac{4}{3}\right)^i$$

$$T(n) = n^2 + 2\left(\tfrac{2}{3}\right)^2 n^2 + 4\left(\tfrac{2}{3}\right)^3 n^2 + \dots$$

Since the dominant term is n^2, we can argue that the overall order is $O(n^2)$