Problem Set 2

$$T(n) = \sqrt{n}T(\sqrt{n}) + n, where\ T(2) = 4$$

We can rewrite the equations as :

$T(n) = n^{1/2}T(n^{1/2}) + n$

Then the next progression for $T(n^{1/2})$ will be :

$T(n^{1/2}) = n^{1/4}T(n^{1/4}) + n^{1/2}$

Substituting the value of $T(n^{1/2})$ into $T(n)$

$T(n) = n^{1/2}(n^{1/4}T(n^{1/4}) + n^{1/2}) + n \Rightarrow n^{3/4}T(n^{1/4}) + 2n$

Now, the power of n is increasing with $1/2^k$ at the kth level.

According to our base case, $T(2) = 4$, so we must divide the tree till 2 values are left.

Hence at the kth level, when only 2 values are left, $n^{1/2^k} = 2$.

so $k = loglogn$

Now we can perform summation of all the values

$\sum_{i=0}^{i=k} = loglogn$