Northeastern University
CS5800 – Discrete Structures
Fall 2023 Semester, Jonathan Mwaura

Problem Set #6

This Problem Set is due at **11:30PM on Friday Nov** 3^{rd}, and will be submitted on GRADESCOPE.

This Problem Set will be marked out of 30. There are three problems, each worth 10 points.

Please type (or neatly handwrite) your solutions on standard 8.5×11 paper, with your name at the top of each solution. Ensure that you submit you solutions in one file PDF file on Gradescope. **each problem sets solution should be on in its own individual page, Gradescope will help ensure you submit each solution under its correct problem number**

While a solution must be absolutely perfect to receive full marks, I will be generous in awarding partial marks for incomplete solutions that demonstrate progress.

So that there is no ambiguity, there are two non-negotiable rules. A violation of either rule constitutes plagiarism and will result in you receiving an F for this course.

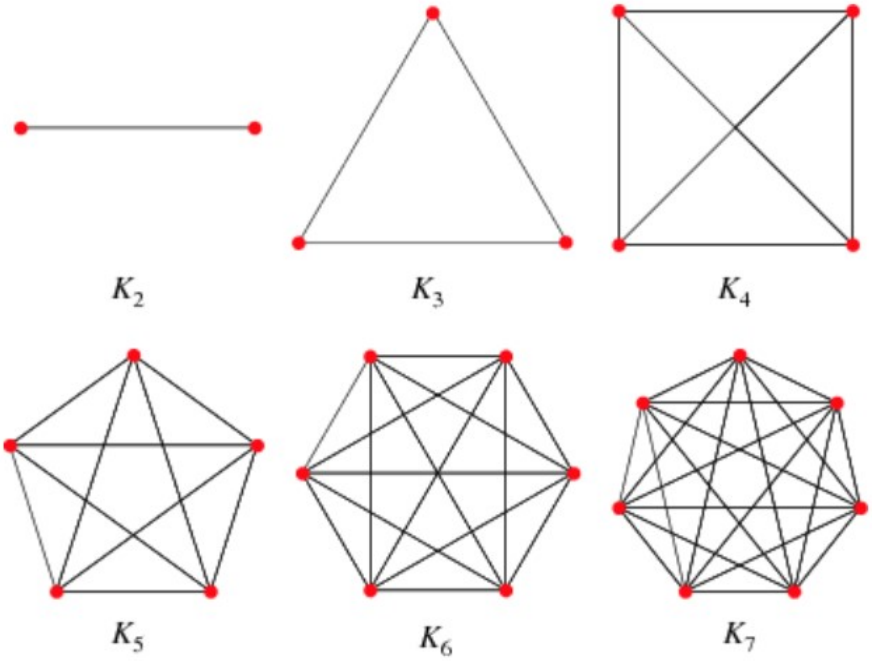
(a) If you meet with a classmate to discuss any of the Individual Problems, your submission must be an individual activity, done in your own words, away from others. The process of finding a solution might take 3 - 5 iterations or even more BUT you learn from all these attempts and your confidence grows with each iteration.

(b) These problem sets might seem hard on a first look. They are designed to be so. We learn by attempting problems, struggling through them and coming on top. I encourage you to make this learning exercise worth your while. What do I mean? Open the problem sets as early as you get them, then do not look at hints or answers any where (including on the internet and consulting other students for direct answers), give it the best shot you can. If you get stuck come to Professor or TA's office hour and we shall be glad to listen to your rationale and work with you till you are able to tackle the problem sets.

Problem #1

Define K_n to the graph on n vertices, where each pair of vertices is connected by an edge. K_n is known as the *complete graph* on n vertices.

To illustrate, here are the complete graphs K_n, for $n = 2, 3, 4, 5, 6, 7$.



(a) Draw the complete graph K_{10}, and determine the total number of edges in this graph. Briefly explain how you calculated this total.

(b) The complete graph K_n has exactly 120 edges.

Determine the value of n. Clearly justify your answer.

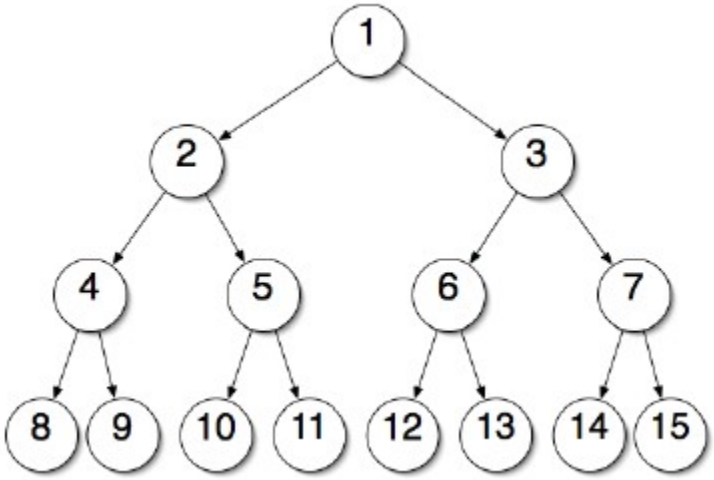
(c) Sometime in 2021 (or 2022), a group of CS5800 students meet at the Northeastern campus for the first time, to have a post-Covid celebration party. Each pair of students shakes hands.

Unfortunately, Paul walks in late. As a result, Paul is only able to shake hands with *some* of the other students at the celebration party.

If there are exactly 42 handshakes in total, determine the number of hands that Paul shook. Clearly and carefully justify your answer.

Problem #2

Consider this binary tree, where each vertex is labelled with a positive integer. The root vertex is 1.



For all positive integers $k \geq 1$, vertex k has two children: $2k$ (Left) and $2k + 1$ (Right).

(a) In your own words, describe how Breadth-First Search (BFS) and Depth-First Search (DFS) work. Does one search algorithm *always* reach the destination faster than the other? Explain.

(b) Suppose we want to determine a path from vertex 1 (start vertex) to vertex 10 (end vertex).

Using BFS, determine the order in which the vertices will be visited. Using DFS, determine the order in which the vertices will be visited. Briefly explain your answers.

(c) Suppose that we extend this binary tree to infinitely many levels, so that each vertex k has two children: $2k$ (Left) and $2k + 1$ (Right).

The path from vertex 1 to vertex 10 can be described by a sequence of Left and Right moves, namely **Left, Right, Left**.

Consider the path from vertex 1 to vertex 2021. Determine the sequence of Left and Right moves for this path. Clearly justify your answer.

Problem #3

In this question, you will create a **mini-portfolio** consisting of any **two** LeetCode problems on Graphs, chosen from the four problems below:

`https://leetcode.com/problems/clone-graph/`
`https://leetcode.com/problems/is-graph-bipartite/`
`https://leetcode.com/problems/find-the-town-judge/`
`https://leetcode.com/problems/find-if-path-exists-in-graph/`

As always, you may code your algorithms in the programming language of your choice.

Here is how your mini-portfolio will be graded.

There will be a total of 10 points for any of the combination of problems in your mini-portfolio: For each of these, provide the problem number, problem title, difficulty level, and the screenshot of you getting your solution accepted by LeetCode (10 points).

Note that you are allowed to work with Teammates on this part of the problem. Make sure you write all names of the collaborators.