

# Problem Set 2

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1)  $T(n) = 2T(\frac{2}{3}n) + n^2$

1.2) Give asymptotic upper bounds using Master Method

The master method can be described through the form :

$$T(n) = aT(n/b) + f(n)$$

Fitting the constants  $a$  and  $b$  into the given equation from the question we can conclude :

We have a number  $a = 2$  subproblems, each dividing the root input into  $b = 3/2$ .

Now,  $\log_{\frac{2}{3}}(2) = 1.70951129$

let  $\epsilon = 0.1$

Now,  $\because n^2 > n^{\log_{\frac{2}{3}}(2)+0.1} \Rightarrow n^2 > n^{1.80951..}$

This falls under **case 3** of the master theorem. This means that  $f(n) = \Omega(n^{\log_{\frac{2}{3}}(2)+0.1})$  so  $T(n) = \Theta(f(n))$

Since we have discovered the tight bound on  $T(n)$ , the upper bound  $O(n)$  will be the same as the tight bound.