

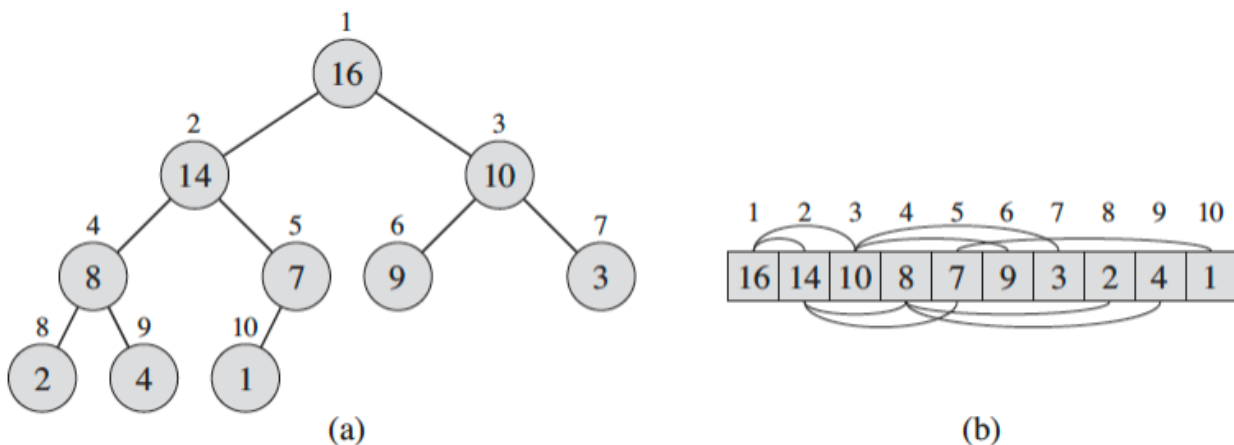
Q3

5 Points

Grading comment:

A binary heap is called a **max heap** if it has the property that for every node i other than the root, the value of the node is at most the value of its parent.

Here is an example of a max heap with 1010 nodes (i.e., 1010 elements), presented both as a binary tree and as an array.



The height of a heap is defined to the number of edges on the longest downward path from the root node to a leaf node. Thus, in the example above, the height of the heap is $h=3$.

If a (binary) heap has height $h=6$, determine the minimum number and maximum number of elements that can be in this heap. Clearly justify your answer.

Solution 3(a):

Since the heap is a binary tree, a perfect binary tree will be the condition when a heap contains the maximum amount of nodes. Number of nodes in a perfect binary tree are $2^{h+1} - 1$.

Since $h = 6$, this is equal to $2^7 - 1 = 127$

Hence the maximum number of nodes in a heap of height 6 is 127.

The minimum number of nodes must be when there is 1 leaf in the last level. This equals to the height of a perfect tree with $h - 1$ leaves $-1 + 1 =$ height of a tree with $h - 1$ leaves, which is equal to $2^h - 1$.

Hence, when $h = 6$, the minimum number of leaves is $2^6 - 1 = 64$.

