Masters 1 Physics PICS/COMPUPHYS

PROBLEM SET ON LASER DYNAMICS AND ULTRAFAST OPTICS

DUE FRIDAY 26 MAY 2023

Note: some of these questions are quite long. So even if you have trouble answering the questions, please make an attempt or write down notes about how you would approach the problems. I will award marks based on the general approach even if you do not manage to carry out a full calculation.

Question 1

(i) Briefly explain the operation of a mode-locked laser in physical terms. Give a brief comparison of the main techniques used to induce mode-locked operation in a laser.

(ii) Consider a Titanium Sapphire laser operating around 800 nm, and assume the spectral FWHM extends from 790 nm to 810 nm. If the laser has a standard two-mirror cavity of length L = 2 m and is operated far above threshold, estimate how many longitudinal cavity modes would oscillate underneath the spectral FWHM. What is the expected pulse duration and repetition rate if the laser is ideally mode-locked?

(iii) Sketch the output temporal and spectral characteristics.

(iv) Imagine that we now pass the laser output through a narrowband optical filter of linewidth 1 GHz. What is the expected pulse duration and repetition rate of the pulses after the filter? Is there any physical contradiction in this result?

Question 2

A **soliton molecule** is a pair of pulses with the same amplitude but a delay T and a relative phase shift ϕ.

$$E(t) = E_0(t) + E_0(t - T)\exp(i\phi)$$

(i) Derive a general expression of the spectrum $|E(\nu)|^2$ of this soliton molecule. Use the shift theorem for the Fourier transform that states: If $E(t) \leftrightarrow E(\nu)$ then $E(t-T) \leftrightarrow e^{-i2\pi T\nu}E(\nu)$

(ii) Plot the spectrum of the soliton molecule supposing that $E_0(t)$ is a gaussian pulse of intensity FWHM 10 ps, the delay T = 100 ps, and for three cases of relative phase $\phi = 0$, π/2, π. You can use either the analytic form for the spectrum of a gaussian pulse (Question 2 above) or you can compute it numerically via an fft operation. Plot the spectra as a function of frequency on the x-axis.

(iii) Does this result suggest that two pulses that are temporally separated by 10 times their width can actually interfere in a spectrometer?

(iv) Can this result be interpreted as a form of Young's experiment in the temporal domain? Justify.

Question 3

(i) Many optical fibre lasers produce soliton pulses whose temporal field profile is $E(t) = \mathrm{sech}(\alpha t)$ Derive an expression for the intensity $|E(t)|^2$ in terms of the intensity full width at half-maximum (FWHM) Δτ, and show that: $\Delta\tau = \frac{1}{\alpha}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \approx \frac{1.7627}{\alpha}$

(ii) Using the Fourier transform relation for a hyperbolic secant $\mathrm{sech}(\alpha t) \longleftrightarrow \sqrt{\frac{\pi}{2\alpha^2}}\ \mathrm{sech}\left(\frac{\pi}{2\alpha}2\pi\nu\right)$

compute the spectrum and the time-bandwidth product ΔτΔν where Δν is the FWHM of the spectrum.

(iii) Consider gaussian and sech pulses of intensity FWHM 100 fs centered on 800 nm. Plot the temporal intensity profiles of both pulses on the same axes. How might be able to distinguish them? Also plot the spectral intensity profiles $|E(\nu)|^2$ of both pulses as a function of wavelength on the x-axis.

Question 4

A saturable absorber is a laser element that is used to discriminate between pulsed and noise-like operation in a laser. We write the time-dependent transmission of a saturable absorber as follows:

$$T(t) = 1 - \frac{q_0}{1 + I(t)/I_{sat}}$$

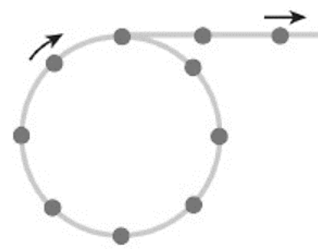
Here q_0 is a modulation depth, I(t) is the power profile of the incident pulse, and P_{sat} is a saturation power parameter.

(i) Take $q_0 = 0.75$ and consider 1 ps FWHM Gaussian pulses of peak power 10 W, 100 W and 1000 W. For $I_{sat} = 500$ W, plot the transmission function T(t) for these three cases on the same axes. Interpret the plot in terms of the operation of a saturable absorber

(ii) Try to write some code to show the effect of a saturable absorber in discriminating between noise and pulses. For example, take a random temporal field with peak power of say 1000 W. Then propagate this through the saturable absorber a very large number of times and see how this influences the temporal shape. Note that since the saturable absorber will introduce loss with each pass, you will need to compensate this. A convenient way that works is just to renormalize the amplitude of the output to have the same energy as the input. In a physical sense this is simulating the steady state regime of a laser so that we can just examine the influence of the saturable absorber on the dynamics.

Question 5.

Usually when we consider a mode-locked laser, we assume that there is only one pulse circulating in the cavity over one round-trip. But sometimes a laser can operate in a **harmonic mode-locked regime** with multiple pulses simultaneously circulating. The figure below illustrates this for a ring laser consisting of a single unidirectional loop of optical fibre of length L and refractive index n.



Suppose a laser is operating on its Nth-harmonic so we have N circulating pulses. If each pulse has energy E, derive an expression for the laser average power in terms of E, L, n and N. How do you think we might generate such a harmonically-mode locked output?