Estimating Unit Level Causal Effect

- Given a binary treatment setting, the unit level causal effect is the difference in potential outcomes for the same unit:

↳ Unit level causal effect $= Y_{1i} - Y_{0i} = \tau_i$

- Recall the fundamental problem of causal inference says that we cannot compute the unit level causal effect as it is impossible to observe the outcome of unit under both control and treatment at the same time.
- A surprising fact (noted in Spiess JMP, 2018 and elsewhere) is that it is still possible to unbiasedly estimate the unit level causal effect using the following estimator:

$$\hat{\tau}_i = \frac{D_i - p}{p(1-p)} Y_i, \quad \underbrace{D_i \in \{0,1\}}_{\text{Treat status}}, \quad p = Pr(D_i = 1)$$

- Recall the potential outcome model can be used to write the observed outcome in terms of the potential outcomes:

$$Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}, \quad (Y_{0i}, Y_{1i}) \text{ are potential outcomes}$$

- We can show that the above estimator is a unbiased estimator for the unit level causal effect:

$$E[\hat{\tau}_i] = E[\hat{\tau}_i | D_i = 1] \underbrace{P(D_i = 1)}_{p} + E[\hat{\tau}_i | D_i = 0](1 - P(D_i = 1))$$

$$= \frac{1}{p} \cdot p \cdot E[Y_i | D_i = 1] - \frac{1}{1-p} \cdot (1-p) \cdot E[Y_i | D_i = 0]$$

$$= E(Y_{1i} - Y_{0i}) = Y_{1i} - Y_{0i}$$

Fix i

- Suppose p = 1/2 then the unit level causal effect estimator is:

$$\hat{\tau}_i = \begin{cases} 2Y_i, & D_i = 1 \\ -2Y_i, & D_i = 0 \end{cases}, \quad Y_i = \text{obs. outcome}$$

- Although the above estimate is unbiased, it is not practically usefully as it is not precise since it has a very high variance.

- Let us compute the variance of the unit level causal effect estimator:

$$Var(\hat{\tau}_i) = E(\hat{\tau}_i^2) - E(\hat{\tau}_i)^2$$

$$= \left(\frac{Y_{i1}}{p}\right)^2 p + (1-p)\left(\frac{Y_{i0}}{1-p}\right)^2 - (Y_{i1} - Y_{i0})^2$$

$$= p(1-p)\left[\frac{Y_{i1}^2}{p^2} + \frac{Y_{i0}^2}{(1-p)^2} + 2\frac{Y_{i1}Y_{i0}}{p(1-p)}\right]$$

$$= p(1-p)\left(\frac{Y_{i1}}{p} + \frac{Y_{i0}}{1-p}\right)^2$$

- The variance of the estimator is a quadratic function of the potential outcomes, and hence the standard errors of the unit level causal effect estimates will be huge.
- Although there exists an unbiased estimator for the unit level causal effects, inference is difficult relative to the ATE as limit theorems do not apply.