Dynamic Demand III: Hendel Nevo

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Grad IO

Today's Readings

- Melnikov (Yale PhD Thesis 2001)
- Gowrisankaran Rysman (JPE)
- Hendel and Nevo (Econometrica)
- Erdem Imai Keane (2003) also look at a similar problem in Marketing (I am going to skip it this year).

Hendel and Nevo (2006)

- When a supermarket cuts the price of laundry detergent for a week there is a huge increase in sales.
- This leads us to conclude consumers are extremely elastic with respect to price
- When a supermarket makes a permanent price cut to laundry detergent, there is little sales impact in the long run.
- Now consumers look highly inelastic with respect to price
- Often we use average prices which include high and low periods in regression studies – does this make sense?
- How can we resolve this puzzle?

Hendel and Nevo (2006)

- Hendel and Nevo suggest that consumers respond by temporary price reductions by stockpiling inventories.
- Consumers spend down their inventories during periods of high prices
- Consumers have variable storage costs and price sensitivities. Why?
- This has implications for inter temporal price discrimination and retail High-Low pricing strategies.

Data

- 9 Supermarkets in a large midwest city (Dominick's in Chicago)
- Store-level: for each brand 13 (j) size x: 32-256oz in each store, each week (t)
  1. Price p_{jxt}
  2. Quantity q_{jxt}
  3. Promotions a_{jxt} (binary for feature/display)
- Consumers are of type h with utility: $u(c_{ht} + \nu_{ht}; \theta_h)$
- Current consumption is $c_{ht} = \sum_j c_{jht}$ not brand specific!
- There is a shock affected marginal utility of consumption ν_{ht}.
- Decision: $d_{hjxt} = 1$ is a purchase of h of brand j and size x at t. (includes outside option $= 0$).

Table 3: Sales

	Quantity Discount (%)	Quantity Sold on Sale (%)	Weeks on Sale (%)	Average Sale Discount (%)	Quantity Share (%)
Liquid					
32 oz.	—	2.6	2.0	11.0	1.6
64 oz.	18.1	27.6	11.5	15.7	30.9
96 oz.	22.5	16.3	7.6	14.4	7.8
128 oz.	22.8	45.6	16.6	18.1	54.7
256 oz.	29.0	20.0	9.3	11.8	1.6
Powder					
32 oz.	—	16.0	7.7	14.5	10.1
64 oz.	10.0	30.5	16.6	12.9	20.3
96 oz.	14.9	17.1	11.5	11.7	14.4
128 oz.	30.0	36.1	20.8	15.1	23.2
256 oz.	48.7	12.9	10.8	10.3	17.3

Dynamic Discrete Choice

$$
\begin{aligned}
V(s_t) &= \max_{c_h(s_t), d_{jxt}(s_t)} \sum_t \beta^{t-1} E[u(c_{ht} + \nu_{ht}; \theta_h) - C_h(i_{h,t+1}; \theta_h) \\
&+ \sum_j d_{hjxt}(\alpha_h^p p_{jxt} + \xi_{hjx} + \alpha_h^a a_{jxt} + \epsilon_{hjxt}) | s_t] \\
i_{h,t+1} &= i_{ht} + x_{ht} - c_{ht} \\
\sum_{j,x} d_{hjxt} &= 1
\end{aligned}
$$

- Abuse of notation: x_{ht} is size of the choice
- $C_h(i; \theta_h)$ is cost of storage
- s_t contains current inventory i_t, current prices, and consumption shock ν_t as well as ϵ_{ht}.
- ξ_{jhxt} captures expected future differences in utility of x units of j at time of purchase.
    1. as long as discounting is low
    2. brand-specific differences in utility (but not consumption) enter linearly.

Model Assumptions

Assumption 1
ν_t is independently distributed over time and across consumers.

No serial correlation!

Assumption 2
Prices p_{jxt} and advertising a_{jxt} follow an exogenous first-order Markov process.

Hard to justify this with a model of profit maximizing supply!

Assumption 3
ϵ_{jxt} is i.i.d. extreme value type 1.

Likelihood

Conditional on Household we can write the probability of a sequence of purchase decisions:

$$P(d_1, \ldots, d_t | p_1, \ldots, p_T) = \int \prod_t P(d_t | p_t,$$
$$i_t(d_{t-1}, \ldots, d_1, \nu_{t-1}, \ldots, \nu_1, i_1)) dF(\nu_1, \ldots, \nu_T) dF(i_1)$$

- Beginning of period inventory depends on previous decisions, previous shocks, and initial inventory.
- p_t now includes all observed state variables not just prices

Choice Problem

$$
\begin{aligned}
Pr(d_{jx}|p_t, i_t, \nu_t) &= \frac{\exp[\alpha p_{jxt} + \xi_{jx} + \beta a_{jxt} + M(s_t, j, x)]}{\sum_{k,y} \exp[\alpha p_{kyt} + \xi_{jy} + \beta a_{kyt} + M(s_t, k, y)]} \\
M(s_t, j, x) &= \max_c [u(c + \nu_t) - C(i_{t+1}) + \beta E[V(s_{t+1}|d_{jx}, c, s_t]]
\end{aligned}
$$

- State space has very high dimension. (Lots of brand-size combos at different prices)
- Keeping track of all brands/prices would be very costly

3-step Procedure

To reduce complexity, Hendel and Nevo propose a 3-step estimator

- Maximize likelihood of observed brand choice conditional on size in order to recover the (α, ξ) parameters.
- This avoids solving MDP but instead is just static discrete choice problem (efficiency loss!)
- Second step: compute inclusive values for each size and transition probability matrix.
- Now solve a quantity choice only nested fixed point problem. The key is that there is only one "index price" per size.
- The reason this is feasible is our old friend, the conditional independence assumption (of what?)

Step 1: Brand Choice

$$
\begin{aligned}
Pr(d_{jx}|x_t, p_t, i_t, \nu_t) &= \frac{\exp[\alpha^p p_{jxt} + \xi_{jx} + \alpha^a a_{jxt}]}{\sum_{k,y} \exp[\alpha^p p_{kyt} + \xi_{jy} + \alpha^a a_{kyt}]} \\
&= Pr(d_{jx}|x_t, p_t)
\end{aligned}
$$

- The trick is that $M(s_t, j, x)$ is the same for all products of the same size x.
- This means the dynamics drop out of the brand-choice equation conditional on x_t.
- We can recover (α, ξ) from static demand estimation!

Step 2: Inclusive Values

$$\omega_{xt} \quad = \quad \log\left(\sum_k \exp(\alpha^p p_{kxt} + \xi_{xt} + \alpha^a a_{kxt})\right)$$

Assumption 4: IVS

$$F(\omega_t | s_{t-1}) = F(\omega_t | \omega_{t-1})$$

- Compute ex-ante expected utility of purchasing size x in period t
- Does not depend on which j is purchased.
- IVS means we can keep track of a lot less information!
- Same as G&R two price vectors with same inclusive values must have same transition probabilities.
- Do individual prices still matter? (Test)

Step 3: Dynamic Choice of Size

$$V(i,\omega_t,\epsilon_t,\nu_t) = \max_{c,x}[u(c+\nu_t) - C(i_{t+1}) + \omega_{xt} + \epsilon_{xt} +$$
$$\beta E[V(i_{t+1},\omega_{t+1},\epsilon_{t+1},\nu_{t+1})|i_t,\omega_t,\epsilon_t,\nu_t,c,x]]$$

- Compute ex-ante expected utility of purchasing size x in period t
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Key Proposition

How do we know that the simplified problem has the same solution as the original dynamic problem?

$$P(x_t|i_t, p_t, \nu_t) = P(x_t|i_t, \omega(p_t), \nu_t)$$

- Before we got to see the entire state s_t
- Now we only see the expected utility of x_t aka ω_{xt}
- The proof relies on Assumption 3(IID Logit errors) and Assumption 4 (IVS).

Computational Details

Iterate policy evaluation and policy improvement

1. Approximate the value function by a polynomial function of s_t. (Logarithmic)
2. Guess an optimal policy and minimized (LSQ) deviation between the value function and expected future value
3. Update the policy function for every state
4. Update expectation with coefficients and expected value of state variables
5. Repeat until value function coefficients converge

This is a Smooth Approximation approach

Table 4: Brand Choice Estimates

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Price	−0.51	−1.06	−0.49	−0.26	−0.27	−0.38	−0.38	−0.57	−1.41	−0.75
	(0.022)	(0.038)	(0.043)	(0.050)	(0.052)	(0.055)	(0.056)	(0.085)	(0.092)	(0.098)
*Suburban dummy				−0.33	−0.30	−0.34	−0.33	−0.25	−0.45	−0.19
				(0.055)	(0.061)	(0.055)	(0.056)	(0.113)	(0.127)	(0.127)
*Nonwhite dummy				−0.34	−0.39	−0.38	−0.33	−0.34	−0.33	−0.26
				(0.075)	(0.083)	(0.076)	(0.076)	(0.152)	(0.166)	(0.168)
Large family				−0.23	−0.13	−0.21	−0.22	−0.46	−0.38	−0.43
				(0.080)	(0.107)	(0.080)	(0.082)	(0.181)	(0.192)	(0.195)
Feature			1.06	1.05	1.08	0.92	0.93	1.08		1.05
			(0.095)	(0.096)	(0.097)	(0.099)	(0.100)	(0.123)		(0.126)
Display			1.19	1.17	1.20	1.14	1.15	1.55		1.52
			(0.069)	(0.070)	(0.071)	(0.071)	(0.072)	(0.093)		(0.093)
Brand dummy variable		✓	✓	✓	✓					
*Demographics					✓					
*Size						✓				
Brand–size dummy variable							✓			
Brand-HH dummy variable								✓		
*Size									✓	✓

Table 5: Belief Process Estimates

	Same Process for All Types				Different Process for Each Type			
	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}
$\omega_{1,t-1}$	0.003	−0.014	0.005	0.014	−0.023	−0.005	−0.019	0.007
	(0.012)	(0.011)	(0.014)	(0.014)	(0.017)	(0.014)	(0.019)	(0.015)
$\omega_{2,t-1}$	0.413	0.033	0.295	0.025	0.575	−0.003	0.520	0.011
	(0.007)	(0.010)	(0.008)	(0.007)	(0.013)	(0.010)	(0.016)	(0.013)
$\omega_{3,t-1}$	0.003	−0.034	0.041	−0.006	0.027	−0.072	0.051	−0.018
	(0.007)	(0.007)	(0.009)	(0.009)	(0.020)	(0.016)	(0.025)	(0.020)
$\omega_{4,t-1}$	0.029	0.249	0.026	0.236	−0.018	0.336	−0.018	0.274
	(0.008)	(0.008)	(0.008)	(0.017)	(0.020)	(0.016)	(0.021)	(0.017)
$\sum_{\tau=2}^{5} \omega_{1,t-\tau}$			−0.003	−0.012			−0.008	−0.003
			(0.005)	(0.004)			(0.006)	(0.005)
$\sum_{\tau=2}^{5} \omega_{2,t-\tau}$			0.089	0.006			0.073	−0.004
			(0.003)	(0.002)			(0.005)	(0.004)
$\sum_{\tau=2}^{5} \omega_{3,t-\tau}$			−0.008	−0.009			−0.004	−0.016
			(0.003)	(0.003)			(0.008)	(0.006)
$\sum_{\tau=2}^{5} \omega_{4,t-\tau}$			−0.013	0.018			−0.008	0.056
			(0.003)	(0.003)			(0.007)	(0.005)

Table 5: Dynamic Problem Estimates

Household Type:	1	2	3	4	5	6
	Urban Market			Suburban Market		
Household Size:	1–2	3–4	5+	1–2	3–4	5+
Cost of inventory						
Linear	9.24	6.49	21.96	4.24	4.13	11.75
	(0.01)	(0.02)	(0.09)	(0.01)	(0.17)	(5.3)
Quadratic	−3.82	1.80	−35.86	−8.20	−6.14	−0.73
	(29.8)	(1.77)	(0.19)	(0.03)	(1.69)	(1.53)
Utility from consumption	1.31	0.75	0.51	0.08	0.92	3.80
	(0.02)	(0.09)	(0.21)	(0.03)	(0.18)	(0.38)
Log likelihood	365.6	926.8	1,530.1	1,037.1	543.6	1,086.1

Table 8: Elasticities Compared to Static Model

AVERAGE RATIOS OF ELASTICITIES COMPUTED FROM A STATIC MODEL TO LONG-RUN ELASTICITIES COMPUTED FROM THE DYNAMIC MODEL[a]

		64 oz.						128 oz.					
Brand	Size (oz.)	All[b]	Wisk	Surf	Cheer	Tide	Private Label	All[b]	Wisk	Surf	Cheer	Tide	Private Label
All[b]	64	1.03	0.13	0.14	0.12	0.13	0.15	0.14	0.17	0.17	0.18	0.21	0.34
	128	0.17	0.24	0.26	0.20	0.28	0.35	1.23	0.09	0.11	0.09	0.15	0.22
Wisk	64	0.14	1.20	0.13	0.17	0.12	0.13	0.16	0.22	0.14	0.22	0.25	0.20
	128	0.25	0.27	0.23	0.31	0.26	0.28	0.08	1.42	0.08	0.13	0.18	0.11
Surf	64	0.14	0.13	0.93	0.16	0.13	0.14	0.18	0.18	0.12	0.18	0.22	0.28
	128	0.25	0.22	0.18	0.27	0.25	0.18	0.12	0.11	1.20	0.08	0.15	0.14
Cheer	64	0.12	0.17	0.16	0.84	0.09	0.13	0.14	0.24	0.16	0.14	0.22	0.24
	128	0.25	0.26	0.26	0.12	0.23	0.22	0.09	0.12	0.06	0.89	0.15	0.07
Tide	64	0.16	0.17	0.13	0.13	1.26	0.15	0.22	0.28	0.16	0.26	0.22	0.37
	128	0.25	0.31	0.22	0.24	0.22	0.31	0.11	0.16	0.08	0.13	1.44	0.31
Solo	64	0.15	0.12	0.15	0.14	0.12	0.14	0.17	0.15	0.15	0.30	0.30	0.28
	128	0.23	0.20	0.24	0.21	0.21	0.25	0.07	0.07	0.06	0.16	0.17	0.21
Era	64	0.21	0.12	0.13	0.13	0.10	0.19	0.43	0.17	0.15	0.22	0.19	0.35
	128	0.31	0.22	0.24	0.25	0.17	0.38	0.19	0.08	0.09	0.11	0.10	0.22
Private	64	0.19	0.15	0.14	0.17	0.17	1.02	0.32	0.22	0.15	0.26	0.31	0.25
label	128	0.29	0.28	0.34	0.30	0.39	0.29	0.16	0.12	0.13	0.10	0.27	1.29
No purchase		2.12	1.13	1.15	1.40	1.27	2.39	1.80	7.60	2.26	14.11	2.38	10.86

[a] Cell entries i and j, where i indexes row and j indexes column, give the ratio of the (short-run) elasticities computed from a static model divided by the long-run elasticities computed from the dynamic model. The elasticities for both models are the percent change in market share of brand i with a 1 percent change in the price of j. The static model is identical to the model estimated in the first step, except that brands of all sizes are included as well as a no-purchase decision, not just products of the same size as the chosen option. The results from the dynamic model are based on the results presented in Tables IV–VI.