Estimating Single-Agent Dynamic Models II

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A "macroeconomic" introduction: Euler equations

- For a consumption-savings problem with CRRA utility, the following Euler equation describes optimal savings behavior:

$$c_t^\alpha = \beta R_t E_t\left[c_{t+1}^\alpha\right],$$

or

$$E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^\alpha\right] = (\beta R_t)^{-1},$$

where

  - c_t is time-t consumption,
  - α is the parameter of interest to be estimated,
  - β is the discount factor,
  - R_t is the rate of return on time t savings

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where

  - c_t is time-t consumption,
  - α is the parameter of interest to be estimated,
  - β is the discount factor,
  - R_t is the rate of return on time t savings
    - Known at t
    - Time varying

Euler equations review II

- Baseline Euler equation:

$$E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{\alpha} \right] = (\beta R_t)^{-1},$$

- We can drop the expectation operator, writing

$$\left(\frac{c_{t+1}}{c_t} \right)^{\alpha} = (\beta R_t)^{-1} + e_t,$$

where $e_t = \left(\frac{c_{t+1}}{c_t} \right)^{\alpha} - E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{\alpha} \right]$.

Euler equations review III

- As observed by Hall (1978), e_t is uncorrelated with everything in the time-t information set by construction:

$$
\begin{aligned}
E\left[e_t x_t\right] &= E\left[\left(\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} - E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{\alpha}\right]\right) x_t\right] \\
&= E\left[\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} x_t\right] - E\left[E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{\alpha}\right] x_t\right] \\
&= E\left[\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} x_t\right] - E\left[E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} x_t\right]\right] \\
&= E\left[\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} x_t\right] - E\left[\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} x_t\right] \\
&= 0,
\end{aligned}
$$

where the third line follows from x_t's being in the time t information set, and the fourth follows from the law of iterated expectations.

Euler equations review IV

- In particular, $E\left(R_t^{-1}e_t\right) = 0$. It follows that

$$\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} = (\beta R_t)^{-1} + e_t \qquad (1)$$

  is a valid (nonlinear) regression equation.

- Following Hansen and Singleton (1982), we could use an IV approach to estimating (1).

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- Following Hansen and Singleton (1982), we could use an IV approach to estimating (1).
  - E.g., suppose we have measurement error in $R_t = R_t^* + \nu_t$. As usual, we want an instrumental variable z_t that is correlated with the true value R_t^* and not correlated with the measurement error ν_t.
  - The instrument z_t should also be in the time-t information set so that it's not correlated with e_t.

Euler equations review IV

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  - The instrument z_t should also be in the time-t information set so that it's not correlated with e_t.
  - We could also deal with other endogeneity problems with instruments, e.g.

ECCP estimators

- **ECCP**: Euler Equations in Conditional Choice Probabilities
  - Derived from standard dynamic discrete choice tools (Hotz and Miller, 1993; Arcidiacono and Miller, 2011).
  - Like Euler equations from continuous choice contexts, ECCP equations relate observed behavior in successive time periods.
  - Aguirregabiria and Magesan (2013) formalize the analogy to continuous choice Euler equations: treat choice probabilities as choice variables.
- ECCP estimators make IV's tractable in dynamic discrete models.
- Easy to estimate (OLS or linear IV)
- No need to fully specify how all state variables evolve (unlike Hotz and Miller (1993), Aguirregabiria and Mira (2002), Pesendorfer and Schmidt-Dengler (2008))

Literature

- ECCP framework: Kalouptsidi, Scott, and Souza-Rodrigues (2018).
- Applied papers: Scott (2013), Traiberman (2018), De Groote and Verboven (2018), Diamond et al (2017), Almagro and Dominguez-Iino (2019).
- Basic DDC tools: Rust (1987), Hotz and Miller (1993), Arcidiacono and Miller (2011), Arcidiacono and Ellickson (2011)
- Euler equation connection: Aguirregabiria and Magesan (2013)
- Related non-stationary framework: Arcidiacono and Miller (2017)
- Other approaches to unobservable state variables in the estimation of dynamic models: Arcidiacono and Miller (2011), Hu and Shum (2012), Berry and Compiani (2019), Kasahara and Shimotsu (2009)

Outline

1. General framework for ECCP equations
   - Examples: land use change, durable demand
2. Derivation of ECCP regression equation
3. Identification and Asymptotics (very briefly)
   - See Kalouptsidi, Scott, and Souza-Rodrigues (2018) for details
4. Monte Carlo study with durable demand example

Example 1: Land Use Change

Applied question

- What are the effects of biofuels policy?

Methodological issue

- How to estimate long-run elasticities of crop supply?
  - Relevant for many agricultural and environmental policy questions

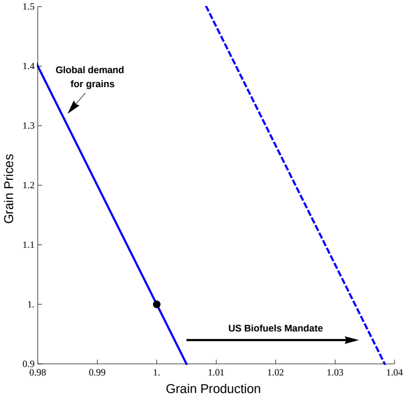
Contributions

- I develop a tractable and flexible empirical dynamic model of land use
- Taking dynamics into account implies larger environmental impacts, smaller price impacts from biofuels

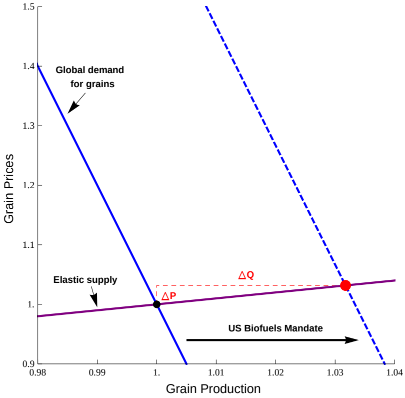
Motivation: biofuels policy

- US biofuels mandate: about 10% of gasoline must come from biofuels (Renewable Fuels Standard)
- Appeal of biofuels: closing the carbon cycle
  - But what is the opportunity cost of the feedstock?
- Biofuels mandate $\Rightarrow$ a long-run increase in demand for grains
  - 35-40% of US corn production used to for ethanol recently
- Increased demand $\Rightarrow$ higher food prices and/or environmentally destructive land use change

Effects of the US biofuels mandate



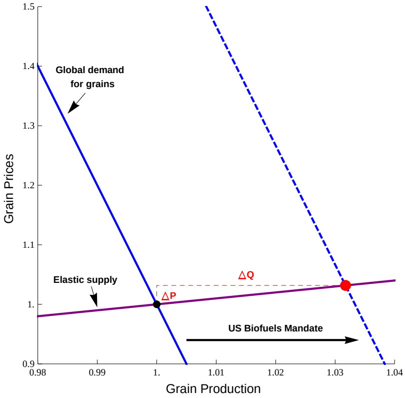
Effects of the US biofuels mandate

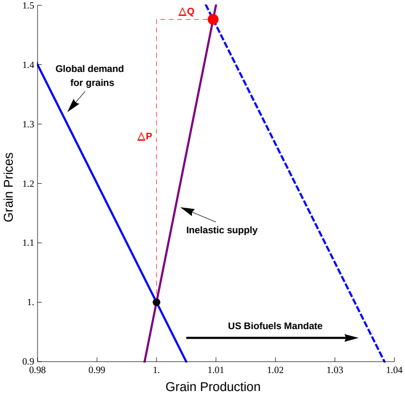


Elastic supply ⇒

Environmental Destruction

Effects of the US biofuels mandate





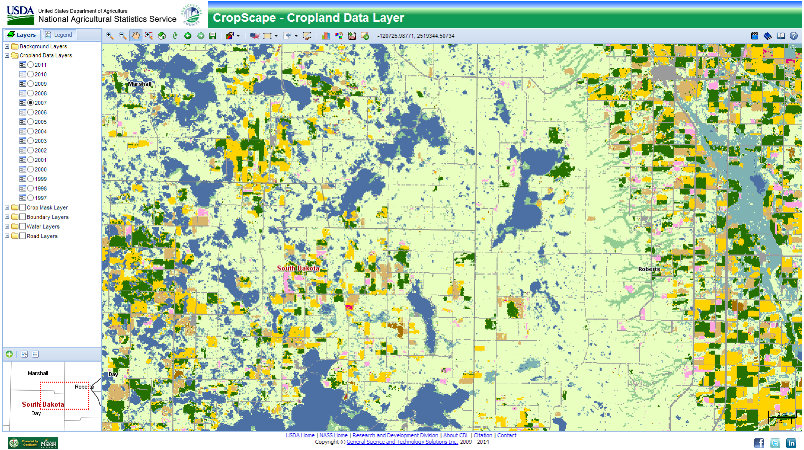
Elastic supply $\Rightarrow$
Environmental Destruction

Inelastic supply $\Rightarrow$
Starvation

Corn Prices

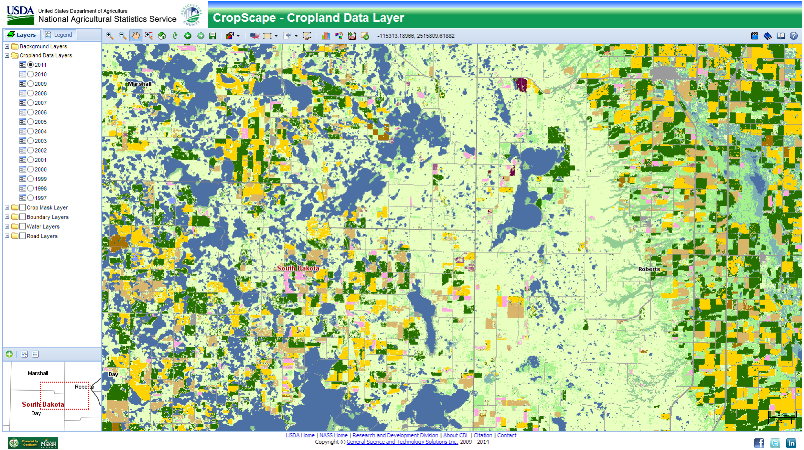
US Dollars per Metric Ton
349.6
322.54
295.48
268.42
241.36
214.3
187.24
160.18
133.12
106.06
79
Oct-2000
Oct-2001
Oct-2002
Oct-2003
Oct-2004
Oct-2005
Oct-2006
Oct-2007
Oct-2008
Oct-2009
Oct-2010
Oct-2011
Oct-2012
Oct-2013
Oct-2014

Choice data preview



Roberts County, SD, 2007

Choice data preview



Roberts County, SD, 2011

Dynamics in agricultural supply estimation

- Common in empirical agricultural economics are models featuring state dependence: Nerlove (1956), Lubowski (2002)
  - distinction between short- and long-run comes from changes compounding over time, behavior in current period is function of current price and current state
- My estimation strategy differs in allowing for dynamically optimizing agents
  - landowners may respond differently to different types of price variation
  - important if we want to predict response to counterfactual price variation that is (potentially) different than the type of price variation in the data
- Relative to adaptive expectations models,
  - My model is computationally simpler to estimate
  - My framework does not require the econometrician to specify the full state space and determine how all state variables evolve

Model I

- Agents indexed by i; markets, by m, time, by t.
- Agent chooses an action $a_{imt} \in \mathcal{A} = \{0, \ldots, A\}$, to maximize her expected discounted sum of payoffs, Π_{imt},

$$E\left[\sum_{\tau=0}^{\infty} \beta^{\tau} \Pi_{im,t+\tau} | \mathcal{I}_{imt}\right],$$

where β is the discount factor, and $E\left[\cdot|\mathcal{I}_{imt}\right]$ denotes the expectation operator conditioned on the information set $\mathcal{I}_{imt}$.

Model II

- Per-period utility payoff function:

$$\Pi_{imt}(a) = \overline{\pi}(a, k_{imt}, w_{mt}) + \xi(a, k_{imt}, \omega_{mt}) + \varepsilon_{imt}(a).$$

  where

  - k_{imt} is an agent-level state variable, observed by econometrician,
  - w_{mt} is a market-level state variable, observed by econometrician,
  - ω_{mt} is a market-level state variable, not observed by econometrician,
  - ε_{imt} is an i.i.d. shock.

- State variables evolve as follows:

$$F\left(k', w', \omega', \varepsilon' | a, k, w, \omega, \varepsilon\right) = F^{\omega}\left(w', \omega' | w, \omega\right) F^{k}\left(k' | a, k, w\right) F^{\varepsilon}\left(\varepsilon'\right)$$

- Note: agents are "small"

Model III

- Per-period utility payoff function:

$$\Pi_{imt}(a) = \overline{\pi}(a, k_{imt}, w_{mt}) + \xi(a, k_{imt}, \omega_{mt}) + \varepsilon_{imt}(a).$$

- WLOG, assume $E(\xi) = 0$ and define $\pi = \overline{\pi} + \xi$
- Note: no restrictions on dimension of or relationship between w and ω; $\overline{\pi}$ and ξ may be correlated
  - Hence, the potential need for instrumental variables
- ξ may be serially correlated; no need to specify an explicit process for how it evolves
- β and F^{ε} are assumed to be known.

Example 1: Land Use Change Model

$$\overline{\pi}\left(a_{imt}, k_{imt}, w_{mt}\right) + \xi\left(a_{imt}, k_{imt}, \omega_{mt}\right) + \varepsilon_{imt}\left(a_{imt}\right).$$

- Each period, each farmer chooses whether to plant crops or not.
- Agent-level state variable k_{imt} denotes condition of field – has it been cleared, or is it covered in rough vegetation?
- Observed market-level state variable w_{mt} includes input and output prices, technological conditions that affect yield/productivity.
- Unobserved market-level state variable ω_{mt} may capture unobserved local price variation and/or components of farmer's returns that weren't measured.

Example 2: Durable Demand

$$\overline{\pi}\left(a_{imt}, k_{imt}, w_{mt}\right) + \xi\left(a_{imt}, k_{imt}, \omega_{mt}\right) + \varepsilon_{imt}\left(a_{imt}\right).$$

- Each period, a consumer decides whether to buy a good or not.
- Individual-level state k_{imt} indicates whether consumer already owns a working unit of the good or not.
- Observed market-level state variable w_{mt} is the price of the good
- Unobserved component of payoffs ξ represents quality/demand shock.

Value Functions and Choice Probabilities

- Let $V(k, \omega, \varepsilon)$ be the agent's value function. Note: absorbing w_{mt} into ω_{mt}
- Ex ante value function:

$$V(k_{imt}, \omega_{mt}) \equiv \int V(k_{imt}, \omega_{mt}, \varepsilon_{imt})\, dF^{\varepsilon}(\varepsilon_{imt})$$

- Conditional value function:

$$v_a(k_{imt}, \omega_{mt}) = \pi(a, k_{imt}, \omega_{mt}) + \beta E\left[V(k_{imt+1}, \omega_{mt+1}) \,|a, k_{imt}, \omega_{mt}\right].$$

- Conditional choice probabilities:

$$p_a(k, \omega) = \int 1\{v_a(k, \omega) + \varepsilon(a) \geq v_j(k, \omega) + \varepsilon(j), \forall j \in \mathcal{A}\}\, dF^{\varepsilon}(\varepsilon)$$

Rational Expectations

Rational Expectations

Agent's expectations conditional on the information set $\mathcal{I}_{imt}$ correspond to the conditional expectations of the true data generating process given $\mathcal{I}_{imt}$.

Forecast errors

For a particular realization $\omega^* \in \Omega$,

$$e^V\left(k', \omega, \omega^*\right) \equiv E_{\omega'|\omega}\left[V\left(k', \omega'\right)|\omega\right] - V\left(k', \omega^*\right),$$

$$e^V\left(a, k, \omega, \omega^*\right) \equiv \sum_{k'} e^V\left(k', \omega, \omega^*\right) F^k\left(k'|a, k, w\right),$$

where k' and ω' denote next period values for k and ω.

Some background: Logit Inclusive Value

- For a static logit model, suppose mean utilities (before the logit shocks) are $(u_1, u_2, \dots, u_J)$
- The logit inclusive value (i.e., ex ante value of the choice set) is

$$U = \ln \sum_j \exp(u_j) + \gamma$$

- Adding and subtracting u_1,

$$U = \ln\left(\exp(u_1)^{-1} \sum_j \exp(u_j)\right) + \gamma + u_1$$

- Given that the probability of choosing $j = 1$ is $p_1 = \exp(u_1) / \sum_j \exp(u_j)$,

$$U = -\ln(p_1) + \gamma + u_1$$

Useful Tool: The Arcidiacono-Miller Lemma

- From Arcidiacono and Miller (2011, Lemma 1):

$$V(k,\omega) = v_a(k,\omega) + \psi_a(p(k,\omega))$$

  where ψ_a is a known function as long as the distribution of idiosyncratic shocks F^ε is known.

- Note: this equation holds for any action a. By differencing the equation across actions, we get the better-known Hotz-Miller inversion.

- This is an implication of dynamic optimization. It's the starting point for ECCP equations just as first-order conditions are the starting point for standard Euler equations in a continuous choice setting.

Regression Equation Derivation I

- Recall the conditional value function:

$$v_a(k_{imt}, \omega_{mt}) = \pi(a, k_{imt}, \omega_{mt}) + \beta E\left[V(k_{imt+1}, \omega_{mt+1})\,|a, k_{imt}, \omega_{mt}\right].$$

- After substituting with the Arcidiacono-Miller Lemma:

$$\begin{array}{ccl} V(k_{imt}, \omega_{mt}) & = & \pi(a, k_{imt}, \omega_{mt}) \\ -\psi_a(k_{imt}, \omega_{mt}) & & +\beta E\left[V(k_{imt+1}, \omega_{mt+1})\,|a, k_{imt}, \omega_{mt}\right]. \end{array}$$

- And then substituting for the expectation using the forecast error:

$$\begin{array}{l} \pi(a, k_{imt}, \omega_{mt}) + \beta e^{V}(a, k_{imt}, \omega_{mt}, \omega_{mt+1}) \\ \quad = V(k_{imt}, \omega_{mt}) - \psi_a(k_{imt}, \omega_{mt}) \\ \qquad -\beta \sum_{k'} V(k', \omega_{mt+1})\, F^k(k'|a, k_{imt}, w_{mt}) \end{array}$$

Regression Equation Derivation II

- Letting mt subscripts stand for dependence on ω_{mt} and stacking equations across k, we have

$$\overline{\pi}_{amt} + \xi_{amt} + \beta e^{V}_{am,t,t+1} = V_{mt} - \beta F^{k}_{amt} V_{mt+1} - \psi_{amt}$$

- The V_{mt} term can be by differencing across actions a and j:

$$\begin{aligned}
\psi_{jmt} - \psi_{amt} \quad = \quad & \overline{\pi}_{amt} - \overline{\pi}_{jmt} \\
& + \xi_{amt} - \xi_{jmt} + \beta \left(e^{V}_{am,t,t+1} - e^{V}_{jm,t,t+1} \right) \\
& + \beta \left(F^{k}_{amt} - F^{k}_{jmt} \right) V_{mt+1}.
\end{aligned}$$

- Notice that the LHS can be estimated in a first stage. Before we have a regression equation, we need to deal with the V_{mt+1} term.

Regression Equation Derivation III

$$\begin{aligned}\psi_{jmt} - \psi_{amt} &= \overline{\pi}_{amt} - \overline{\pi}_{jmt} \\ &\quad + \xi_{amt} - \xi_{jmt} + \beta \left(e^V_{am,t,t+1} - e^V_{jm,t,t+1} \right) \\ &\quad + \beta \left(F^k_{amt} - F^k_{jmt} \right) V_{mt+1}.\end{aligned}$$

- Using the Arcidiacono and Miller Lemma to substitute for V_{mt+1},

$$\begin{aligned}&\psi_{jmt} - \psi_{amt} = \\ &\quad \overline{\pi}_{amt} - \overline{\pi}_{jmt} \\ &\quad + \xi_{amt} - \xi_{jmt} + \beta \left(e^V_{am,t,t+1} - e^V_{jm,t,t+1} \right) \\ &\quad + \beta \left(F^k_{amt} - F^k_{jmt} \right) \left(\overline{\pi}_{Jmt+1} + \xi_{Jmt+1} + \psi_{Jmt+1} \right) \\ &\quad + \beta \left(F^k_{amt} - F^k_{jmt} \right) \left(F^k_{Jmt} E_{t+1} \left[V_{mt+2} \right] \right)\end{aligned}$$

Regression Equation Derivation IV

$$
\begin{aligned}
&\psi_{jmt} - \psi_{amt} = \\
&\qquad \overline{\pi}_{amt} - \overline{\pi}_{jmt} \\
&\qquad + \xi_{amt} - \xi_{jmt} + \beta \left(e^{V}_{am,t,t+1} - e^{V}_{jm,t,t+1} \right) \\
&\qquad + \beta \left(F^{k}_{amt} - F^{k}_{jmt} \right) \left(\overline{\pi}_{Jmt+1} + \xi_{Jmt+1} + \psi_{Jmt+1} \right) \\
&\qquad + \beta \left(F^{k}_{amt} - F^{k}_{jmt} \right) \left(F^{k}_{Jmt} E_{t+1} \left[V_{mt+2} \right] \right)
\end{aligned}
$$

- If J is a renewal (or terminal) action, then F^{k}_{J} has constant columns. Consequently,

$$F^{k}_{amt} F^{k}_{Jmt} = F^{k}_{jmt} F^{k}_{Jmt}.$$

and

$$\left(F^{k}_{amt} - F^{k}_{jmt} \right) \left(F^{k}_{Jmt} E_{t+1} \left[V_{mt+2} \right] \right) = 0$$

Regression Equation Derivation V

- Thus, if J is a terminal or renewal action, we have the following regression equation:

$$\begin{aligned} \psi_{jmt} - \psi_{amt} - \beta\left(F^k_{amt} - F^k_{jmt}\right)\psi_{Jmt+1} = \\ \overline{\pi}_{amt} - \overline{\pi}_{jmt} + \beta\left(F^k_{amt} - F^k_{jmt}\right)\overline{\pi}_{Jmt+1} \\ + \xi_{amt} - \xi_{jmt} + \beta\left(F^k_{amt} - F^k_{jmt}\right)\xi_{Jmt+1} \\ + \beta\left(e^V_{am,t,t+1} - e^V_{jm,t,t+1}\right) \end{aligned}$$

- Note LHS is observable (with an imputed discount factor); RHS has only payoff function that we want to estimate and error terms.

Finite Dependence

Finite Dependence

A pair of choices a and j satisfies τ-period finite dependence if there exists two sequences of actions $(a, a_1, \ldots, a_\tau)$ and $(j, j_1, \ldots, j_\tau)$ such that, for all t,

$$F^k_{amt} F^k_{a_1 mt+1} \ldots F^k_{a_\tau mt+\tau} = F^k_{jmt} F^k_{j_1 mt+1} \ldots F^k_{j_\tau t+\tau}.$$

- With finite dependence, we can eventually get cancellation of continuation values with repeated substitution with the Arcidiacono-Miller Lemma.

Example: Durable Demand

- $a_{imt} = b$ indicates buying; $a_{imt} = nb$, not buying.
- $k_{imt} = 0$ if the consumer does not have a unit of the good, and $k_{imt} = 1$ when she already owns it.
- The state evolves as follows, where buying is a renewal action:

$$Pr\left(k_{imt+1} = 1 | k_{imt}, a_{imt}\right) = \begin{cases} 1 & \text{if } a_{imt} = b \\ 0 & \text{if } a_{imt} = nb, k_{imt} = 0 \\ 1 - \phi & \text{if } a_{imt} = nb, k_{imt} = 1. \end{cases}$$

where ϕ is the rate of product failure/death.

Example: Durable Demand

- The consumer enjoys the following flow utility if purchasing the product:
$$\pi\left(b, k_{imt}, \omega_{mt}\right) = \theta_0 + \theta_1 w_{mt} + \xi_{mt},$$
where w_{mt} is the price of the good and ξ_{mt} is a demand shock.
- Payoffs when not purchasing:
$$\pi\left(nb, k_{imt}, \omega_{mt}\right) = \begin{cases} \theta_0 & \text{if } k_{imt} = 1 \\ 0 & \text{if } k_{imt} = 0. \end{cases}$$

Example: Durable Demand

- The Hotz-Miller inversion (differenced Arcidiacono-Miller Lemma):

$$\ln\left(\frac{p_{b,mt}\left(k_{imt}\right)}{p_{nb,mt}\left(k_{imt}\right)}\right) = v_{b,mt}\left(k_{imt}\right) - v_{nb,mt}\left(k_{imt}\right).$$

- Focusing on $k = 0$,

$$\begin{aligned}\ln\left(\frac{p_{b,mt}\left(k_{imt}\right)}{p_{nb,mt}\left(k_{imt}\right)}\right) &= \theta_0 + \theta_1 w_{mt} + \xi_{mt} \\ &\quad + \beta\left(V_{mt+1}(1) - V_{mt+1}(0)\right) \\ &\quad + e^V\left(1, \omega_{mt}, \omega_{mt+1}\right) - e^V\left(0, \omega_{mt}, \omega_{mt+1}\right).\end{aligned}$$

Example: Durable Demand

- Focusing on $k = 0$,

$$\ln\left(\frac{p_{b,mt}\left(k_{imt}\right)}{p_{nb,mt}\left(k_{imt}\right)}\right) = \theta_0 + \theta_1 w_{mt} + \xi_{mt} + \beta\left(V_{mt+1}\left(1\right) - V_{mt+1}\left(0\right)\right) + e^V\left(1, \omega_{mt}, \omega_{mt+1}\right) - e^V\left(0, \omega_{mt}, \omega_{mt+1}\right).$$

- Substituting for V using the Arcidiacono-Miller Lemma:

$$\ln\left(\frac{p_{b,mt}\left(k_{imt}\right)}{p_{nb,mt}\left(k_{imt}\right)}\right) = \theta_0 + \theta_1 w_{mt} + \xi_{mt} + \beta\left[-\ln p_{b,mt+1}\left(1\right) + \ln p_{b,mt+1}\left(0\right)\right] + e^V\left(1, \omega_{mt}, \omega_{mt+1}\right) - e^V\left(0, \omega_{mt}, \omega_{mt+1}\right).$$

noting that the π_{mt+1} terms completely cancel.

Example: Durable Demand

$$
\begin{aligned}
\ln\left(\frac{p_{b,mt}\left(k_{imt}\right)}{p_{nb,mt}\left(k_{imt}\right)}\right) \quad = \quad & \theta_0 + \theta_1 w_{mt} + \xi_{mt} + \\
& \beta\left[-\ln p_{b,mt+1}\left(1\right) + \ln p_{b,mt+1}\left(0\right)\right] \\
& + e^V\left(1, \omega_{mt}, \omega_{mt+1}\right) - e^V\left(0, \omega_{mt}, \omega_{mt+1}\right).
\end{aligned}
$$

- Rewriting this, we have a regression equation

$$Y_{mt} = \theta_0 + \theta_1 w_{mt} + u_{mt},$$

where

$$Y_{mt} = \ln\left(\frac{p_{b,mt}\left(0\right)}{p_{nb,mt}\left(0\right)}\right) + \beta \ln\left(\frac{p_{b,mt+1}\left(1\right)}{p_{b,mt+1}\left(0\right)}\right),$$

and

$$u_{mt} = \xi_{mt} + e^V\left(1, \omega_{mt}, \omega_{mt+1}\right) - e^V\left(0, \omega_{mt}, \omega_{mt+1}\right).$$

Example: Durable Demand

The regression equation:

$$Y_{mt} = \theta_0 + \theta_1 w_{mt} + u_{mt},$$

where

$$Y_{mt} = \ln\left(\frac{p_{b,mt}(0)}{p_{nb,mt}(0)}\right) + \beta \ln\left(\frac{p_{b,mt+1}(1)}{p_{b,mt+1}(0)}\right),$$

and

$$u_{mt} = \xi_{mt} + e^V(1, \omega_{mt}, \omega_{mt+1}) - e^V(0, \omega_{mt}, \omega_{mt+1}).$$

- In our Monte Carlo simulations, we will assume the existence of an instrument z_{mt} (a cost shifter) that is correlated with the price w_{mt} but not the demand shock ξ_{mt}.
- Note that z_{mt} is automatically uncorrelated with $e^V(k, \omega_{mt}, \omega_{mt+1})$ as long as it is in the consumer's time t information set.

Instrumental variables

IV Assumption

There exist instruments z_{mt} such that:

(i) For all functions $q(w_{mt})$ with finite expectation, if $E[q(w_{mt})|z_{mt}] = 0$ almost surely, then $q(w_{mt}) = 0$ almost surely,

(ii) $E[\widetilde{\xi}_{ajmt}|z_{mt}] = 0$, for all a and j, and

(iii) $E[\widetilde{e}^{V}_{ajmt}|z_{mt}] = 0$, for all a and j.

Identification and Asymptotics

Proposition 1

Suppose (β, F^{ε}) are known and Rational Expectations and the IV Assumption hold. Assume that, for all pair of actions a and j, the single-action τ-period finite dependence property holds for action J, and that the payoff $\overline{\pi}(J, k, w)$ is known for all (k, w). Then, given the joint distribution of observables $\Pr(y)$, where $y_{imt} = (a_{imt}, k_{imt}, w_{mt}, z_{mt})$, the flow payoff $\overline{\pi}(a, k, w)$ is identified for all (a, k, w).

- We also show that, given a payoff function that is linear in parameters, there is no need to restrict/know the payoffs of the action involved in finite dependence J (the other requirements are still needed).
- We also show that two-step GMM estimates of will be consistent and asymptotically normal under standard conditions.

Rational Expectations: Implications

Lemma 1

Assume Rational Expectations. For any action a, the forecast error term $e^h\left(a, k_{imt}, \omega_{mt}, \omega^*_{mt+1}\right)$ is mean zero given the information set available to the agent $\mathcal{I}_{imt}$: $E\left[e^h\left(a, k_{imt}, \omega_{mt}, \omega^*_{mt+1}\right)|\mathcal{I}_{imt}\right] = 0$.

Lemma 2

Assume Rational Expectations, and assume $z_{mt} \in \mathcal{I}_{imt}$. Then,
$E[\widetilde{e}^V_{ajmt}|z_{mt}] = 0$, for all a and j.
Furthermore, expectational errors are serially uncorrelated.

Short and Long Panels

- Lemma 2 establishes that, given RE, the terms in our moments involving forecast errors are mean zero.
- Furthermore, given the lack of serial correlation, we can be confident that as $T \to \infty$, the sample average of the $\widetilde{e}^V_{ajmt}$ terms in our moments will converge to zero.
- However, with finite T, there is nothing to suggest that the forecast errors will average to zero even as $M \to \infty$ (aggregate shocks)
- Thus, Assumption 2 is less plausible in the context of a short panel.

Monte Carlo: Setup I

$$\pi_{mt}\left(b, k_{imt}\right) = \theta_0 + \theta_1 w_{mt} + \xi_{mt}$$
$$\pi_{mt}\left(nb, k_{imt}\right) = \theta_0 \mathbb{1}\left[k_{imt} = 1\right]$$
$$Y_{mt} = \theta_0 + \theta_1 w_{mt} + u_{mt}$$

- Evolution of prices:

$$w_{mt} = \gamma_0 + \gamma_1 z_{mt} + \gamma_2 \xi_{mt} + \varepsilon^{w}_{mt}$$

- Demand shock:

$$\xi_{mt+1} = \rho_1 + \rho_2 \xi_{mt} + \varepsilon^{\xi}_{mt}$$

- Cost shifter (instrument):

$$z_{mt+1} = \rho_3 + \rho_4 z_{mt} + \varepsilon^{z}_{mt}$$

- Each of the ε^x terms is an i.i.d. shock with variance σ^2_x

Monte Carlo: Setup II

Payoff Parameters:	θ_0	1	$\xi \sim$ Normal AR(1)	ρ_1	0
	θ_1	$-.1$		ρ_2	.2
				σ_ξ^2	0 or 16
Prob. of Product Failure:	δ	.1			
			$z \sim$ Normal AR(1)	ρ_3	0
Discount Factor:	β	.95		ρ_4	.7
				σ_z^2	25
Process for price w_{mt}:	γ_0	40			
	γ_1	1	Aggregate Shocks	λ_z	0 or .7
	γ_2	1			
	σ_w^2	4			

- λ_z represents the fraction of ε_{mt}^z that comes from an aggregate shock.

Estimation Approaches

- We will estimate the model using the ECCP equation,

$$Y_{mt} = \theta_0 + \theta_1 w_{mt} + u_{mt},$$

  estimated using OLS or 2SLS.

- We also use a "Standard CCP" estimation approach (more structural) that requires a full model of how all state variables evolve.
    - Similar to Hotz and Miller (1993), Aguirregabiria and Mira (2002), and Pesendorfer and Schmidt-Dengler (2008)

- The Standard CCP estimator assumes that w_{mt} is the only market-level state variable and is first-order Markov process.

- 5000 replications per specification

ECCP-OLS Estimates						
$\sigma_\xi^2 = 0$		True Parameters: $\theta_0 = 1$, $\theta_1 = -.1$				
		$\lambda_z = 0$		$\lambda_z = .7$		
	T	160	10	160	10	160
	M	10	160	10	160	160
θ_0	Mean Est.	1.01	1.01	0.99	0.75	0.99
	Rel. Bias	0.74%	0.81%	-1.49%	-24.6%	-1.38%
	SD	0.04	0.04	0.09	0.29	0.09
	RMSE	0.04	0.04	0.10	0.38	0.09
θ_1	Mean Est.	-0.10	-0.10	-0.10	-0.09	-0.10
	Rel. Bias	0.18%	0.20%	-0.38%	-6.20%	-0.33%
	SD	1.04e-3	9.97e-4	2.22e-3	6.78e-3	2.05e-3
	RMSE	1.06e-3	1.02e-3	2.26e-3	9.19e-3	2.07e-3

	ECCP-2SLS Estimates					
$\sigma^2_\xi = 0$	True Parameters: $\theta_0 = 1$, $\theta_1 = -.1$					
		$\lambda_z = 0$		$\lambda_z = .7$		
	T	160	10	160	10	160
	M	10	160	10	160	160
θ_0	Mean Est.	1.01	1.01	0.98	0.72	0.98
	Rel. Bias	0.79%	0.87%	-1.66%	-28.2%	-1.57%
	SD	0.04	0.04	0.10	0.33	0.09
	RMSE	0.05	0.04	0.10	0.43	0.10
θ_1	Mean Est.	-0.10	-0.10	-0.10	-0.09	-0.10
	Rel. Bias	0.19%	0.22%	-0.42%	-7.08%	-0.38%
	SD	1.08e-3	1.03e-3	2.40e-3	7.67e-3	2.23e-3
	RMSE	1.10e-3	1.06e-3	2.44e-3	0.01	2.26e-3

Standard CCP Estimates						
$\sigma_\xi^2 = 0$		True Parameters: $\theta_0 = 1$, $\theta_1 = -.1$				
		$\lambda_z = 0$		$\lambda_z = .7$		
	T	160	10	160	10	160
	M	10	160	10	160	160
θ_0	Mean Est.	1.00	1.00	1.00	1.00	1.00
	Rel. Bias	-0.27%	-0.11%	-0.31%	-0.02%	-0.19%
	SD	8.45e-3	0.01	0.02	0.07	0.02
	RMSE	8.87e-3	0.01	0.02	0.07	0.02
θ_1	Mean Est.	-0.10	-0.10	-0.10	-0.10	-0.10
	Rel. Bias	-0.51%	-0.30%	-0.60%	-0.64%	-0.25%
	SD	3.88e-4	6.41e-4	5.27e-4	1.33e-3	5.18e-4
	RMSE	6.42e-4	7.08e-4	7.95e-4	1.48e-3	5.73e-4

ECCP-OLS Estimates

$\sigma_\xi^2 = 16$ — True Parameters: $\theta_0 = 1$, $\theta_1 = -.1$

		$\lambda_z = 0$		$\lambda_z = .7$		
	T	160	10	160	10	160
	M	10	160	10	160	160
θ_0	Mean Est.	-8.63	-8.61	-8.81	-11.60	-8.85
	Rel. Bias	-963%	-961%	-981%	-1260%	-985%
	SD	0.58	0.57	1.03	2.77	0.89
	RMSE	9.64	9.62	9.86	12.90	9.89
θ_1	Mean Est.	0.14	0.14	0.15	0.22	0.15
	Rel. Bias	-241%	-240%	-245%	-315%	-246%
	SD	0.01	0.01	0.02	0.06	0.02
	RMSE	0.24	0.24	0.25	0.32	0.25

ECCP-2SLS Estimates						
$\sigma_\xi^2 = 16$	True Parameters: $\theta_0 = 1$, $\theta_1 = -.1$					
		$\lambda_z = 0$		$\lambda_z = .7$		
	T	160	10	160	10	160
	M	10	160	10	160	160
θ_0	Mean Est.	1.00	1.02	1.02	0.87	0.99
	Rel. Bias	-0.20%	1.82%	1.97%	-12.8%	-0.76%
	SD	0.77	0.75	0.78	0.90	0.20
	RMSE	0.77	0.75	0.78	0.91	0.20
θ_1	Mean Est.	-0.10	-0.10	-0.10	-0.10	-0.10
	Rel. Bias	-0.05%	0.42%	0.44%	-3.25%	-0.19%
	SD	0.02	0.02	0.02	0.02	4.96e-3
	RMSE	0.02	0.02	0.02	0.02	4.96e-3

Standard CCP Estimates						
$\sigma_{\xi}^2 = 16$	True Parameters: $\theta_0 = 1$, $\theta_1 = -.1$					
		$\lambda_z = 0$		$\lambda_z = .7$		
	T	160	10	160	10	160
	M	10	160	10	160	160
θ_0	Mean Est.	0.26	0.25	0.26	0.26	0.17
	Rel. Bias	-74.4%	-74.7%	-74.3%	-73.5%	-83.3%
	SD	0.03	0.03	0.03	0.05	0.02
	RMSE	0.75	0.75	0.74	0.74	0.83
θ_1	Mean Est.	-0.01	-0.01	-0.01	-0.01	2.12e-3
	Rel. Bias	-87.8%	-89.2%	-87.8%	-89.9%	-102%
	SD	5.12e-3	5.74e-3	5.41e-3	9.24e-3	3.66e-3
	RMSE	0.09	0.09	0.09	0.09	0.10

Counterfactuals: real vs feasible

- It's great to estimate parameters without needing a fully specified model, but what does this mean for counterfactuals?
- The measurement issues that motivate the ECCP seem to pose problems for counterfactuals: if it's difficult to say exactly how ξ evolves, how do we think about counterfactuals?
- We perform counterfactuals in two ways:
  - *Real*: simulated using true data generating process for ξ.
  - *Feasible*: simulated assuming $\xi_{m,t} = 0$ for all m, t.

Counterfactuals: long-run elasticity

- The counterfactual of interest is the impact of a permanent increase in the mean price. We calculate the long-run elasticity of the unconditional probability of purchase.
- That is, we solve the agent's problem given mean price level w_0 and compute the long-run unconditional probability of purchase:

$$Pr\left(a_{it} = 1 | \overline{w} = w_0\right)$$

- And then we define the long-run elasticity as follows:

$$\mathcal{E} = \frac{Pr\left(a_{it} = 1 | \overline{w} = w_0\right) - Pr\left(a_{it} = 1 | \overline{w} = w_0 + \varepsilon\right)}{\varepsilon} \frac{w_0}{Pr\left(a_{it} = 1 | \overline{w} = w_0\right)}$$

	True value		ECCP OLS	ECCP IV	Standard CCP
Real LRE	-1.106	Mean Estimate	60.15	-1.104	0.01471
		Relative Bias	-5540%	-0.1561%	-101.3%
		SD	16.62	0.04227	0.02545
		RMSE	63.48	0.04231	1.121
Feasible LRE	-1.022	Mean Estimate	-1.187e4	-1.064	0.03888
		Relative Bias	1.162e6%	4.114%	-103.8%
		SD	1.382e6	0.1184	0.06774
		RMSE	1.382e6	0.1256	1.063

Notes: 5000 replications with sample structure $M = T = 160$. *SD is the standard deviation across replications. RMSE is root-mean squared error. Relative Bias is bias as percentage of the true parameter.*

Conclusion

- ECCP estimators make estimating single-agent DDC models easy:
  - First-stage estimation of CCP's – like traditional CCP approach, but with time-t subscripts
  - Second stage is linear regression
- ECCP estimators offer important modeling advantages:
  - Easy to incorporate instrumental variables
  - Does not require model of how market-level states evolve
- The measurement issues that motivate ECCP estimators don't necessarily undermine using estimates from them for counterfactuals.
- Practitioners should be careful about short panels.