Dynamic Demand I:
Durable Goods

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Grad IO

Today's Readings

- Melnikov (Yale PhD Thesis 2001)
- Gowrisankaran Rysman (JPE)
- Hendel and Nevo (Econometrica)

G+R: Assumptions

We can formally write down a dynamic programming problem that consumers solve:

$$\begin{aligned} V_i(f_{i0t}, \varepsilon_{it}, \Omega_t) &= \max\{f_{i0t} + \beta E_\Omega[E_\varepsilon V_i(f_{i0t}, \varepsilon_{it}, \Omega_{t+1})|\Omega_t], \\ &\quad \max_j f_{ijt} - \alpha_i p_{jt} + \beta E_\Omega[E_\varepsilon V_i(f_{ijt}, \varepsilon_{it}, \Omega_{t+1})|\Omega_t]\} \end{aligned}$$

For a dynamic model to make sense we may want to place some restrictions:

- Rational Expectations
- Dynamic Consistency
- Law of motion for consumer types: $w_{i,t+1} = h(w_{i,t}, s_{ijt})$

Replacement Problem

This Bellman has defined a *Replacement Problem*.

- You own a single durable good with the option to *upgrade* each period.
- When you upgrade you throw away the old durable and get nothing in exchange.
- After a purchase j you receive flow utility $f_{i0t+1} = f_{ijt}$ each period if you don't make a new purchase.
- We could add in depreciation or probabilistic failure if we wanted to.
- No resale market (reasonable for high-tech).

Inclusive Value

Helpful to write: $EV_i(\Omega_t) = \int V_i(\varepsilon_{it}, \Omega_t) f(\varepsilon)$ Rust's Trick

$$\begin{aligned} V_i(f_{i0t}, \varepsilon_{it}, \Omega_t) &= \max\{f_{i0t} + \beta E_\Omega[EV_i(f_{i0t}, \Omega_{t+1})|\Omega_t] + \varepsilon_{i0t}, \\ &\quad \max_j f_{ijt} - \alpha_i p_{jt} + \beta E_\Omega[EV_i(f_{ijt}, \Omega_{t+1})|\Omega_t] + \varepsilon_{ijt}\} \end{aligned}$$

We can write the ex-ante expected utility of purchasing in period t without having to condition on which good you purchase:

$$\begin{aligned} \delta_i(\Omega_t) &= E_\varepsilon[\max_j f_{ijt} - \alpha_i p_{jt} + \beta E_\Omega[EV_i(f_{ijt}, \Omega_{t+1})|\Omega_t] + \varepsilon_{ijt}] \\ &= \log\left(\sum_j \exp[f_{ijt} - \alpha_i p_{jt} + \beta E_\Omega[EV_i(f_{ijt}, \Omega_{t+1})|\Omega_t]]\right) \end{aligned}$$

Inclusive Value Sufficiency

$$EV_i(f_{i0}, \Omega) = \log\left(\exp[f_{i0} + \beta E_{\Omega'}[EV_i(f_{i0}, \Omega')|\Omega]] + \exp(\delta_i(\Omega))\right) + \eta$$

where $\eta = 0.577215665$ (Euler's Constant).

The fact that the expected value function depends recursively on itself and $\delta_i(\Omega_t)$ (Inclusive Value) leads to the following assumption.

Inclusive Value Sufficiency
If $\delta_i(\Omega) = \delta_i(\tilde{\Omega})$ then $g(\delta_i(\Omega')|\Omega) = g(\delta_i(\tilde{\Omega}')|\tilde{\Omega})$ for all $\Omega, \tilde{\Omega}$.

- The idea is that δ tells me everything about the future evolution of the states
- More restrictive than it looks. δ is low because quality is low? or because prices are high? Is this the result of a dynamic pricing equilibrium? (No!)

Inclusive Value Sufficiency

Under IVS the problem reduces to

$$\begin{aligned} EV_i(f_{i0}, \delta_i) &= \log\left[\exp(f_{i0} + \beta E_{\Omega'}[EV_i(f_{i0}, \delta_i')|\delta_i]) + \exp(\delta_i)\right] \\ \delta_i &= \log\left(\sum_j \exp[f_{ijt} - \alpha_i p_{jt} + \beta E_{\delta'}[EV_i(f_{ijt}, \delta_i')|\delta_i]\right) \end{aligned}$$

The idea is that the inclusive value δ_{it} IS the state space, along with his current holding of the durable f_{i0t}.

Rational Expectations

We still have the expectation to deal with:

$$E_{\delta'}[EV_i(f_{ijt}, \delta_i')|\delta_i]$$

We need to take a stand on $g_i(\delta_i'|\delta_i)$ the anticipated law of motion for δ_i. G&R assume it follows an $AR(1)$ process.

$$\delta_{it+1} = \gamma_0 + \gamma_1 \delta_{it} + \nu_{it} \text{ with } \nu_{it} \sim N(0, \sigma_\nu^2)$$

If we see δ_{it} we could just run the $AR(1)$ regression to get consumer belief's $\hat{\gamma}$

Rational Expectations-Interpolation

I still haven't told you how to compute

$$E_{\delta'}[EV_i(f_{ijt},\delta_i')|\delta_i,\gamma] = \int EV_i(f_{ijt},\delta_i')g(\delta'|\delta,\gamma)$$

1. We need to integrate $EV(f_{ijt},\delta_i)$ (a function) over a normal density.
2. But we don't observe $EV(f_{ijt},\delta_i)$ everywhere, only on the grid points of our state space.
3. We can fit a linear function, cubic spline, etc. over δ_i to EV_i at each value of f_{ijt} on our grid.
4. We need to interpolate $\widehat{EV}_i(\delta_i^s)$ (Linear, Cubic Spline, etc.)
5. We might as well interpolate the function at the *Gauss-Hermite* quadrature nodes and weights, recentered at $\gamma_0 + \gamma_1\delta$ in order to reduce the number of places we interpolate $\widehat{EV_i}$.

Rational Expectations-Alternative

There is an alternative method that is likely to be less accurate

$$E_{\delta'}[EV_i(f_{ijt}, \delta_i')|\delta_i, \gamma] = \int EV_i(f_{ijt}, \delta_i')g(\delta'|\delta, \gamma)$$

1. We need to integrate $EV(f_{ijt}, \delta_i)$ (a function) over a normal density but we only see it at the grid points of our state space.
2. We could discretize $g(\delta'|\delta, \gamma)$ so that it is a valid markov transition probability matrix (TPM) evaluated only at the grid points.
3. Now computing the expectation is just matrix multiplication.

I am a bit nervous about whether two discrete approximations will get the continuous integral correct.

The Estimation Problem

We need to solve $\forall i, t$:

$$
\begin{aligned}
S_{jt} &= \sum_i w_i s_{ijt}(f_{i0t}, \delta_{it}) \\
f_{ijt} &= \overline{\alpha} x_{jt} + \xi_{jt} + \sum_l \sigma_l x_{jl} \nu_{il} \\
s_{ijt}(f_{i0t}, \delta_{it}) &= \frac{\exp[f_{ijt} - \alpha_i p_{jt} + \beta E_{\Omega'}[EV_i(f_{ijt}, \delta_i')|\delta_i]}{\exp[EV_i(f_{i0t}, \delta_{it})]} \\
EV_i(f_{i0}, \delta_i) &= \log\left[\exp(f_{i0} + \beta E_{\Omega'}[EV_i(f_{i0}, \delta_i')|\delta_i]) + \exp(\delta_i)\right] \\
\delta_i(EV_i) &= \log\left(\sum_j \exp[f_{ijt} - \alpha_i p_{jt} + \beta E_{\delta'}[EV_i(f_{ijt}, \delta_i')|\delta_i]\right) \\
E[\delta_{it+1}|\delta_{it}] &= \gamma_0 + \gamma_1 \delta_{it} \\
w_{i,t+1} &= h(w_{i,t}, s_{ijt})
\end{aligned}
$$

The Estimation Problem

1. Like BLP we guess the nonlinear parameters of the model θ
2. For a guess of the ξ_{jt}'s we can solve for EV_i by iteratively computing δ, and running the γ regression for each i and spline/interpolating to compute $E[EV_i]$. (Inner Loop)
3. G&R show how the contraction mapping of BLP can be modified to find a fixed point of the δ, ξ, γ relationship to find f_{ijt} (Middle Loop).
4. We need to make sure to update the $w_{i,t}$ via $h(\cdot)$. (This is a TPM that tells maps the transition probabilities of type i holding f_{i0t} to $f_{i0,t+1}$).
5. Once we've solved this whole system of equations, we use ξ to form moments just like BLP and do GMM. (Outer Loop)

G&R Parameters

Table 1: Parameter estimates

Parameter	Base dynamic model	Dynamic model without repurchases	Static model	Dynamic model with micro-moment
	(1)	(2)	(3)	(4)
Mean coefficients (α)				
Constant	-.092 (.029) *	-.093 (7.24)	-6.86 (358)	-.367 (.065) *
Log price	-3.30 (1.03) *	-.543 (3.09)	-.099 (148)	-3.43 (.225) *
Log size	-.007 (.001) *	-.002 (.116)	-.159 (.051) *	-.021 (.003) *
Log pixel	.010 (.003) *	-.002 (.441)	-.329 (.053) *	.027 (.003) *
Log zoom	.005 (.002) *	.006 (.104)	.608 (.075) *	.018 (.004) *
Log LCD size	.003 (.002) *	.000 (.141)	-.073 (.093)	.004 (.005)
Media: DVD	.033 (.006) *	.004 (1.16)	.074 (.332)	.060 (.019) *
Media: tape	.012 (.005) *	-.005 (.683)	-.667 (.318) *	.015 (.018)
Media: HD	.036 (.009) *	-.002 (1.55)	-.647 (.420)	.057 (.022) *
Lamp	.005 (.002) *	-.001 (.229)	-.219 (.061) *	.002 (.003)
Night shot	.003 (.001) *	.004 (.074)	.430 (.060) *	.015 (.004) *
Photo capable	-.007 (.002) *	-.002 (.143)	-.171 (.173)	-.010 (.006)
Standard deviation coefficients ($\Sigma^{1/2}$)				
Constant	.079 (.021) *	.038 (1.06)	.001 (1147)	.087 (.038) *
Log price	.345 (.115) *	.001 (1.94)	-.001 (427)	.820 (.084) *

Standard errors in parentheses; statistical significance at 5% level indicated with *. All models include brand dummies, with Sony excluded. There are 4436 observations.

G&R Robustness

Parameter	State space includes number of products	Perfect foresight	Dynamic model with extra random coefficients	Linear price	Melnikov's model	Month dummies
	(1)	(2)	(3)	(4)	(5)	(6)
Mean coefficients (α)						
Constant	-.098 (.026) *	-.129 (.108)	-.103 (.037) *	-.170 (.149)	-6.61 (.815) *	-.114 (.024) *
Log price	-3.31 (1.04) *	-2.53 (.940) *	-3.01 (.717) *	-6.94 (.822) *	-.189 (.079) *	-3.06 (.678) *
Log size	-.007 (.001) *	-.006 (.001) *	-.015 (.007) *	.057 (.008) *	-.175 (.049) *	-.007 (.001) *
Log pixel	.010 (.003) *	.008 (.001) *	.009 (.002) *	.037 (.012) *	-.288 (.053) *	.010 (.002) *
Log zoom	.005 (.002) *	.004 (.002) *	.004 (.002)	-.117 (.012) *	.609 (.074) *	.005 (.002)*
Log LCD size	.004 (.002) *	.004 (.001) *	.004 (.002) *	.098 (.010) *	-.064 (.088)	.003 (.001) *
Media: DVD	.033 (.006) *	.025 (.004) *	.044 (.018) *	.211 (.053) *	.147 (.332)	.031 (.005) *
Media: tape	.013 (.005) *	.010 (.004) *	.024 (.016)	.200 (.051) *	-.632 (.318) *	.012 (.004) *
Media: HD	.036 (.009) *	.026 (.005) *	.047 (.019) *	.349 (.063) *	-.545 (.419)	.034 (.007) *
Lamp	.005 (.002) *	.003 (.001) *	.005 (.002) *	.077 (.011) *	-.200 (.058) *	.004 (.001) *
Night shot	.003 (.001) *	.004 (.001) *	.003 (.001) *	-.062 (.008) *	.427 (.058) *	.003 (.001) *
Photo capable	-.007 (.002) *	-.005 (.002) *	-.007 (.002) *	-.061 (.019) *	-.189 (.142)	-.007 (.008)
Standard deviation coefficients ($\Sigma^{1/2}$)						
Constant	.085 (.019) *	.130 (.098)	.081 (.025) *	.022 (.004) *		.087 (.013) *
Log price	.349 (.108) *	2.41e-9 (.919)	1.06e-7 (.522)	1.68 (.319) *		.287 (.078) *
Log size			-.011 (.007)			
Log pixel			1.58e-10 (.002)			

Standard errors in parentheses; statistical significance at 5% level indicated with *. All models include brand dummies, with Sony excluded. There are 4436 observations, except in the yearly model, in which there are 505.

Results

- Contrary to the static model, price coefficient is negative (as one would expect).
- Coefficients on many product characteristics are intuitively appealing.
- Allowing for repeated purchases generates more “sensible” results.
- “Better results” from a dynamic model may be due to the fact that people wait to purchase because of the expectations of price declines and not directly because of high prices.
- Unlike the static model, in dynamic setup the explanation of waiting does not conflict with consumers buying relatively high-priced products.
- A variety of robustness measures show that the major simplifying assumptions about the dynamics in the model are broadly consistent with the data.

Perfect Foresight

G&R Report similar elasticities in the perfect foresight case. We make the following simplification

$$E_{\Omega'}[EV_i(f_{i0}, \delta_{i,t+1})|\delta_{i,t}])] = EV_i(f_{i0}, \delta_{i,t+1})$$

This saves us a lot of headaches:

- No more integration/interpolation
- We can solve the problem on the grid!
- No more belief regressions

Alternative Perspective on Beliefs

Recall our objective:

- Plug in an unbiased estimate for the "no-purchase" utility.
- Under perfect foresight this is just the inclusive value of tomorrow's market $\delta_{i,t+1}$ appropriately discounted: $\sum_{k=1}^{T-k} \beta^{t+k} \delta_{i,t+k}$.
- Different ways to think about rational expectations
  - Expectational error of some or all of $\delta_{i,t+k}$'s.
  - Expectational error in today's reservation utility.

Endogeneity and Instruments

- Dynamics mean we lean harder on the assumption of exogenous product characteristics
- In one period we can take characteristics as given, but in many periods this becomes less palatable (Do cameras exogenously improve over time?).
- Endogeneity: price is endogenous while other product characteristics are not, i.e. x_{jt}. (Size,Resolution, etc.)
- Price is chosen by the firms possibly after observing ξ_{jt} and, hence, is endogenous.
- Instruments: use variables that affect the price-cost margin, e.g. measures of how crowded a product is in characteristics space, which effects price-cost margin and the substitutability across products.
  1. all of the product characteristics in x;
  2. mean product characteristics for a given firm;
  3. mean product characteristics for all firms;
  4. the count of products offered by the firm and by all firms.
  5. changes in costs over time?