Entry Models

C.Conlon

Grad IO

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Short run vs. long run

- Mostly we've looked at competition over the short run where we hold the number/identity of competitors fixed.
- In the long run firms may enter or exit depending on their fixed/sunk costs.
- When we teach undergrads we talk about long run average cost curves in addition to short run (price) competition.
- Market structure will obviously change over time.
- Entry is a inherently dynamic process. First we are going to think about entry in a static or two period framework
    1. M Firms make simultaneous entry decision and pay sunk cost F so that we have n entrants.
    2. Firms see number/identity of competitors and set prices/quantities. $\pi_i(n)$

Some Questions

- Why are some industries concentrated and others not? Barriers to Entry? Returns to Scale? Productivity Dispersion?
- How many firms does it take for a market to be "comepetitive"? Three? Four?
- Should we worry about concentrated industries?
  - Baulmol (1982): Theory of contestable markets. Firms keep innovation and market share high and profits low in order to deter prospective entrants.
  - Schumpeter (1942): Monopoly rents provide an incentive for innovation, which competition may reduce. (patents!)
  - Arrow 1963: dominant firms may have dis-incentive to invest in innovation.
  - Barriers to Entry create rents (Sutton 1991), Fewer firms facilitates collusion (Green and Porter 1984).

Basic Theoretical Framework

- Begin by assuming symmetry so that $\pi_i(n) = \pi(n)$. (think of π as a continuation value rather than per-period profits).
- First stage equilibria: Pure Strategies
  - In equilibrium n^* firms enter where

$$\pi(n^*) - F \geq 0 > \pi(n^* + 1) - F$$

  - Does not say anything about identity of entrants. How do firms coordinate on whether to enter or not?
- First stage equilibria: Mixed Strategies:
  - All firms enter with probability p so that expected value of entry is F

$$\sum_{i=1}^{M} \binom{M-1}{n-1} p^{n-1}(1-p)^{M-n}\pi(n) - F = 0$$

  - There may be *ex post* regret in this case.

What is the optimal number of firms?

- First best says: $P = MC$ with a single firm.
- Take second period equilibrium as given (Bertrand, Cournot, etc.).
- Compare free-entry equilibrium $\pi(n) = F$ with social planner's optimal number of firms

$$n^* = \arg\max_n \left(n \cdot (\pi(n) - F) + CS(n)\right)$$

Business Stealing: each entrant imposes negative externality, leading too too much entry.

Consumer Surplus/Market Expansion: each entrant creates a positive externality, leading to too little entry.

- Which effect dominates? Depends on the market conditions!

Mankiw and Whinston (1986)

- Symmetric, homogenous goods $p(n) = p(n \cdot q(n))$.
- Marginal private return to entry

$$\pi(n) - F = p(n)q(n) - c(q(n)) - F$$

- Social Planner solves:

$$\begin{aligned} S(n) &= \max_n \left[\int_0^{nq(n)} p(x)d\,x \right] - nc(q(n)) - nF \\ S'(n) &= p(n)[q(n) + nq'(n)] - c(q(n)) - nc'(q(n))q'(n) - F \\ S'(n) &= \pi(n) - F + \underbrace{nq'(n)[p(n) - c'(q(n))]}_{\geq 0} \geq \pi(n) - F \end{aligned}$$

- Social return $\leq$ Private Return (Excess entry!).
- Business stealing dominates market expansion.

A General Framework

$$\pi(x_i, F_i; n^*) \quad \equiv \quad \underbrace{v(x_i; n)}_{\text{Variable Profit}} \quad - \quad \underbrace{F_i}_{\text{fixed costs}}$$

- Assume that we have lots of cross sectional data so that probability of entry $p(x)$ is observed without error
- Goal: can we identify $v(x)$ and distribution of F_i,$\Phi(\cdot)$.
- Easy case:
  - Monopoly entry
  - Assume that $F_i \perp v(x_i)$.
  - enter when $F_i \leq v(x_i)$.

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- Easy case:
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  - Assume that $F_i \perp v(x_i)$.
  - enter when $F_i \leq v(x_i)$.
  - But, we can take a monotone transform of both sides without changing $p(x)$.

A General Framework

Continue with the "easy case":

- Assume $F \sim U[0,1]$ then $v(x) = p(x)$. so that our condition is $F \leq p(x)$.
- Seems bad if $v(x)$ is some monotone transform of $p(x)$.
- Can still answer questions about "sources of profitability"

$$\frac{\partial v/\partial x^2}{\partial v/\partial x^1} = \frac{\partial p(x)/\partial x^2}{\partial p(x)/\partial x^1}$$

is also invariant to monotone transforms of v.

How to proceed?

- Normalize v using data (prices, quantities, etc.) then $p(x) = \Phi(v(x))$ to recover ϕ.
- Or, place some structure on Φ such as assuming $\sim N(\mu, \sigma)$ with (un) known parameters.

Nonparametric Identification

Can we proceed under weaker assumptions (Matzkin 1992):

- Assume that $v(x)$ is homogenous of degree 1 and $\exists\, x_0$ so that $v_1(x_0) = 1$ then $v(\cdot)$ and $\Phi(\cdot)$ are identified.
- Suppose demand is purely proportional to the population (for dentists?) and that marginal cost is constant.

$$z_i v(x_i) - F_i$$

- Now v is per-capita profit and z is population. We can just normalize $v(x_0) = 1$ for some arbitrary x_0

$$p(z, x_0) = Pr(F < zv(x_0)) = Pr(F < z) = \Phi(z)$$

- We need variation in z so that $p(z, x_0)$ takes on every value between zero and one, then we can recover the entire Φ.
- If you care about non-parametrics there are other versions for "partially linear models" or "median independence" (Manski 1988) instead of strict independence. [possibly less relevant for entry].

Bresnahan and Reiss (1991a/b)

- Assume identical potential entrants with identical fixed costs.
- Assume many (possibly heterogeneous markets) so that F_m.
- Goal recover the shape of the profit function w.r.t. # of firms. (v_2/v_1, v_3/v_2, etc.).

$$\pi(n_m, x_m, F_m) = v_{n_m}(x_m) - F_m$$

- Unique N.E. for # firms (not identity): two inequalities: $v_n(x_m) \geq F_m$ and $v_{n+1}(x_m) < F_m$.

$$\begin{aligned}
Pr(n = 0|x) &= 1 - \Phi(v_1(x)) \\
Pr(n = 1|x) &= \Phi(v_1(x)) - \Phi(v_2(x)) \\
Pr(n = 2|x) &= \Phi(v_2(x)) - \Phi(v_3(x)) \ldots
\end{aligned}$$

Rewrite so that $G(x)$ is known (just like monopoly!):

$$\begin{aligned}
\Phi(v_1(x)) &= 1 - Pr(n = 0|x) = G_1(x) \\
\Phi(v_2(x)) &= 1 - Pr(n = 0|x) - Pr(n = 1|x) = G_2(x) \\
\Phi(v_3(x)) &= 1 - Pr(n = 0|x) - Pr(n = 1|x) - Pr(n = 2|x) = G_2(x)
\end{aligned}$$

Bresnahan and Reiss (1991a/b)

- Identification is parametric. $G(x)$ is known. BR make assumptions on both $v(\cdot)$ and $\Phi(\cdot)$ and estimate the resulting model by MLE.
- Assume that Φ is lognormal so that n follows an ordered probit. and that $v(x_i) = x_i\beta$.
- Interested in the ratio $\frac{v_2(x)}{v_1(x)}$.
  - Fixed prices: $\frac{v_2(x)}{v_1(x)} = \frac{1}{2}$
  - Cournot: $\frac{v_2(x)}{v_1(x)} \in (0, \frac{1}{2})$
  - Bertrand: $\frac{v_2(x)}{v_1(x)} = 0$
- But this ratio is NOT robust to monotone transforms of v_n and F
  - The economic parameter of interest is NOT non-parametrically identified without shape restrictions!

Result 3: Berry and Tamer (2006)

Key: homogeneity of degree one in population z (Per-capita profits)

1. Suppose there exists (x, x') so that $G_i(x') = G_j(x)$ for $i \neq j$ then $v_i(x') = v_j(x)$. (ie: $G_1(x') = G_2(x)$ then $v_1(x') = v_2(x)$.
   Match choice probabilities from markets with different entrants to recover regressor values for which variable profits match.
2. If $v_n(x) = z v_n(\tilde{x})$ and $x = (z, \tilde{x})$ and $x' = (z', \tilde{x}')$ so that $G_2(x') = G_1(x)$ then the ratio of interest is identified.
   $$\frac{v_2(\tilde{x})}{v_1(\tilde{x})} = \frac{z}{z'}$$
   Nonparametric point identification given variation in (z, x) with the proportional to population assumption
3. If $v_n(x) = z v_n(\tilde{x})$ and $v_1(x_0) = a \equiv 1$ then $\Phi(z) = G_1(z, x_0)$ and hte function $\Phi(\cdot)$ is identified on the support of z. Also for all y
   $$v_n(\tilde{x}) = \frac{F^{-1} G_n(z, \tilde{x})}{z}$$
   The function $v_n(\cdot)$ is identified on the support of (x, z)

Result 3: Berry and Tamer (2006) Extension

1. With proportional to population assumption, can relax the model further to allow for $\Phi_n(F)$ (potentially increasing fixed costs with number of entrants – higher rent?). [BR do this one]
2. If we had additional data to pin down levels of $v_n(x)$ we could do even better.

$$Pr(y \geq n|x) = \Phi(v_n(x))$$

(See Reiss and Spiller (1989) or Berry and Waldfogel (1999).

Bresnahan and Reiss: Data

- Make your RA drive around isolated towns in the south-western US.
- Idea: each town is an isolated market.
- Confirm the number of dentists, plumbers, tire-dealers, etc. in the phonebook
- How many people do you need for a second dentist? $s_n(x_m)$.
  - Twice as many as for the first dentist?
  - More than that? Less than that?
- Actual functional form (ordered probit)

$$\underbrace{S_m}_{\text{market size}} \cdot \underbrace{\left(X_m \beta - \sum_{i=1}^{n} \alpha_i \right)}_{\text{per capita profit}} - \underbrace{W_m \gamma + \varepsilon_m}_{\text{sunk cost}} > 0$$

Bresnahan and Reiss: Results

Estimated ratios of per-firm thresholds. B & R interpret the ratios greater than one as evidence of prices declining in N.

Profession	m_2/m_1	m_3/m_2	m_4/m_3	m_5/m_4
Doctors	1.98	1.10	1.00	0.95
Dentists	1.78	0.79	0.97	0.94
Plumbers	1.06	1.00	1.02	0.96
Tire Dealers	1.81	1.28	1.04	1.03

(Bresnahan and Reiss, 1991 Table 5)

- Prices seem to decline a lot when moving from one to two doctors, tire dealers or dentists.
- But further increases in N do not seem to increase competition much.
- Consistent with old jokes, plumbers' prices never fall.

The Model: Results cntd.

The caveats above may of course apply.

But there is good intuition behind the answers and none of our caveats explains why so many of the estimated threshold ratios are close to exactly one.

B & R also look at the observed prices of tire dealers in a simple regression context. These prices do seem to fall with the first few entrants and then level out. This is consistent with the estimated entry thresholds. They also note the interesting fact that the prices in these small towns appear to level out at a much higher level than is observed at large, big-city tire dealers.

Adding Price and Quantity

- Berry-Waldfogel (RAND 1999) consider entry into the radio industry, where both price and quantity data are available.
- They assume a simple parametric relationship between continuation values $v_n(x)$ and current profits (largely implicit in B&R anyway) and use prices and quantities to estimate $v_n(x)$.
- Entry data are only needed to estimate distribution of fixed costs (which can be done non-parametrically).
- This works because they can normalize the scale of $v_n(x)$ instead of just relying on revealed preference inequalities.

Berry and Waldfogel (1999): Overview

A three-equation model:

1. Slope of listeners as a function of N. Stations "produce" listeners, who make a free choice over stations. They use a nested logit. Each station produces an idiosyncratic benefit, so listening increases in N; but there is still a business stealing effect generated by entry.The logit functional form determines the effect of N on total listening (so one should think hard about its implications).
2. Revenues as a function of number of listeners (and therefore of N). Stations sell listeners to advertisers at some price per listener-hour. Advertisers' demand is downward sloping with a simple constant elasticity functional form.
3. Entry equation. Free entry with identical firms: so given estimates of revenue as a function of N from equations 1-2, the entry equation is the same as in Bresnahan and Reiss. If the (identical) fixed costs distribute (log) normally across markets, the entry equation is an ordered probit.

Berry and Waldfogel, cntd.

- Data: a cross section of 135 metro areas
- Results are used to look at relationship between PS and entry.
- Idea: as in Mankiw and Whinston (1986), there is a " business stealing" effect of new entry on the profits of existing firms
- Empirical results show a lot of business stealing by new stations, implying a large PS loss from free entry (approx 40% of industry revenue).
- Implies a large potential gain from cartelization

Berry and Waldfogel, cntd.

These results are interesting, but problematic for several reasons in addition to the general problems with the two period models.

- The results consider only PS. For a social planner CS matters too.
- Authors partially address this by trying to calculate externalities to listeners as a result of entry.
- But we've already discussed problems of estimating CS from logit estimates, plus here there's no mechanism for advertising to affect consumers and not enough flexibility to allow product diversity to affect utility.
- Note: the FCC established new, more liberal regulations on ownership in the radio industry from 1996 and there's debate on further liberalization. It would be interesting to investigate likely welfare effects.
- Waldfogel (2003) and Sweeting (2012) have done relevant work here.

Simple Bivariate Game

		Player Y	
		A	B
Player X	A	(x, y)	(x, y)
	B	(x, y)	(x, y)

Firm Heterogeneity

- Two different ways of allowing for firm heterogeneity have been introduced into the entry models.
- The first allows for fixed costs: this was discussed in B&R (who have some early results) and was extended by Berry (1992) who showed how to use simulation to ameliorate the computational problems with using the model.
- The second is heterogeneity in variable costs or in continuation values. There have been a number of ways recently proposed to do this; we will focus on Seim (2005), Mazzeo(2004) and Toivanen and Waterson (1999).

Heterogeneity in Fixed Costs: Berry (1992)

- Berry considers entry of airlines operating on city-pair routes soon after deregulation of the U.S. airline industry.
- Keeps the assumption that continuation values $VC(N)$ are the same across firms
- But lets FC vary across firms. Entry values are given by

$$EV(N, x_m, z_f, \varepsilon_{mf}, \theta) = VC(N, x_m, \theta) - FC(z_f, \varepsilon_{mf}, \theta) \tag{1}$$

where x_m are market-specific variables, z_f are firm specific variables, ε is a firm/market specific profit shock and θ is a vector of parameters to be estimated. Note the restriction that firm-specific variables do not enter continuation values (as before). Once we relax this assumption we will require both a different "notion" of equilibrium, and a different estimation routine.

Berry (1992), cntd.

To parametrize even further, assume that:

$$EV_{mf}(N) = x_m\beta + \delta \ln(N) - z_{mf}\alpha - \varepsilon_{mf} \tag{2}$$

with ε modeled as having a market specific component, which allows for correlation in unobservables across firms within the market,

$$\varepsilon_{mf} = \rho u_{0m} + \sqrt{1-\rho^2}u_{mf}$$

(Note the constraint on the variances to set the variance of ε equal to one; similar to any discrete choice model.)

Berry (1992), cntd.

- The ε's are known to the firms (but not the econometrician).
- This is a full information Nash equilibrium (all firms know each other's entry costs).
- The probability of an N firm equilibrium is not an ordered probit because the unobservables are now a vector of dimension equal to the number of potential firms.
- Deriving the likelihood requires us to
  - Find the ε's that generate a particular outcome variable (this will be the number of entrants),
  - Find the probability of observing these ε's (i.e. the probability of the observed outcome) as a function of the parameters.

Berry (1992), cntd.

- In these more general discrete entry models, it is not clear that an equilibrium exists or is unique.
- But Berry shows that if the continuation values are symmetric (depend on N but not z_{mf}) and there is a finite number of potential entrants, then there is a simple constructive proof of an equilibrium which detemines a unique N (though not a unique set of identities to the entrants).
- *Proof* of unique N (sketch): Order firms in decreasing entry values, let entry occur until last profitable entry. Call this last firm N^*. This allocation of firms to "in" and "out" is an equilibrium. Can't have fewer firms in equilibrium, because firm N^* would then enter. Can't have more because N^*+1'th firm won't be profitable in an N^*+1 equilibrium.

Berry (1992), cntd.

(See the paper for details on existence and uniqueness. Existence comes from assuming that firm profits decrease in rivals' entry decisions and that we can rank firms in order of profitability. Uniqueness comes from firm heterogeneity entering only through the fixed portion of profits, ie through FC. Ranking by decreasing profitability therefore implies a ranking by increasing FC and the paper contains a proof that this is enough. (If not it could be that, if j entered then $j + 1$ could enter too but if $j + 1$ entered first then j could not.))

Berry (1992), cntd.

- While the number of firms is unique, the identities of the entering firms are not.
- It can happen that in equilibrium either firm A or firm B could enter, but not both. (This happens when both firms would make profits in a $N^* - 1$ equilibrium but neither would in a $N^* + 1$ equilibrium.)
- So the model does not identify the likelihood function for the identities of the entering firms without some further assumption (e.g. on the order of entry.)
- We therefore lose what information there might be in who enters, and just keep the information on the number of firms that enter.

Berry (1992), cntd.

The likelihood (the probability of drawing the ε's that lead to N entrants) is hard to calculate because the region of the ε space that leads to an N-firm equilibrium is hard to describe. But Berry notes that it's easy to use simulation methods to solve the problem.

- Begin by taking S draws on the underlying random variables u in equations (6)-(7).
- For each guess of $\theta = (\alpha, \beta, \rho, \delta)$ and each draw of the u's, construct the (unique) equilibrium number of firms, $\hat{N}(u_s, \theta)$, via the constructive method of the equilibrium proof just given. Holding θ fixed, integrate over the draws on u to find the probability of N entrants for any N. (Show a picture in two dimensional space.)
- Iterate over guesses for θ to maximize the probability of the observed N (simulated MLE) or to fit the observed to the predicted N (method of simulated moments) where $\hat{N} = E(N)$ is a continuous variable. Berry does the latter.

Berry (1992), cntd.

- Note that for all this to work, we need to hold the random draws constant as we move the parameter vector, just as we always do when we use other simulation estimators (McFadden, 1986, Pakes and Pollard, 1986).
- Simulation has wide applicability in estimating game-theoretic models where the controls take on discrete values, because
  - Even if we know a characteristic of the equilibrium (e.g. N) which is uniquely determined by the (observed and unobserved) characteristics of firms and the parameters, often the set of unobservables that lead to that characteristic has complicated boundaries and it's hard to integrate over that set.
  - But it's easy to *simulate* equilibrium realizations for this characteristic conditional on parameter values and base our estimation method on that simulation.
- (The empirical application is to the importance of airline hubs in early deregulated airline industry. The data are a cross-section by city-pair market. But the application clearly isn't the focus here.)

A Start on Variable Continuation Values: Seim (2006)

- Seim models geographic differentiation in the video retail market
- N^* firms decide whether to enter a given market (indexed by m); those that enter choose their location from a finite set of L locations
- Each firm makes its choices based on the expected post entry value.
- Seim assumes asymmetric information (firms know their own unobservable η_i^a but not other firms' η_i^{-a}). This makes verifying and solving for equilibria much easier than in the Berry case.
    - Existence follows easily from Brouwer's fixed point theorem
    - The Bayesian equilibrium concept outlined below makes solving the model much easier.
- So we can include more product characteristics in the model.
- Application: video rental stores. Ideal for this type of model because the good is homogeneous, non-storable and relatively inexpensive and the main aspect of differentiation comes from geographic location.

Seim (2006), cntd.

Post entry value in location i for firm a is

$$\begin{aligned} v_i^a &= \xi^m + x_i^m\beta + n_i^m\theta + \sum_{j\in d1i,j} x_j^m\beta_1 + \sum_{j\in d2i,j} x_j^m\beta_2 + \\ &\quad \sum_{j\in d1i,j} n_{1,j}^m\theta_1 + \sum_{j\in d2i,j} n_{2,j}^m\theta_2 + \eta_i^a \\ &= \tilde{v}_i^a + \eta_i^a \end{aligned}$$

where

- $d1i,j$ is an indicator for regions adjacent to location i (one to three miles from), and $d2i,j$ is an indicator for locations somewhat farther away from i (three to ten miles from location)
- n_i^m is the number of entering firms in the immediate "tract",
- $n_{1,j}^m$ is the number of firms within the distance defined from $d1i,j$ etc.
- x_i^m and x_j^m are location characteristics
- η_i^a is the only difference between firms: unsystematic, firm-specific characteristics.

Seim (2006), cntd.

- The mean profitability from not entering is normalized to zero (this is a "linear" discrete choice model so we can only define things up to an affine transformation).
- Unobservables are nested logit: the outer nest is "in" or "out", inner nest is the choice of location.
- (This is the same as pure logit on the inner nest choices. But there's an extra unobservable with the same value for all the "in" choices, so they have a correlation that isn't shared by the "out" choice.)
- The model determines both how many firms enter, say $\hat{N}$, and a probability distribution for their locations.
- We will focus on the equilibrium probabilities of firms entering at different locations conditional on $\hat{N}$. (We won't run through the section that's more similar to previous entry models: given these probabilities, use revealed preference and fit observed to predicted N.)

Seim (2006), cntd.

Note that:

- You might think of estimating a demand equation, using it to determine the profit function and deriving the value function from that.
- This would give you a functional form with micro foundations. If individual utilities were derived from distance and other x's, and we knew the distribution of locations, this would connect the β and θ to $\beta(d)$ and $\theta(d)$ and the characteristics of the adjacent locations.
- It would save on parameters, but would be harder to compute:
    - we would need to aggregate up from location-level demand
    - we would have to think seriously about how to move from static profits to value functions.

The fact that we don't do this seriously means that we should be primarily thinking in terms of a reduced form value function.

Seim (2006), cntd.

- By going to the reduced form we introduce the "too many parameters" problem that we saw in demand models in product space: the number of parameters grows geometrically in the number of possible locations.
- So she simplifies by saying the effect of a competitor is the same for all locations that are equidistant from the firm.
- The only difference between firms is η_i^a which distributes extreme value. (Could be an idiosyncratic costs of operating for different potential firms?)
- The only unobservable that is correlated across firms is ξ^m which is market-specific. Implicit assumption that we have very good data on within market differences in profit shifters.

Seim (2006): Methodology

- The firm *only knows* its *own* unobservable. To form expected profits for any location it has to form the expected number of firms at all locations. These are expectations, rather than realizations, because no firm knows the other firm's values for η.
- The equilibrium expectation of the fraction that choose location i is just the expected fraction of firms for whom the max profits come from location i.
- Since all firms are symmetric in observables, this has a simple form.
- The true fraction of firms that will choose location i is given by the usual logit probabilities; i.e.

$$s_i = \frac{\exp(\rho \tilde{v}_i^a)}{\sum_k \exp(\rho \tilde{v}_k^a)}$$

where the ρ comes from the use of a nested logit specification.

Seim (2006), cntd.

- However, firms don't observe their rivals' η's so have to form an expectation: $p_i^{a,b}$= firm a's conjecture of the probability with which firm b chooses location i:
$$p_i^{a,b} = \frac{\exp(\rho E_{\eta-a} \tilde{v}_i^b)}{\sum_k \exp(\rho E_{\eta-a} \tilde{v}_k^b)}$$
where $E_{\eta-a}$ means we have integrated out over the unobservable firm specific probabilities and the implied number of entrants.
- Now say that a's perceived probability that any one of the other entering firms will choose location i is p_i^a (the same for all b since the value function depends on firm characteristics only through η), and that there are $\hat{N}$ firms that enter.
- Then the total number of competitors, excluding itself, that firm a expects to find in location i is $E(n_i^m) = \sum_{b \neq a} p_i^{a,b} = (\hat{N} - 1)p_i^a$

Seim (2006), cntd.

- Then firm a's expected payoff in cell i is:

$$\begin{aligned} E_{\eta^{-a}} v_i^a &= \xi^m + x_m^i \beta + [p_i^a(\hat{N}-1)+1]\theta + \sum_{j \in d1i,j} x_m^j \beta_1 + \sum_{j \in d2i,j} x_m^j \beta_2 + \\ & \sum_{j \in d1i,j} p_j^a(\hat{N}-1)\theta_1 + \sum_{j \in d2i,j} p_j^a(\hat{N}-1)\theta_2 + \eta_i^a \\ &= E_{\eta^{-a}} \tilde{v}_i^a + \eta_i^a \end{aligned}$$

- Now assume a symmetric equilibrium so that $p_i^a = p_i^b = p_i$. Substituting the above expression into the definition of p_i^a gives us

$$p_i = \frac{\exp(\rho E_{\eta^{-a}} \tilde{v}_i^a)}{\sum_k \exp(\rho E_{\eta^{-a}} \tilde{v}_i^a)}$$

- This gives us a fixed point for equilibrium conjectures on the p_i.
- Note that this fixed point does not depend on any of the market characteristics; that is choices among locations within the market depend only on location-specific variables (e.g. ξ_m cancel out).

Seim (2006), cntd.

So there are 2 steps:

- Given $\hat{N}$, we can solve for equilibrium conjectures p_i as a function of the parameters of the model.
  - There are p_i's on both sides of the equation ($p_i = f(\hat{N}, x, \theta, \beta, p_i)$; we iterate until the LHS equals the RHS.
  - This is done using the method of "successive approximations". First pick values of (θ, β). Start with conjectures for p_i, calculate their implications and then use those implications for the next iteration's conjectures. Repeat until we converge on a value for p_i for this (θ, β).
- Iterate over (θ, β) to fit these equilibrium conjectures to data on the shares of the entrants in each tract (M.L.E.). This provides estimates of all but the market specific parameters. See the paper for details on the latter.

Seim (2006): Notes

- The model requires that $\hat{N}$ is not a function of η_{-a} - we need this to do the integral for expected values.
  - One possible assumption: firms pay an initial entry cost before they decide where they are going to enter and before they know their η_a^i.
- Another issue: after choices are made there is ex post regret; i.e. firms will have made choices which ex post are not profitable.
  - This would be OK in a dynamic framework
  - But it's odd in a 2-stage game where firms can't correct their mistakes (e.g. through exit).
- We have not considered existence or uniqueness. Existence follows from continuity. There is no proof of uniqueness, and there will not be one – at least not without further special assumptions.

(Both of the last comments apply to all or many papers in this literature.)

Entry Models with Symmetric Information and Variable Continuation Values.

- Mazzeo (2003, RAND) and Toivanen and Waterson(1999) extend the simple B & R model to consider firm heterogeneity in continuation values keeping fixed costs constant.
- They assume symmetric information (unlike Seim).
- Both assume a small number of entry locations which are fixed exogeneously.
- They then consider entry into all locations.
- Again we work with a reduced form continuation value, but now it varies with the locations of all entrants, and depends on the number of entrants at each location.

Symmetric Information and Variable Continuation Values, cntd.

- Toivanen and Waterson deal with fast food outlets in Britain, distinguishing between McDonalds and Burger King. Each chain decides how many of its outlets should enter.
- Mazzeo models independent motels choosing which interstate exits to enter and with which quality level (high, medium or low).
- Both assume that quality levels/types belong to a small discrete set (with 2 or 3 elements) and these are the only differences among firms.
- Continuation values for any firm choosing quality level $t = (1, ..., T)$ in market m are assumed to be

$$V_{tm} = x_m \beta_t + g(N_1, ..N_T, \theta_t) + \varepsilon_{tm.}$$

where $\bar{N} = N_1, ..N_T$ is the vector of the number of firms of each type.

Symmetric Information and Variable Continuation Values, cntd.

- The parameters (specific to each type) are β_t on the market level variables and θ_t which parameterizes the effect of own-type and other-type competition.
- Both papers approximate g with a set of dummy variables (since $\bar{N}$ takes on discrete values, so does $g(., \theta_t)$).
- Note that the unobservables are constant across firms within quality type, so that the model is symmetric conditional on type.
- Unfortunately, even this simple type of heterogeneity in variable profits leads to both possible nonexistence, and possible non-uniqueness when equilibrium exists – at least in a simultaneous move game.
- Both authors resolve this by assuming that choices are sequential.

Toivanen and Waterson

Assume Burger King moves first and the ε's are draws from a known distribution.

- Take a set of ns draws of $(\varepsilon_b, \varepsilon_m)$. Pick a θ to evaluate.
- Let $EV^j(b, m)$ be the total entry value of Burger King (if $j = b$) or McDonald's (if $j = m$) when there are b Burger King outlets and m McDonald's outlets.
- Now solve the problem backwards, for BK moving first.
  - If BK chooses b, predict McDonald's choice $m(b)$ to $max_m EV^m(b, m)$.
  - Given that response, BK chooses b to $max_b EV^b(b, m(b))$.
- This gives us (b, m) for this (ε, θ)
- Repeat for every draw of ε to generate an expected $(\hat{b}(\theta), \hat{m}(\theta))$ given this θ
- Iterate over θ to minimize $E[((\hat{b}(\theta), \hat{m}(\theta)) - (b, m)) * f(x)]$ for exogenous entry determinants x.

Mazzeo

We simplify by assuming there are only two types, L and H.

- Let $EV^l(l,h)$ be the entry value of next type l firm when already l low type and h high type firms are active $(j = L, H)$.
- Assume that firms play sequentially and make irrevocable decisions about entry and product type before the next firm plays.
- Firms anticipate subsequent entry in the usual way.
- The last firm of each product type finds entry profitable and prefers the chosen type to the alternatives.
- Additional entry in either product type is not profitable.

Mazzeo, cntd.

Therefore a Nash equilibrium is an ordered pair (l, h) for which the following inequalities hold

$$V_l(l-1,h) > 0,\ V_l(l,h) < 0,\ V_l(l-1,h) > V_h(l-1,h)$$

and

$$V_h(l,h-1) > 0,\ V_h(l,h) < 0,\ V_h(l,h-1) > V_l(l,h-1)$$

- Mazzeo also considers an equilibrium where firms sink their sunk costs without committing to whether they will be low or high quality.
- Product types are selected among the entrants simultaneously in the second stage.
- He shows that the two equilibria can be different.
- Mazzeo claims that both equilibria are unique, and he can simulate them and match them to data.

Results of these Papers

- Mazzeo finds strong returns to differentiation – all entry by rivals drives down profits but an incumbents profits fall by much more when the rival is offering the same quality type as the incumbent.
- Toivanen and Waterson find that McDonald's is much more likely to enter a location where there are only Burger King outlets then where there are McDonald's outlets.
- I.e. consistent with Seim, and probably not surprisingly, both papers find that differentiation matters for entry decisions, and hence probably for profits and CS calculations.

General Issues in the Analysis of Entry Games.

- *We should be careful about doing either PS or CS calculations without allowing for sufficient product differentiation.*
- There is a real question of which equilibrium we pick out and why, and this issue just gets more worrisome when we increase the number of types.
  - There are a couple of ways around the non-uniqueness problem.
  - We could enumerate all possible equilibria and assign a probability to each (perhaps depending on a vector of parameters), and estimate those probabilities (or the extra parameter vector) along with the other parameters (Tamer, 2003, RESTUD).
  - Or we could investigate whether the data can pick out the equilibrium for us (this requires alternative assumptions).
  - Or we can get just bounds on probabilities (instead of probabilities themselves), and use the bounds for estimation. See next class notes.

General Issues, cntd.

- All reduced form value function models become harder as we increase the number of types, because the number of parameters we estimate increases at about the square of the number of types.
  - One way around this is to define demand and cost functions directly on the underlying "characteristic" space, just as we did in static analysis, and then use those profit functions directly in the empirical analysis.
  - The inequalities estimator we will cover next class is well suited to this approach.
- Lack of True Dynamics.
  - Introducing dynamics would solve the conceptual problems noted above
  - And it would use the information in the sequential nature of most data.
  - I.e., in many of these data sets we see when each firm entered, so we could condition that firm's entry decision on firms existing in the market at the time they enter.
  - The problems here are a mix of computational and conceptual, and they will be covered next semester.