

Tipping and Concentration in Markets with Indirect Network Effects

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This paper develops a framework for measuring “tipping”—the increase in a firm’s market share dominance caused by indirect network effects. Our measure compares the expected concentration in a market to the hypothetical expected concentration that would arise in the absence of indirect network effects. In practice, this measure requires a model that can predict the counterfactual market concentration under different parameter values capturing the strength of indirect network effects. We build such a model for the case of dynamic standards competition in a market characterized by the classic hardware/software paradigm. To demonstrate its applicability, we calibrate it using demand estimates and other data from the 32/64-bit generation of video game consoles, a canonical example of standards competition with indirect network effects. In our example, we find that indirect network effects can lead to a strong, economically significant increase in market concentration. We also find important roles for beliefs on both the demand side, as consumers tend to pick the product they expect to win the standards war, and on the supply side, as firms engage in penetration pricing to invest in growing their networks.

Key words: dynamic oligopoly; indirect network effects; tipping; standards war; high technology

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1. Introduction

We study the diffusion of competing durable goods in a market exhibiting indirect network effects as a result of the classic hardware/software structure (Katz and Shapiro 1985). Of particular interest is whether such markets are prone to *tipping*, which is “the tendency of one system to pull away from its rivals in popularity once it has gained an initial edge” (Katz and Shapiro 1994, p. 106) and, in some instances, to emerge as the de facto industry standard. As incompatible hardware firms vie for market dominance, they may engage in aggressive penetration pricing strategies, battling for the initial advantage that will ultimately tip the market in their favor. These “standards wars” are widely regarded as a “fixture of the information age” (Shapiro and Varian 1999, p. 8). An empirically practical measure of tipping would therefore enable researchers and practitioners to understand better the diffusion dynamics of new hardware/software products and the emergence of a victor during a standards war.

The extant literature has yet to provide an empirically practical measure of tipping. We define tipping herein as the degree of market share concentration due to indirect network effects. It is typically very difficult, if not impossible, to assess tipping directly from field data. In actual markets, product differentiation, differences in costs, and other differences between standards frequently lead to asymmetric market outcomes, even in the absence of indirect network effects. In essence, an empirical measure of tipping would need to compare the expected concentration in a market to the hypothetical expected concentration that would arise if the sources of indirect network effects were reduced or eliminated. The key insight is that tipping generally needs to be measured relative to a well-defined, counterfactual market outcome. Constructing a counterfactual market outcome using a field experiment would be highly impractical. Therefore, for an empirical implementation of this measure, we instead need a model that captures indirect network effects, can be calibrated from actual data, and allows us to make predictions about the equilibrium

adoption of the competing standards under various different parameter values capturing the strength of indirect network effects.¹

We build a dynamic model that captures indirect network effects and gives consumer expectations a central role. Our model involves three types of players: consumers, hardware manufacturers, and software developers. The demand side of our model extends the framework of Nair et al. (2004) and allows for dynamic adoption decisions. Consumers are assumed to “single-home,” meaning they adopt at most one of the competing hardware standards.² The utility of each hardware standard increases in the availability and variety of complementary software. Consumers form beliefs about future hardware prices and software availability. These beliefs influence when consumers adopt (the rate of diffusion) and which standard they adopt (the size of each installed base). On the supply side, forward-looking hardware firms compete in prices while anticipating the impact of hardware sales on the future provision of software and, hence, future hardware sales. Software firms provide a variety of titles that is increasing in the installed base of a hardware standard. Our solution concept for this model is a Markov perfect Bayesian equilibrium. The complexity of the model makes analytical solution methods intractable, and hence we solve the model numerically. Note that this motivation for the research herein was also reported in the discussion piece by Bronnenberg et al. (2008).

To demonstrate our model and how it can be used to measure tipping, we calibrate it with demand parameter estimates and other market data from the 32/64-bit generation of video game consoles.³ The video game console market is a canonical example of indirect network effects. Furthermore, from previous empirical research, the 32/64-bit generation is known to exhibit indirect network effects (Shankar and Bayus 2003, Clements and Ohashi 2005). To obtain our demand estimates, we adapt a two-step

procedure to solve our demand estimation problem (e.g., Hotz and Miller 1993, Hotz et al. 1994, Bajari et al. 2007, Pesendorfer and Schmidt-Dengler 2008). A similar approach has recently been employed for the estimation of a durable good exhibiting network effects by Ryan and Tucker (2008).⁴

The calibrated model reveals that the 32/64-bit video game console market can exhibit economically significant tipping effects, given our model assumptions and the estimated parameter values. The market concentration, as measured by the one-firm concentration ratio in the installed base after 48 months, increases by at least 24 percentage points due to indirect network effects. We confirm the importance of consumer expectations as an important source of indirect network effects; in particular, if neither firm has an initial installed base advantage, we find that tipping occurs at a (monthly) consumer discount factor of 0.9, but not for smaller discount factors. However, if one firm has gained an initial installed base advantage, tipping arises also for smaller discount factors.

Our model also predicts penetration pricing (for small levels of the installed base) if indirect network effects are sufficiently strong. We are specifically interested in the case where a firm prices below marginal cost. In markets with strong network effects, firms literally price below cost during the initial periods of the diffusion to invest in network growth. Interestingly, the emergence of penetration pricing as an equilibrium strategy dissipates as we weaken (but do not eliminate) the indirect network effects. In short, the mere presence of indirect network effects does not per se lead to penetration pricing. These findings build on some of the earlier marketing literature that has discussed situations under which firms set a low price at the launch of a new product and then increase prices over time (e.g., Dean 1976, Jeuland and Dolan 1982, Kalish 1983, Dockner and Jørgensen 1988). Our findings herein help clarify for managers the circumstances under which penetration pricing makes sense from a dynamic perspective.

Our approach for measuring market concentration as a result of tipping contributes to recent antitrust discussions about hardware/software markets, as highlighted in the recent high-profile case surrounding the browser war between Microsoft and Netscape (*United States v. Microsoft*, 87 F. Supp. 2d 30; Bresnahan 2001). The proposed framework also resolves some of the concerns regarding the inability of existing antitrust policies and tools to address the feedback

¹ Note that we focus herein on tipping and market dominance during a specific hardware generation. A related theoretical literature has also studied whether tipping can create inertia across hardware generations when there are innovations (e.g., Farrell and Saloner 1986, Katz and Shapiro 1992, Markovich 2008). Therein, tipping, or “excess inertia,” is defined by the willingness of consumers to trade off the scale benefits of a current standard with a large installed base in favor of a new technology without an installed base. Interestingly, in this type of environment, network effects may also serve as a potential barrier to entry (Cabral 2009).

² Recent literature has begun to study the theoretical implications of multihoming, whereby consumers may adopt multiple standards and software firms may create versions for multiple standards (Armstrong 2006).

³ This approach follows in the tradition of Benkard (2004) and Dubé et al. (2005, 2009) by conducting counterfactual simulations of the market outcomes using empirically obtained parameters.

⁴ An interesting difference is that Ryan and Tucker (2008) use individual-level adoption data, which enables them to accommodate a richer treatment of “observed” consumer heterogeneity. The trade-off from incorporating more heterogeneity is that they are unable to solve the corresponding dynamic hardware pricing game on the supply side.

dynamics in markets with indirect network effects (e.g., Evans 2003, Koski and Kretschmer 2004, Evans and Schmalensee 2007, Rysman 2007). Our dynamic framework extends much of the extant empirical literature that either estimates the effects of indirect network effects using demand only or treats the supply side of the market as static (Gupta et al. 1999; Basu et al. 2003; Bayus and Shankar 2003; Ohashi 2003; Dranove and Gandal 2003; Nair et al. 2004; Karaca-Mandic 2004; Park 2004; Rysman 2004, 2007; Clements and Ohashi 2005; Akerberg and Gowrisankaran 2006; Ryan and Tucker 2008). Gandal et al. (2000) allow for forward-looking consumers but they assume hardware sponsors do not have a strategic role. More recently, Liu (2010) and, most closely related to our work, Jenkins et al. (2004) allow for forward-looking hardware manufacturers. However, both papers treat consumers as myopic.

The rest of this paper is organized as follows. Section 2 lays out how indirect network effects can lead to market concentration. Next, we describe the model in §3, and in §4 we describe the data. In §5 we then discuss how the model parameters are estimated, and §6 details our estimation results. Section 7 demonstrates via numerical experiments how tipping can occur in this market. Section 8 concludes our paper.

2. Market Concentration in the Presence of Indirect Network Effects

We begin with a discussion of the economics of how indirect network effects lead to market concentration. We also provide a measure of the degree of market concentration caused by indirect network effects that should apply to most models of indirect network effects in the literature. In the next section, we develop a model that generalizes many of the theory models and can be calibrated from demand estimates and cost data.

We consider a market where consumers derive utility from two complementary goods: hardware, such as a video game console, and software, such as a video game. Consumers only derive utility from software if they also own compatible hardware. On the supply side, we assume that there are two competing, incompatible hardware standards, each sold by two independent firms. The software market is monopolistically competitive, and no single software firm can strategically influence the sales of either standard.⁵ Furthermore, we assume that there are returns

to scale in the production of software such that the variety of software for a standard is increasing in its market size, i.e., the number of consumers who have adopted the standard.

Even though consumers derive no direct utility from the usage of hardware by other consumers, indirect network effects arise in such a market as follows. Each consumer's utility from hardware is increasing in the amount of available software. As more consumers adopt a given hardware standard, the supply of compatible software increases. Therefore, a consumer's willingness to pay for a given hardware standard is, indirectly, increasing in the cumulative number of adopters of that standard—the current *installed base* (e.g., Chou and Shy 1990, Church and Gandal 1993).⁶ If consumers are forward looking, their adoption decisions will also be influenced by their expectations about the future evolution of the installed base of either standard.

Let y_{jt} be the installed base of standard j in period t . We normalize the market size to one, such that y_{jt} is also the cumulative share of the potential customer base that standard j has obtained by period t . The current state of the market can then be described by the vector $y_t = (y_{1t}, y_{2t})$, where $0 \leq y_{1t} + y_{2t} \leq 1$. All possible states are contained in the shaded triangles in Figure 1. We assume that the demand and cost primitives of the market are symmetric: both firms have the same production costs, the consumers' distribution of tastes for either standard is identical, and the variety of software offered for either standard is the same when both standards have the same installed base.

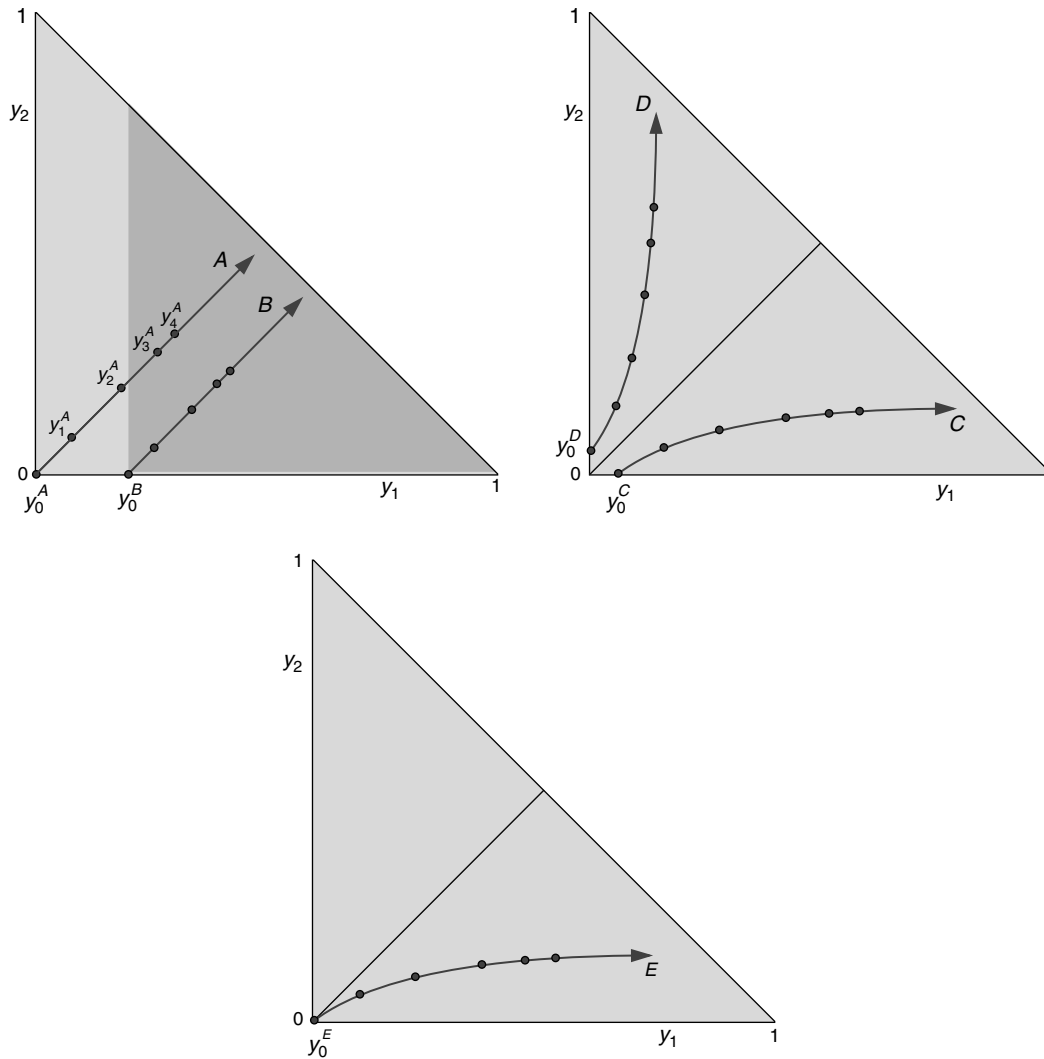
Suppose that there are no indirect network effects. For example, suppose that the supply of software is independent of the installed base or that consumers do not value software. The equilibrium evolution of the market will then be symmetric, provided that a symmetric equilibrium exists: the firms split the market in each period, and both firms will always have the same installed base. This case is depicted by curve A corresponding to the initial state $y_0^A = (0, 0)$ in the top left-hand graph in Figure 1 (the points labeled y_t^A correspond to the realized state vectors in periods $t = 0, 1, 2, \dots$). Hence, in the absence of indirect network effects, this market would not become concentrated. Now suppose that firm 1 has gained an initial advantage, possibly because of early entry, over firm 2 in terms of the installed base, such that $y_{10} > y_{20} = 0$.

⁵ In some industries this assumption would not be valid. For example, the decision made by Warner Brothers in January 2008 to discontinue its support of high-definition (HD) DVD was key to the success of the Blu-ray format over the HD DVD standard.

Moreover, forward-looking software suppliers could be instrumental in tipping the market, much like the forward-looking consumer we study herein.

⁶ Rochet and Tirole (2003) argue that most network effects arise in an indirect manner.

Figure 1 Market Evolution Under Different Scenarios: No Indirect Network Effects (Top Left), Positive Feedback (Top Right), and Multiple Self-Fulfilling Equilibria (Bottom)



This scenario corresponds to the initial state y_0^B in Figure 1. Absent any indirect network effects, the equilibrium evolution of the market will still be symmetric and the firms will split the total remaining market size, $1 - y_{10}$. Hence, in the absence of indirect network effects, the only source of concentration in this market would be due to the initial share advantage.

Now consider the general case with indirect network effects. Two economic mechanisms lead to market concentration (Katz and Shapiro 1994). First, indirect network effects induce *positive feedback*. To illustrate, consider again the case where standard 1 has gained an initial installed base advantage, corresponding to the state y_0^C in the top right-hand graph of Figure 1. This initial advantage leads to the creation of more software variety for standard 1 than for standard 2. Hence, more consumers will adopt standard 1 than standard 2 in the initial period. This asymmetric adoption rate reinforces the initial advantage of standard

1, creating positive feedback that leads to market concentration in favor of standard 1. Similarly, if standard 2 had gained an initial advantage, depicted by the state y_0^D , the market would have become concentrated in favor of standard 2. Such positive feedback due to indirect network effects arises even if there is a unique, symmetric equilibrium characterizing the market. Positive feedback is related to the concept of path dependence, pioneered by Arthur (1989) and David (1985, 2007) and defined as follows: small historical events have an important influence on the eventual outcome in a market.

A second source of market concentration is the potential incidence of *multiple equilibria*, even if no standard has gained an initial advantage. Multiple equilibria arise if consumers are forward looking and make their adoption decisions based on their expectations about the diffusion of each standard. In a rational expectations equilibrium, the consumers' expectations

are consistent with the future evolution of the market, allowing consumers to coordinate their adoption choices. In one possible equilibrium all consumers expect that standard 1 will gain most of the market share. These expectations are self-fulfilling: due to the large expected variety of software for standard 1, most consumers will adopt standard 1. This case corresponds to the initial state y_0^E in the bottom graph of Figure 1. In another equilibrium, the consumers expect that standard 2 will gain most of the market share, and because of the self-fulfilling nature of these expectations, standard 2 will indeed attract most consumers.

2.1. Measuring Market Concentration Due to Indirect Network Effects

The literature broadly refers to the emergence of a dominant firm through positive feedback or multiple equilibria as *tipping*. However, in spite of the widespread use of the term tipping in the literature on network effects, no single, precise definition exists. For example, the survey by Katz and Shapiro (1994) defines tipping as the “tendency of one system to pull away from its rivals in popularity once it has gained an initial edge.” Alternatively, according to the survey by Farrell and Klemperer (2007, p. 2034):

We have seen how early choices are...able either to help coordination or to wield disproportionate influence. Thus any early lead in adoptions (whether strategic or accidental) will tend to expand rather than to dissipate. Network markets are “tippy”: early instability and later lock-in.

Deriving a more precise and empirically practical definition is tricky. Some associate tipping with winner-take-all outcomes, whereby a single dominant firm emerges capturing the entire market. In practice, many of the classic examples of markets that are deemed to have tipped towards a single standard do not have a single dominant firm. Consider for example the standards war between VHS and Betamax in the VCR market. Even though VHS is widely regarded as the victor, Betamax machines continued to be manufactured until 2002. Similarly, we still have a coexistence of AM and FM radio even though the latter is widely regarded as the victor. Therefore, defining tipping as a situation where one standard achieves 100% of the market share would be too restrictive. A more useful and general definition of tipping should be based on measuring the degree of market concentration caused by indirect network effects. In this section, we build an empirically implementable definition of tipping that encompasses 100% market dominance as a special case but is applicable more generally to any situation where concentration occurs as a result of indirect network effects.

In the special case of a market with two symmetric competitors, we could define a measure of tipping by comparing the expected share of the installed base for the larger standard, T periods after the product launch, to a share of 50%, i.e., to the share in a symmetric outcome. That is, we could measure tipping as the extent to which the cumulative, one-firm concentration ratio in period T exceeds 50%. In most actual markets, however, the expected share of the larger standard will exceed 50% even in the absence of indirect network effects as a result of product differentiation, cost differences across the standards, etc. To assess tipping, we need to compare the expected share in the installed base to the hypothetical share that would arise if one or more economic factors that cause indirect network effects were mitigated or entirely eliminated. For this measure, we need a model to predict counterfactual market outcomes and to define the (counterfactual) baseline case relative to which tipping could be measured.⁷

We now provide a formal definition of the proposed tipping measure. Suppose we know the data-generating process of the installed base evolution. Let Θ be the model parameters describing this process. If the installed base evolution is described by an economic model, Θ will contain all demand and cost parameters describing the underlying model primitives.

Let ρ_{jt} be the share of standard j in the installed base t periods after product launch:

$$\rho_{jt} \equiv \frac{y_{j,t+1}}{y_{1,t+1} + y_{2,t+1}}. \quad (1)$$

Here, $y_{j,t+1}$ is the installed base of standard j at the end of period t , including the sales of j during period t . The cumulative one-firm concentration ratio after T periods is then given by

$$\mathcal{C}(y_T) = \max\{\rho_{1T}, \rho_{2T}\}.$$

The realization of $\mathcal{C}(y_T)$ depends on the model parameters Θ , an equilibrium that exists for these parameters $\mathcal{E}(\Theta)$, and a sequence of demand shocks ξ_t . Given Θ and $\mathcal{E}(\Theta)$, the distribution of $(y_t)_{t=0}^T$ is well defined, and we can thus calculate the expected cumulative one-firm concentration ratio

$$C_1(\Theta, \mathcal{E}(\Theta)) \equiv \mathbb{E}(\mathcal{C}(y_T) \mid \Theta, \mathcal{E}(\Theta)). \quad (2)$$

⁷ Note that because of demand shocks or other random events, the expected cumulative one-firm concentration ratio could significantly exceed 50% in a market with symmetric competitors even if there were no indirect network effects. In this situation, the difference between the cumulative one-firm concentration ratio and 50% would not provide a meaningful measure of tipping even in the symmetric case.

Let Θ' be a variation of the model where one or more parameters that govern the strength of indirect network effects are changed compared to the model described by Θ , and let $\mathcal{E}(\Theta')$ be a corresponding equilibrium. We can thus measure tipping, the increase in market concentration as a result of indirect network effects, as follows:

$$\Delta C_1 = C_1(\Theta, \mathcal{E}(\Theta)) - C_1(\Theta', \mathcal{E}(\Theta')). \quad (3)$$

If we knew that the market under investigation was symmetric, then $C_1 \approx 0.5$ in the absence of indirect network effects, and we could measure tipping by $\Delta C_1 = C_1(\Theta, \mathcal{E}(\Theta)) - 0.5$.⁸

Our discussion highlights that the increase in market concentration due to indirect network effects can only be measured relative to a well-defined counterfactual market outcome. In most cases of interest, this counterfactual outcome is not observed. First, we hardly ever observe the evolution of the same standards more than once under identical conditions. Second, we would have to observe some diffusion paths that are not affected by indirect network effects. Therefore, in practice one will seldom be able to measure the degree of tipping or even to assess the incidence of tipping in observed market data. Instead, one will need an economic model that accounts for the underlying causes of indirect network effects and that can be used to simulate the equilibrium evolution of the market. Such a model would need to be calibrated with demand and cost estimates or other data sources. In the next section, we develop a model that satisfies these requirements.

3. Model

We consider a market with competing hardware platforms. A consumer who has adopted one of the available technologies derives utility from the available software for that platform. Software titles are incompatible across platforms. Consumers are assumed to choose at most one of the competing hardware platforms and to purchase software compatible with the chosen hardware, a behavior Rochet and Tirole (2003) term “single-homing.” There are indirect network effects in this market, which are due to the dependence of the number of available software titles for a given platform on that platform’s installed base. The consumers in this market have expectations about the evolution of hardware prices and the future availability of software when making their adoption decisions. Correspondingly, the hardware manufacturers anticipate the consumer’s adoption decisions and set prices for their platforms accordingly. The software market

is monopolistically competitive, and the supply of software titles for any given platform is increasing in the platform’s installed base.

Time is discrete; $t = 0, 1, \dots$. The market is populated by a mass $M = 1$ of consumers. There are $J = 2$ competing firms, each offering one distinct hardware platform. The installed base of platform j in period t —i.e., the fraction of consumers who have adopted j in any period previous to t —is denoted by $y_{jt} \in [0, 1]$. The state of the market is described by $y_t = (y_{1t}, y_{2t})$.

In each period, platform-specific demand shocks ξ_{jt} are realized. ξ_{jt} is private information to firm j ; i.e., firm j learns the value of ξ_{jt} before setting its price but learns the demand shock of its competitor only once sales are realized. As we shall see later, ξ_{jt} can strongly influence the final distribution of shares in the installed base. In particular, the realizations of ξ_{jt} in the initial periods of competition can lead the market to “tip” in favor of one standard. Also, ξ_{jt} will typically ensure that the best response of each firm is unique, and thus ensures the existence of a pure strategy equilibrium.⁹ We assume that the demand shocks are independent and identically distributed (i.i.d.) through time; $\phi_j(\cdot)$ denotes the probability density function (pdf) of ξ_j , and $\phi(\cdot)$ denotes the pdf of $\xi = (\xi_1, \xi_2)$.

The timing of the game is as follows:

1. Firms learn their demand shock ξ_{jt} and set a product price, p_{jt} .
2. Consumers adopt one of the available platforms or delay their purchase decisions.
3. For each platform j , software firms supply a given number of titles, n_{jt} .
4. Sales are realized, and firms receive their profits. Consumers derive utility from the available software titles and—in the case of new adopters—from the chosen platform.

3.1. Software Market

The number of available software titles for platform j in each period is a function of the installed base of platform j : $n_{jt} = h_j(y_{j,t+1})$. To see why n_{jt} is a function of $y_{j,t+1}$ and not y_{jt} , note that y_{jt} denotes the installed base at the beginning of period t , whereas $y_{j,t+1}$ denotes the total installed base after the potential adopters have made a purchase decision. The software producers observe this total installed base before they supply a given number of titles.

⁸ Once again, this would not be accurate if random shocks had a significant impact on the market evolution.

⁹ We are not able to prove this statement in general but could easily verify it across all versions of our model that we solved numerically on a computer. In general, the right-hand side of the firm’s Bellman equation, regarded as a function of price of firm j at time t p_{jt} , has two local maxima. The realization of ξ_{jt} ensures that these local maxima are not equal.

Although this relationship may seem ad hoc, we show in Appendix A how this relationship between n_{jt} and $y_{j,t+1}$ can be derived from a structural model of monopolistic competition and constant elasticity of substitution (CES) software demand in the software market. These assumptions abstract away from some of the dynamic aspects of game demand (e.g., Nair 2007), but they retain the fundamental interdependence between software and hardware.

3.2. Consumer Decisions

Consumers make their adoption decisions based on current prices and their expectation of future prices and the availability of compatible software titles. Consumers expect that the installed hardware evolves according to $y_{t+1} = f^e(y_t, \xi_t)$ and that firms set prices according to the policy function $p_{jt} = \sigma_j^e(y_t, \xi_{jt})$. Consumers observe both ξ_t and the current price vector p_t before making their decisions.

Consumers who have already adopted one of the platforms receive utility from the available software in each period. Because the supply of software is a function of the installed base at the end of a period, we can denote this utility as $u_j(y_{j,t+1}) = \gamma n_{jt} = \gamma h_j(y_{j,t+1})$. The present discounted software value is then defined as

$$\omega_j(y_{t+1}) = \mathbb{E} \left[\sum_{k=0}^{\infty} \beta^k u_j(y_{j,t+1+k}) \mid y_{t+1} \right].$$

This value follows the recursion

$$\omega_j(y_{t+1}) = u_j(y_{j,t+1}) + \beta \int \omega_j(f^e(y_{t+1}, \xi)) \phi(\xi) d\xi. \quad (4)$$

Consumers who have not yet adopted either buy one of the hardware platforms or delay adoption. The choice-specific value of adopting hardware platform j is given by

$$v_j(y_t, \xi_t, p_t) = \delta_j + \omega_j(f^e(y_t, \xi_t)) - \alpha p_{jt} + \xi_{jt}. \quad (5)$$

Here, δ_j is the value of owning a specific hardware platform or the value of bundled software; α is the marginal utility of income. The realized utility from adopting j also includes a random utility component ϵ_{jt} . That is, the total utility from the choice of j is given by $v_j(y_t, \xi_t, p_t) + \epsilon_{jt}$. We assume that ϵ_j is i.i.d. type I extreme value distributed.

The value of waiting is given by

$$\begin{aligned} v_0(y_t, \xi_t) &= \beta \int \max \left\{ v_0(y_{t+1}, \xi) + \epsilon_0, \right. \\ &\quad \left. \max_j \left\{ v_j(y_{t+1}, \xi, \sigma^e(y_{t+1}, \xi)) + \epsilon_j \right\} \right\} \\ &\quad \cdot \phi_\epsilon(\epsilon) \phi(\xi) d(\epsilon, \xi). \end{aligned} \quad (6)$$

In this equation, $y_{t+1} = f^e(y_t, \xi_t)$.

Consumers choose the option that yields the highest choice-specific value, including ϵ_{jt} . That is, option j is chosen if and only if for all $k \neq j$, $v_j(y_t, \xi_t, p_t) + \epsilon_{jt} \geq v_k(y_t, \xi_t, p_t) + \epsilon_{kt}$.¹⁰ Given the distributional assumption on the random utility component, the market share of option j is

$$s_j(y_t, \xi_t, p_t) = \frac{\exp(v_j(y_t, \xi_t, p_t))}{\exp(v_0(y_t, \xi_t)) + \sum_{k=1}^J \exp(v_k(y_t, \xi_t, p_t))}. \quad (7)$$

Furthermore, the installed base of platform j evolves according to

$$y_{j,t+1} = y_{jt} + \left(1 - \sum_{k=1}^J y_{kt} \right) s_j(y_t, \xi_t, p_t) = f_j(y_t, \xi_t, p_t). \quad (8)$$

3.3. Firms

Firms set prices according to the Markovian strategies $p_j = \sigma_j(y, \xi_j)$; i.e., prices depend only on the current payoff-relevant information observed to each competitor. Firms expect that the consumers make adoption decisions according to the value functions v_0, \dots, v_J , and accordingly that market shares are realized according to Equation (7) and that the installed base evolves according to (8).

The marginal cost of hardware production is c_j , which we assume to be constant through time. The firms also collect royalty fees from the software manufacturers at the rate of r_j per unit of software. Let $q_j(y_{t+1})$ be the total number of software titles sold in period t .¹¹ The per-period expected profit function is then given by

$$\begin{aligned} \pi_j(y, \xi_j, p_j) &= (p_j - c_j) \cdot \left(1 - \sum_{k=1}^J y_{kt} \right) \\ &\quad \cdot \int s_j(y, \xi_j, \xi_{-j}, p_j, \sigma_{-j}(y, \xi_{-j})) \phi_j(\xi_{-j}) d\xi_{-j} \\ &\quad + r_j \int q_j(f_j(y, \xi_j, \xi_{-j}, p_j, \sigma_{-j}(y, \xi_{-j}))) \phi_j(\xi_{-j}) d\xi_{-j}. \end{aligned}$$

Each competitor maximizes the expected present discounted value of profits. Associated with the solution of the intertemporal pricing problem is the Bellman equation

$$V_j(y, \xi_j) = \sup_{p_j \geq 0} \left\{ \pi_j(y, \xi_j, p_j) + \beta_f \int V_j(f(y, \xi_j, \xi_{-j}, p_j, \sigma_{-j}(y, \xi_{-j})), \xi'_j) \phi(\xi_{-j}) \phi(\xi'_j) d(\xi_{-j}, \xi'_j) \right\}. \quad (9)$$

¹⁰ These inequalities involve some slight abuse of notation, as $v_0(y, \xi)$ is not a function of p .

¹¹ $q_j(y_{t+1}) = \bar{q}_j(n_{jt}) \cdot y_{j,t+1}$, where \bar{q}_j denotes the average number of titles bought by a consumer. See Appendix A for the derivation of this equation within the context of a specific model. For our model simulations, we estimate $q_j(y)$ directly from the data.

Here, we allow for the possibility that firms and consumers discount the future at different rates, $\beta \neq \beta_f$.

3.4. Equilibrium

We seek a Markov perfect Bayesian equilibrium, where firms and consumers base their decision only on the current payoff-relevant information. Consumers have expectations about future hardware prices and the evolution of the installed base of each platform and the associated supply of software. The adoption decisions are dependent on these expectations. Firms have expectations about the adoption decisions of the consumers, the evolution of the installed base, and the pricing decisions of their competitors. Pricing decisions are made accordingly. In equilibrium, these expectations need to be mutually consistent.

Formally, a Markov perfect Bayesian equilibrium in pure strategies of the network game consists of consumer expectations f^e and σ^e , consumer value functions v_k , pricing policies σ_j , and the firm's value function V_j such that

1. The consumer's choice-specific value functions v_1, \dots, v_J satisfy (5), and the value of waiting, v_0 , satisfies (6).
2. The firm's value functions V_1, \dots, V_J satisfy the Bellman equations (9).
3. $p_j = \sigma_j(y, \xi_j)$ maximizes the right-hand side of the Bellman equation (9) for each $j = 1, \dots, J$.
4. The consumer's expectations are rational: $\sigma_j^e \equiv \sigma_j$ for $j = 1, \dots, J$, and $f^e(y, \xi) = f(y, \xi, \sigma(y, \xi))$, where f is as defined by Equation (8).

In the Markov perfect equilibrium, all players—firms and consumers—act rationally given their expectations about the strategies of the other market participants. Furthermore, expectations and actually realized actions are consistent.

4. Data

To make our computational results more realistic, we use data from the 32/64-bit generation of video game consoles, one of the canonical examples of indirect network effects. To understand the relevance of this case study to our model and our more general point about tipping in two-sided markets, we briefly outline some of the institutional details of the industry. We then discuss the data.

4.1. The U.S. Video Game Console Industry

The market for home video game systems has exhibited a two-sided structure since the launch of Atari's popular 2600 VCS console in 1977 (Williams 2002). Much like the systems today, the VCS consisted of a console capable of playing multiple games, each on interchangeable cartridges. Although Atari initially developed its own proprietary games, ultimately more

than 100 independent developers produced games for Atari and more than 1,000 games were released for Atari 2600 VCS (Coughlan 2001a). This same two-sided market structure has characterized all subsequent console generations, including the 32/64-bit generation we study herein.

The 32/64-bit generation was novel in several ways. None of the consoles was backward compatible, eliminating concerns about a previously existing installed base of consumers that might have given a firm an advantage. This was also the first generation to adopt CD-ROM technology although early entrants Philips and 3DO failed as a result of their high console prices of \$1,000 and \$700, respectively. In contrast, the September 1995 U.S. launch of Sony's 32-bit CD-ROM console PlayStation was an instant success. So much so that its competitors, Sega's 32-bit Saturn console, and later Nintendo's 64-bit N64 cartridge console, failed to recapture Sony's lead. In fact, Sega's early exit from the market implied a duopoly console market between Sony's first-generation PlayStation and Nintendo's N64.

PlayStation's success reflected several changes in the management of the console side of the market. From the start, Sony's strategy was to supply as many games as possible, a lesson it learned from its experience with Betamax video technology:

Sony's primary goal with respect to PlayStation was to maximize the number and variety of games.... Sony was willing to license any PlayStation software that didn't cause the hardware to "crash." Coughlan (2001b)

To stimulate independent game development, Sony charged substantially lower game royalties of \$9, in contrast with Nintendo's \$18 (Coughlan 2001c). Sony's CD-based platform also lowered game development costs, in contrast with Nintendo's cartridge based system. Although the PlayStation console failed to produce any truly blockbuster games during its first year (Kirkpatrick 1996), after three months, PlayStation's games outnumbered those of Sega's Saturn by three to one. By 1998, more than 400 PlayStation titles were available in the United States. In addition, Sony engaged in aggressive penetration pricing of the console early on, hoping to make its money back on game royalties (Cobb 2003).

In contrast, Nintendo maintained very stringent conditions over its game licensees, a legacy from its management of game licensees during earlier generations when Nintendo was dominant.¹² By Christmas of 1996, N64 only had eight games in contrast with

¹² The dominance of Nintendo's 8-bit NES console during the 1980s allowed it to command 20% royalties in addition to a manufacturing fee of \$14 per game cartridge. Licensees were also restricted to five new NES titles per year. Nevertheless, by 1991, less than

roughly 200 PlayStation titles (Rigdon 1996). By June 1997, N64 still had only 17 games whereas PlayStation had 285. Nintendo insisted that it competed on quality rather than quantity, and in 1997 its CEO claimed, “Sony could kill off the industry with all of ‘its garbage’” (Kunii 1998). In the end, the dominance of Sony Playstation in the 32/64-bit console generation was attributed primarily to its vast library of games rather than to specific game content.

Recall that our case study focuses only on the 32/64-bit console generation. The success of PlayStation’s game proliferation strategy makes us comfortable with the assumption that game variety proxies meaningfully for the indirect network effects. This assumption would be more tenuous for more recent console generations now that blockbuster games have become more substantial. For example, the blockbuster game *Halo 3*, for Microsoft’s Xbox 360, generated \$300 million in sales during its first week (Blakely 2007). At the same time, monthly Xbox 360 console sales nearly doubled in contrast with two months previously, selling 527,800 units in October 2007 (Gallagher 2007). Similarly, PlayStation 3’s *Spiderman 3* grossed \$151 million during its first week (Blakely 2007). The blockbuster games of the 32/64-bit generation were smaller in magnitude. Only three N64 games garnered over 4% of total U.S. game unit sales on the N64 platform (*GoldenEye 007*, *Mario Kart 64*, and *Super Mario 64*), whereas an additional 21 games captured over 1% of total game sales. Only five PlayStation titles captured over 1% of total PlayStation game sales, none capturing over 2%.¹³ Nair (2007) tests for blockbuster game effects during this generation. He finds no material impact on sales or prices of games in the months leading up to the launch of a best-selling game. Therefore, Nair ignores competitive effects in his analysis of video game pricing during this generation.

4.2. Data

Our data are obtained from NPD Techworld’s point of sale database. The database consists of a monthly report of total sales and average prices for each video game console across a sample of participating U.S. retailers from September 1995 to September 2002. NPD states that the sheer size of the participating firms represent about 84% of the U.S. retail market. We also observe the monthly number of game titles available during the same period. We define the

10% of titles were produced by Nintendo, and the system had over 450 titles in the United States. In addition, one in three U.S. households had an NES console by 1991, with the average console owner purchasing eight or nine games (Coughlan 2001b).

¹³ These numbers are based on U.S. game sales data collected by NPD.

potential market size as the 97 million U.S. households as reported by the U.S. Census.

In the data, we observe a steady decline in console prices over time. At first glance, this pattern seems inconsistent with the penetration pricing motive one would expect from our model. However, Playstation is estimated to have launched at a price roughly \$40 below marginal cost (Coughlan 2001b), and console prices have been documented to have fallen more slowly than costs over time, the latter as a result of falling costs of chips (Liu 2010). The rising margins over time are consistent with penetration pricing. Although we do not observe marginal costs, we control for falling costs by including a time trend as a state in the empirical model. Thus, our empirical model is consistent with a richer game in which firms face falling marginal costs. We include this time trend in the set of exogenous variables that drive both the pricing strategies z_t^p and the demand function z_t^d , thereby treating the time trend as a commonly observed state. In addition, we experiment with producer price indices (PPIs) from the U.S. Bureau of Labor Statistics for computers, computer storage devices, and audio/video equipment to control for technology costs associated with a console. Finally, we also experiment with the inclusion of the exchange rate (Japanese yen per U.S. dollar) to control for the fact that parts of the console are sourced from Japan. These two sets of cost-shifting variables, PPIs and exchange rates, are included in z_t^p . However, because we do not expect these costs to be observed by consumers when they make console purchase decisions, we exclude them from z_t^d .

The empirical model also includes monthly fixed-effects to control for the fact that there are peak periods in console demand (e.g., around Christmas). These states are observed by both firms and consumers and, hence, enter both z_t^p and z_t^d . For the policy simulations, we will ignore the effects of time, month, and cost shifters because they are incidental to our theoretical interest in tipping.

Descriptive statistics of the data are provided in Table 1. The descriptive statistics indicate a striking fact about competition between Sony PlayStation and Nintendo 64. On average, the two consoles charged roughly the same prices. However, Sony outsold

Table 1 Descriptive Statistics

	Console	Mean	Std. dev.	Min	Max
Sales	PlayStation	275,409	288,675	26,938	1,608,967
	Nintendo	192,488	201,669	1,795	1,005,166
Price	PlayStation	119.9	30.3	55.7	200.6
	Nintendo	117.6	33.9	50.3	199.9
Game titles	PlayStation	594.2	381.1	3	1,095
	Nintendo	151.2	109.9	1	281

Nintendo by almost 50%. At the same time, over 3.5 times as many software titles were available for Sony than for Nintendo. Of interest is whether and how much of Sony's share advantage can be attributed to its large pool of software titles.

5. Estimation

5.1. Demand Estimation

Part of the calibration exercise involves estimating the structural demand parameters using the video game data described above in §4. We briefly outline the approach used to obtain demand estimates for our dynamic discrete choice model using field data. Technical details of the estimation are provided in Appendix B.

One of the main elements of a model of durable goods demand is the incorporation of consumer beliefs about their future utilities and, hence, future realizations of the state variables (Horsky 1990, Melnikov 2000, Song and Chintagunta 2003, Nair 2007, Prince 2008, Carranza 2006, Gowrisankaran and Rysman 2009, Gordon 2009, Goettler and Gordon 2009). In practice, consumers' beliefs are not observed by the researcher and need to be estimated. Some elements of the beliefs could be obtained by solving the economic model. For instance, beliefs about future growth in the installed base could be obtained by nesting the solution to the consumers' dynamic adoption decision into the demand estimation procedure, hereafter referred to as the nested fixed-point (NFP) approach (e.g., Melnikov 2000, Nair 2007). However, for our hardware/software model, the dimension of the state space (y_t , ξ_t , and other exogenous states included in the empirical specification) is prohibitively high for this approach.

Instead, we follow a recent tradition in the empirical literature on dynamic games and follow the insight in Hotz and Miller (1993) and Hotz et al. (1994) by estimating the structural parameters of our model in two stages. The goal is to construct moment conditions that match the observed market shares in the data with those predicted by our model; see Equation (7). Rather than computing the choice-specific value functions (5) and (6) needed to evaluate demand, as in the NFP approach, we simulate them. For the simulation, we first need the distribution of consumer beliefs. We assume that consumers form rational expectations. In this manner, we can estimate their beliefs directly from the field data. The estimation of the empirical distribution of the state variables—namely, prices, software, and adoption—constitutes the first stage. Note that previous research using a NFP approach also typically have a first stage in which they estimate beliefs about supply-side state variables, such as prices. The exact

details for both stages of the estimation procedure are provided in Appendix B.

Most empirical applications of the two-step approach for estimating a dynamic model cannot truly estimate beliefs nonparametrically, as would be required theoretically in the Hotz and Miller (1993) framework. Like most applications, our first stage is parametric. A concern in the context of durable goods markets is that the time-series data are inherently nonstationary because of the diffusion process. Therefore, even with an infinitely long time series, one could never estimate beliefs nonparametrically. This problem also applies to NFP applications that estimate portions of consumer beliefs (i.e., beliefs about prices) directly from the data. One solution would be to pool data from multiple independent markets. Pooling markets requires the strong assumptions that all markets are in the same long-run equilibrium and that all markets have the same parameters (e.g., consumer tastes are the same across markets) in order to estimate beliefs. For instance, Nair (2007) assumes that individual video games are sold and marketed independently, but they are priced in a similar fashion. Hence, different games are assumed to constitute different markets. A related solution might consist of pooling independent geographic (e.g., city) markets (e.g., Ryan 2006). The lack of variation in console prices and game supply across U.S. cities limits the advantages of this type of pooling.

5.2. Identification

Like most of the extant literature estimating structural models of durable goods demand, our diffusion data contain only a single time series for the U.S. market.¹⁴ The use of a single time series creates several generic identification concerns for durable goods demand estimation in general. The first and most critical concern is the potential for sales diffusion data to exhibit dependence over time as well as interdependence in the outcome variables. In addition, the diffusion implies that any given state is observed at most once, a property that could complicate the estimation of beliefs. Finally, we also face the usual potential for price endogeneity to bias demand parameters if prices are correlated with the demand shocks ξ (Berry 1994). We now briefly discuss the intuition of our empirical identification strategy.

Diffusion data may naturally exhibit dependence over time in prices, p_t , and an interdependence between prices and the other outcome variables, y_t and n_t . A concern is whether we can separately identify the price coefficient α and the software taste (i.e.,

¹⁴ An interesting exception is Gupta et al. (1999), who use panel data on individual HDTV adoption choices obtained from a conjoint experiment.

the indirect network effect) γ . Our solution consists of adding console cost-shifting variables, PPIs and exchange rates, that vary prices but that are excluded from demand and from software supply. The exclusion restrictions introduce independent variation in prices and, hence, in the term ap_t in the utility function. The exchange rates are particularly helpful in this regard because they introduce independent variation over time—past research has documented that short-run exchange rate innovations follow a random walk (e.g., Meese and Rogoff 1982, Rogoff 2008). The exclusion restrictions embody a plausible assumption that consumers do not observe the PPIs and exchange rates, and hence, they do not adjust their expectations in response to them.

A related concern is whether we can separately identify the role of product differentiation δ_j (i.e., one standard has a higher share because of its superior technology) and the indirect network effects γ (i.e., one standard has a higher share because of its larger installed base, which in turn stimulates more software variety) on demand. Our assumption of single homing (i.e., discrete choice), a reasonable assumption for this generation of video game consoles, enables us to infer preferences from aggregate market shares. In addition, we hold each console's quality fixed over time. Thus, we can identify the current utility of software (i.e., the indirect network effect) using variation in the beginning-of-period installed base y_t .

We also face the usual concerns about endogeneity bias due to prices (e.g., Berry 1994). We do not have a specific console attribute or macro taste shock in mind when we include ξ in the specification; however, we include it as a precautionary measure. We are reasonably confident ξ is not capturing the impact of unmeasured blockbuster games.¹⁵ Nevertheless, to the extent that ξ captures demand information that is observed by firms, any resulting correlation between prices and ξ could introduce endogeneity bias. Our joint-likelihood approach to the first stage does provide a parametric solution to the endogeneity problem through functional form assumptions. We have imposed a structure on the joint distribution of the data that provides us with the relationship between prices and demand shocks, ξ . However, we can relax this strong parametric condition by using our console cost shifters. Both the exchange rate and the PPIs provide sources of exogenous variation in prices that are excluded from demand and that are unlikely to be correlated with consumer tastes for video game consoles, i.e., ξ . In essence, the endogeneity is resolved by

including the control function $X(y_{jt}, p_{jt}, z_t^p)$ in the log-odds of choices; see Equation (A4) in the Appendix A (e.g., Petrin and Train 2006, 2010).

A related software endogeneity concern arises in our model. Recall that the current period's software supply is determined by the end-of-period installed base: $n_{jt} = h_j(y_{j,t+1})$. Therefore, software is correlated with the demand shocks ξ_t indirectly through their correlation with the end-of-period installed base y_{t+1} . Our control function $X(y_{jt}, p_{jt}, z_t^p)$ also resolves this endogeneity.

As a final point, we do not believe that our data are rich enough to estimate the distribution of heterogeneity in consumer tastes. Clearly, we do not believe in practice that consumers have no persistent differences in tastes. However, once we condition on all the state variables, there is little variation left in the data to identify non-IIA comovements in market shares, which in turn would identify heterogeneity. The extant literature that estimates unobserved heterogeneity from aggregate market share data relies on the availability of many independent markets (e.g., Berry et al. 1995 and the related literature). In consumer packaged goods product categories, for instance, researchers can exploit sales from many different stores, each charging different prices for the same goods. Supermarkets also regularly run temporary price discounts, generating large swings in the observed market shares. In the video game console market, such store-level data are unavailable. Moreover, the indirect network effects arise from interconnection between the national supply of games and the national installed base of consoles. Hence, individual stores would not per se constitute independent product markets. In the electronic companion, available as part of the online version that can be found at <http://mktsci.pubs.informs.org>, we attempt to estimate a model that incorporates unobserved heterogeneity. Our findings indicate that model fit worsens once heterogeneity is added: the penalty associated with the extra parameters characterizing heterogeneity outweigh the improvements in the likelihood. We therefore conclude that our data are not sufficiently rich to identify heterogeneity.

6. Estimation Results

6.1. First Stage

During the first stage, we experiment with several specifications. These specifications vary by the manner in which the state variables enter the first stage estimation equations. In Table B.1, which is included in Appendix B, we report the log-likelihood and Bayesian information criterion (BIC) associated with each specification.

¹⁵ We checked the correlation between the ξ estimates from our first stage and the one-firm concentration ratio of video game sales for each console (based on NPD data). Game concentration explains less than 1% of the variation in PlayStation's ξ versus 11% of N64's ξ .

Our findings indicate that allowing the states to enter the first-stage relationships for pricing strategies (\mathcal{P}_j , as per Appendix B) and demand function (\mathcal{L}_j , as per Appendix B) both linearly and quadratically improves fit substantially based on the BIC predictive fit criterion (model 3 versus model 2). Allowing for a time trend also improves fit moderately (model 2 versus model 1). We use a time trend that is truncated after 60 periods because prices roughly level off after that point (i.e., we do not expect costs to decline indefinitely). We also experimented with a more flexible distributional assumption for the demand shocks, ξ . We use a mixture-of-normals specification to check whether the assumption of normality potentially biases our maximum likelihood estimates. However, we find little change in fit from the two-component mixture (model 4 versus model 3). Moving to the last three rows (models 5, 6, and 7), we look at the implications of including additional cost proxies into the pricing function that are excluded from the game supply and from the consumer choices. Recall these are terms we include in z_{it}^p , but we do not include in z_{it}^d . We use a three-month lag and seven-month lag in the exchange rate as they were found to explain more price variation than the contemporaneous exchange rate, which is likely because production is sourced in advance of sales. The inclusion of these terms in the price equation improves the overall likelihood of the first stage (as seen by the BIC for model 7).

Although not reported in the tables, a regression of log-prices on the various price shifters, including the PPIs and the exchange rate, generates an R^2 of 0.9. Similarly, the ordinary least-squares regression for the game titles generates an R^2 of 0.98. In the case of log-odds, the inclusion of ξ makes it hard to interpret an R^2 . Instead, we construct a distribution of ξ using a parametric bootstrap from the asymptotic distribution of the parameters in the price regressions. The mean R^2 of a regression of log-odds on the observed states and the simulated ξ is 0.95. Overall, the first-stage model appears to fit the data well.

A critical aspect of the two-step method is that the first-stage model captures the relationship between the outcome variables and the state variables. To assess the fit of the first-stage estimates, in Appendix B, Tables B.2, B.3, and B.4 report all the first-stage estimates and their standard errors. Most of the estimates are found to be significant at the 95% level. In Table B.4, we find a positive relationship between software variety and the installed base of each standard. Analogous findings are reported in Clements and Ohashi (2005).

Figures B.1, B.2, and B.3, which are also contained in Appendix B, plot the true prices, log-odds, and games, respectively, under each standard. In each case, we plot the outcome variable for a standard

Table 2 Second-Stage Parameter Estimates

	Model 3		Model 7	
	Estimate	Std. error	Estimate	Std. error
δ_{Sony}	−1.21	0.89	−1.119	0.971
δ_{NG4}	−1.34	0.87	−1.119	1.093
α	−1.94	0.52	−1.923	0.460
Time (<60)	−0.04	0.01	−0.049	0.028
γ ($n_{it}/1,000$)	0.09	0.04	0.090	0.040
ψ (std. dev. of ξ_{it})	0.05	0.09	0.028	1.950

Notes. Model 7 uses PPIs and exchange rates as instruments in first stage. $\beta = 0.9$; number of simulations = 60.

against its own installed base (reported as a fraction of the total potential market, $M = 97,000,000$). In addition, we report a 95% prediction interval for each outcome variable based on a parametric bootstrap from the asymptotic distribution of our parameter estimates.¹⁶ In several instances, the observed outcome variable lies slightly outside the prediction interval. However, overall, our first-stage estimates appear to do a reasonably good job preserving the relationship between the outcome variables and the installed base.

6.2. Second Stage

We report the structural parameters from the second stage in Table 2. Note that the parameters of interest here are the structural parameters $\Lambda = (\delta, \alpha, \gamma, \psi)$, where $\delta = \{\delta_s, \delta_N\}$ are the intrinsic preferences for the Sony and Nintendo consoles, α is the price sensitivity parameter, γ is the current period software utility and ψ is the standard deviation of the demand shocks. Results are reported for two specifications: models 3 and 7, as described in §6.1. Recall that model 3 does not have any exclusion restrictions across equations in the first stage. Model 7, the best-fitting model overall in stage 1, includes PPIs and exchange rates in the price equations. To estimate the second stage of the model, we maintain the assumption that consumers do not observe realizations of these costs. Instead, we assume they observe prices each period and can integrate the innovations to prices out of their expected value functions.¹⁷ The results are based on an assumed consumer discount factor of $\beta = 0.9$ and

¹⁶ The prediction intervals are constructed as follows. Five thousand draws are generated from the asymptotic distribution of the first-stage parameter estimates. We then compute the predicted log-price, log-odds, and log of game titles corresponding to each parameter draw. We then plot the 5th and 95th percentiles of these values.

¹⁷ To estimate the distributions of these various costs, we assume they all follow a random walk distribution with drift. Thus, we regress each cost on its one-period lag along with an intercept and an i.i.d. shock. For the PPIs, we obtain an R^2 of 0.99, whereas for the exchange rates, we obtain an R^2 of 0.89.

60 simulated histories¹⁸ of 500 periods in length each. Although not reported, we also included monthly fixed effects in tastes.

First, both model specifications each appear to yield qualitatively similar results. Although the point estimates suggest a slight preference for the Sony PlayStation console, the difference in tastes between the two consoles is statistically insignificant. This finding is consistent with industry observers who noted that the improvements from 32- to 64-bit technology were much less dramatic than in previous generations (Coughlan 2001c). Rather, the variety of availability of games tended to be the main differentiator. Indeed, the taste for software variety, γ , is positive and significant. In both specifications, γ is roughly 0.1. The effective “network effect” in the model arises from the positive (and significant) software taste on the demand side, γ , and the positive (and significant) elasticity of each standard’s supply of software titles with respect to its installed base, λ_{Sony} and $\lambda_{\text{Nintendo}}$ (as in Table B.4). The qualitative implications of these estimates are best understood in the context of our simulations in §7.

7. Model Predictions

In §2, we proposed an approach to measure tipping, or the extent of market concentration caused by indirect network effects. Our measure consists of comparing the expected market concentration in the presence of indirect network effects to the counterfactual market concentration that would arise if indirect network effects were reduced or eliminated:

$$\Delta C_1 = C_1(\Theta, \mathcal{E}(\Theta)) - C_1(\Theta', \mathcal{E}(\Theta')).$$

We illustrate our approach by calibrating the model developed in §3 with empirical estimates from the 32/64-bit video game console market. The parameters consist of the demand estimates and software supply function estimates, from model 7 in §6, and industry estimates of hardware console production costs and royalty fees.¹⁹ For a given set of parameter values, we solve for a Markov perfect Bayesian equilibrium of the model and then simulate the resulting equilibrium price and adoption paths. Appendix C provides details on the algorithm used to compute the equilibria of the model.

Indirect network effects in our model arise both through the consumers’ current marginal utilities of

software γ and their discount factors β . Both these parameters moderate the interaction between consumers’ expectations about the future availability of software for each standard and their current adoption decisions. By conducting numerical experiments that change the values of β and γ , we can assess how the magnitude of indirect network effects affect market concentration as well as equilibrium diffusion and pricing. We also examine how an initial market share advantage, or “first-mover” advantage, for one of the hardware suppliers moderates the impact of indirect network effects on market concentration.

Our model abstracts from certain aspects of the 32/64 hardware market—in particular, learning by doing (declining production costs) and persistent heterogeneity in consumer tastes. In this respect, we caution that our predictions should not be interpreted as attempts to explain literally the observed, historic evolution of the market.

7.1. Preliminaries

We first summarize specific aspects of the model solutions and simulations. Firms and consumers make decisions at the monthly level. Throughout, we assume that firms discount future profits using the factor $\beta_f = 0.99$.²⁰ However, we will consider various consumer discount factors across the different simulations. To simplify the exposition, we also normalize the market size to $M = 1$.²¹

We summarize the firms’ equilibrium pricing strategies by the expected pricing policies $\mathbb{E}(p_{it} | y_t) = \mathbb{E}_{\xi_j}(\sigma_j(y_t, \xi_j) | y_t)$. Here, the expectation is taken over the firm’s private information, the transitory demand component ξ_j . The equilibrium evolution of the state vector is summarized by a vector field, where each state is associated with the expected state in the next period. Thus, for a given current state y_t , we calculate (and plot) a vector describing the expected movement of the state between periods:

$$\vec{z}_t = \mathbb{E}(y_{t+1} | y_t) - y_t = \mathbb{E}_{\xi}(f(y_t, \xi, \sigma(y_t, \xi) | y_t) - y_t).$$

Using the equilibrium policies and equilibrium state transitions, we can simulate a path of prices, software titles, sales, and installed base values given an initial condition y_0 and a sequence of demand shocks, ξ_t . For each set of parameter values, we generate 5,000 simulations of the evolution of the market. Using the simulated values, we can then examine the distribution of prices over time and the distribution of shares in the total installed base at the end of each period, ρ_{jt} , as defined in §2.

¹⁸ Because the second-stage estimator is linear in the simulation error, the choice of the number of draws only influences efficiency.

¹⁹ Cost and royalty data are reported in Liu (2010) and based on various industry reports. The marginal production costs are \$147 (Sony) and \$122 (Nintendo) and correspond to Liu’s cost estimates 20 months after the launch of Nintendo 64. The royalty fees per game sold are \$9 (Sony) and \$18 (Nintendo).

²⁰ This discount factor corresponds to an annual interest rate of 12.8%.

²¹ Note that this normalization also requires rescaling the parameters in the equation describing the predicted flow of titles sold such that the software supply is proportional to the market size M .

7.2. Measuring Tipping: Symmetric Competition

We first analyze a case of symmetric competition, where both competitors have identical demand functions, production costs, and royalty fee structures. The standards are also symmetric in their initial installed base levels, $y_0 = (0, 0)$. In the symmetric case, it is easy to compare the predicted market concentration relative to the benchmark case, where both competitors share the market equally. We assume that both competitors are characterized by the parameter estimates that we obtained for Sony. We refer to these *ex ante* identical competitors as standards 1 and 2.

We first examine how market outcomes are influenced by the consumers' marginal utility of software, γ . We use the parameter estimates obtained for the consumer discount factor $\beta = 0.9$ and then scale the estimated software utility coefficient by the factors 0.25, 0.5, 0.75, and 1. Figure 2 displays the resulting equilibrium pricing policies and expected price paths for each of the different software utility values. The expected price paths are conditional on cases where standard 1 sells at least as many consoles as standard 2 by the end of period $T = 48$, $y_{T1} \geq y_{T2}$. The marginal production costs are indicated by horizontal lines. Figure 3 shows the vector field describing the expected evolution of the state and the distribution of shares of the installed base, ρ_{jt} , $T = 48$ months after both standards were launched.²²

For the scale factors 0.25, 0.5, and 0.75, the results are similar. Prices rise over time as firms compete more aggressively when they have not yet obtained a substantial share of the market. After 48 months, both firms have an approximately equal share of all adopters. Hence, market outcomes are approximately symmetric.

Now compare these results to the model solution obtained for the estimated software utility coefficient (scale factor equals one), indicating a larger indirect network effect than in the previous three model variations. The equilibrium changes both quantitatively and qualitatively. First, unlike in the previous cases, we are no longer able to find a symmetric equilibrium in pure strategies. However, there are at least two asymmetric pure strategy equilibria. The graphs at the bottom of Figure 3 display one of these equilibria, which "favors" standard 1. In this equilibrium, before any consoles have been sold ($y_0 = (0, 0)$), consumers expect that standard 1 will obtain a larger market share than standard 2 (note the direction of the arrow at the origin). These expectations are self-fulfilling, and because of the impact of the expected

future value of software on adoption decisions, standard 1 is expected to achieve a larger share of the installed base than standard 2. However, if standard 2 ever obtains a share of the installed base that is sufficiently larger than that of standard 1 (because of a sequence of favorable demand shocks, for example), then consumers' expectations flip and standard 2 is expected to win. The advantage due to self-fulfilling expectations is increasing in the difference of shares in the installed base, $y_{jt} - y_{-j,t}$.

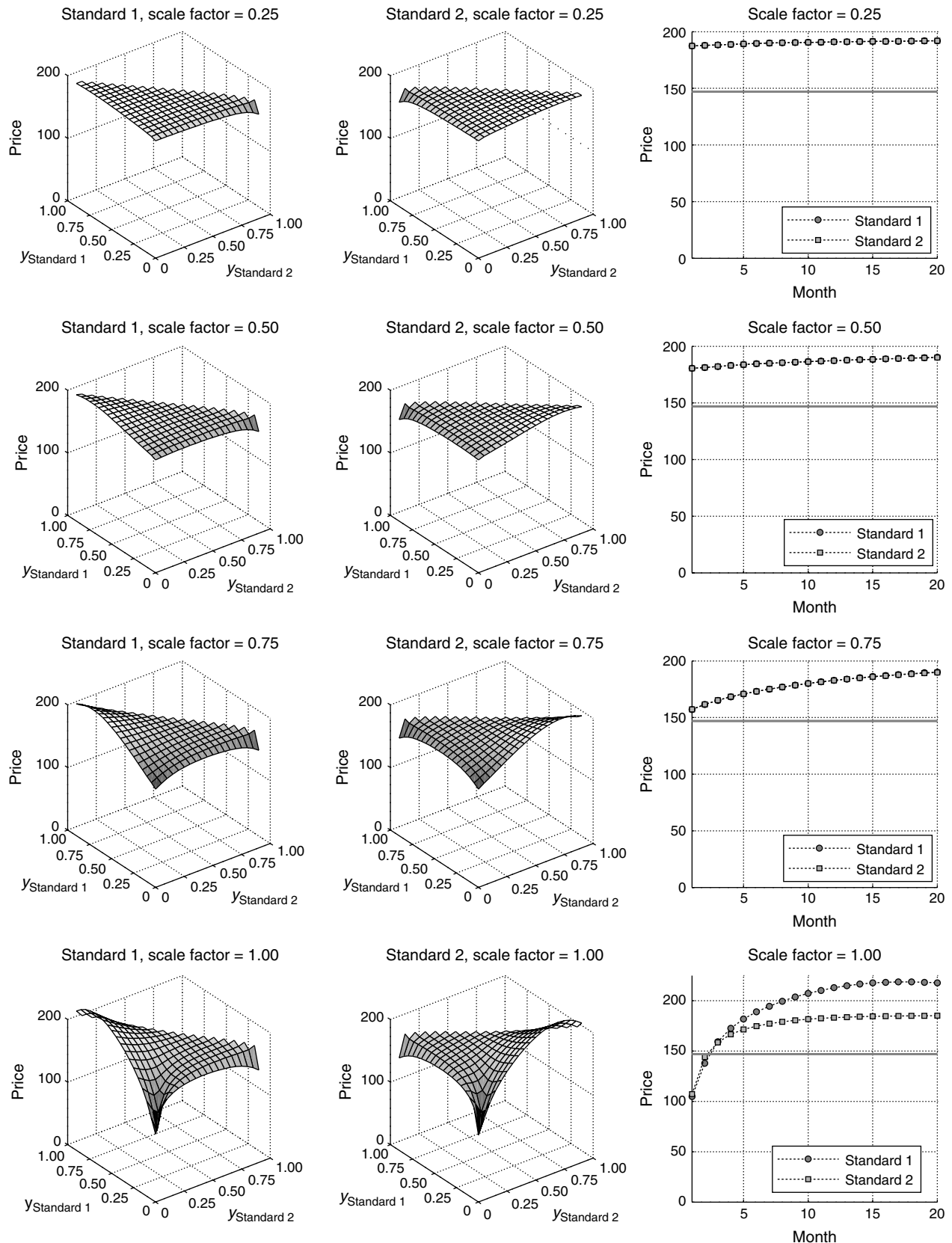
As a consequence of this equilibrium behavior, the market becomes concentrated even though the standards are identical *ex ante*. The expected cumulative one-firm concentration ratio increases from $C_1 = 0.501$, for the scale factor 0.25, to $C_1 = 0.845$, for the scale factor 1 (see Table 3). The distribution of shares in the installed base not only becomes disperse but also asymmetric: in about 55% of all simulations, standard 1 "wins" the market, i.e., obtains a larger share of the installed base than standard 2. Note that there is also another asymmetric equilibrium that favors standard 2. This equilibrium exactly mirrors the one that favors standard 1; for example, standard 2 has a 55% chance of "winning" the market, etc.

Another interesting aspect of the equilibrium is the impact of the magnitude of the marginal utility of software on firms' pricing strategies. As can be seen at the bottom of Figure 2, for a scale factor of one, pricing becomes substantially more aggressive than under the smaller scale factors. For small values of y_{jt} , the firms engage in penetration pricing whereby prices are set below the \$147 marginal production cost of a console.

Next, we examine how market outcomes change under different values of the consumers' discount factor β . The discount factor influences how consumers value software that they expect to become available in the future, and thus it determines the importance of expectations in driving adoption decisions. We choose several discount factors ($\beta = 0.6, 0.7, 0.8, 0.9$) and solve the model for each β , holding the other parameters that were estimated for the discount factor $\beta = 0.9$ constant. Figure 4 shows that the equilibria obtained and the expected concentration of the market are highly sensitive to the magnitude of β . For the smaller discount factors ($\beta < 0.9$), corresponding to relatively small indirect network effects, we obtain a symmetric equilibrium where the expected one-firm concentration ratio C_1 is just slightly larger than 0.5 (Table 3). For $\beta = 0.9$, however, we are unable to compute a symmetric equilibrium, and the expected market concentration increases to $C_1 = 0.845$, as already discussed above.

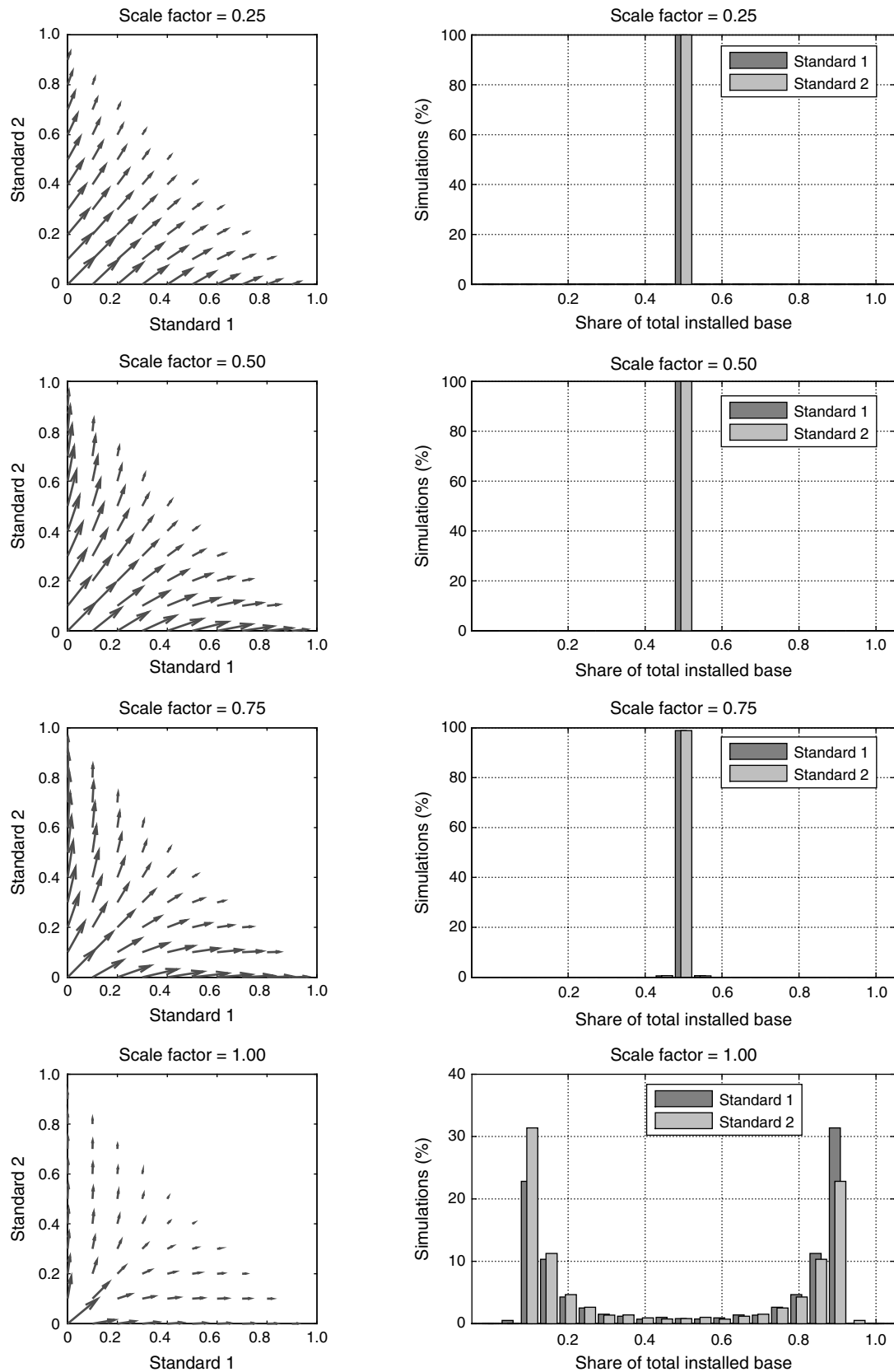
Because the discount factor is typically not identified from field data, in practice it is *assumed* to have a known fixed value when other model parameters

²² The dark bar at the abscissa value ρ depicts the percentage of all model simulations, i.e., approximately the probability that standard 1 accounts for a fraction ρ of all adopters at the end of $T = 48$.

Figure 2 Symmetric Competition: Equilibrium Pricing Policies and Price Paths

Notes. Consumer's software utility coefficient is scaled by different factors. The expected price paths are shown conditional on $y_{T1} \geq y_{T2}$ at the end of period $T = 48$. Marginal production costs are indicated by horizontal lines.

Figure 3 Symmetric Competition: Expected State Evolution and Distribution of Shares in the Installed Base After 48 Months



Note. Consumer's software coefficient is scaled by different factors.

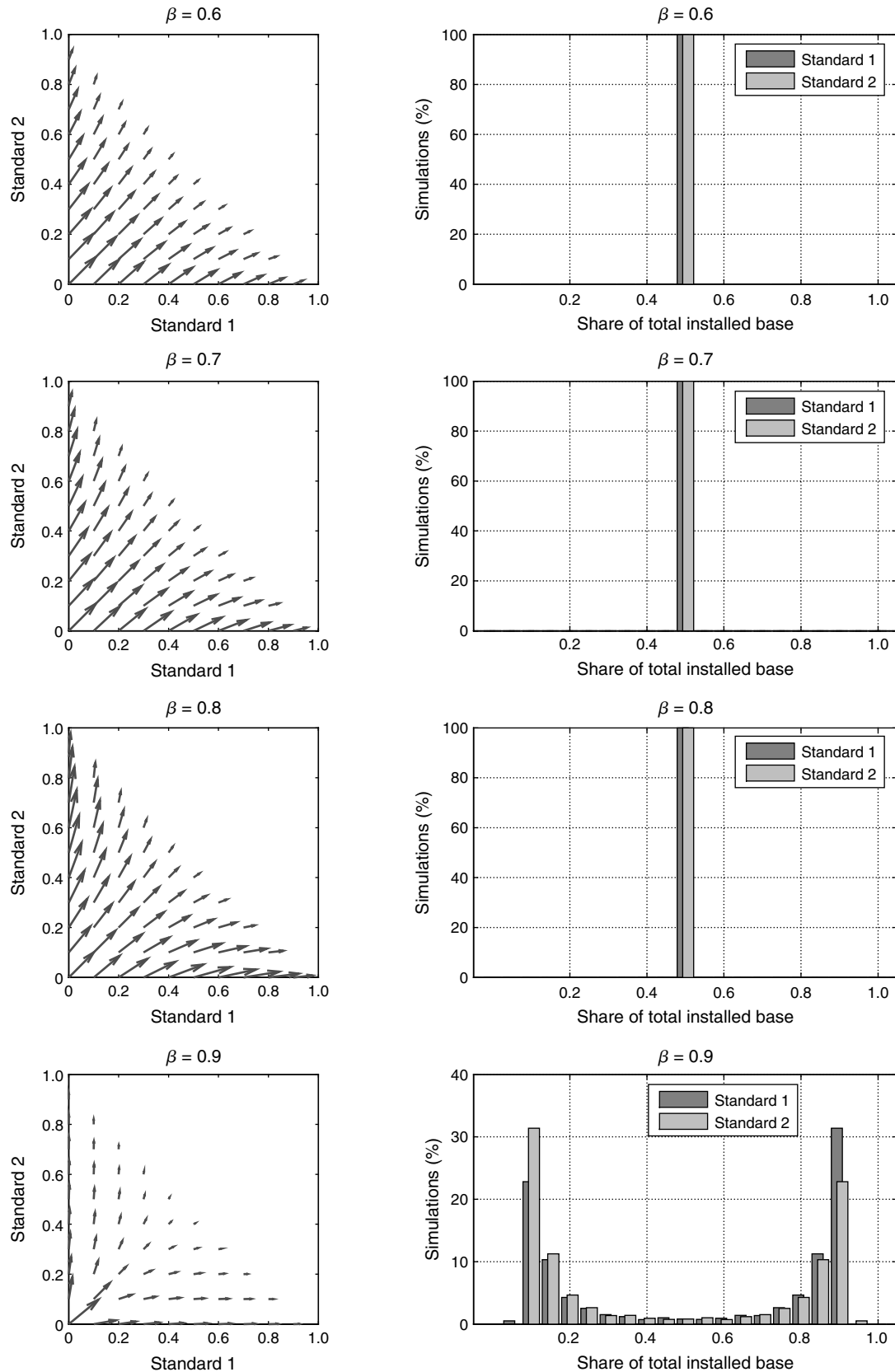
Figure 4 Symmetric Competition: Expected State Evolution and Distribution of Shares in the Installed Base After 48 Months for Different Consumer Discount Factors (β)

Table 3 Predicted One-Firm Concentration Ratios

Model predictions: Symmetric case (parameter estimates for Sony)				
Scale factor for γ	0.25	0.50	0.75	1.00
C_1	0.501	0.503	0.508	0.845
Discount factor (β)	0.600	0.700	0.800	0.900
C_1^a	0.501	0.502	0.508	0.845
C_1^b	0.501	0.501	0.508	0.845
Model predictions: Estimated parameter values				
Scale factor for γ	0.250	0.500	0.750	1.000
C_1	0.600	0.593	0.562	0.843
Discount factor (β)	0.600	0.700	0.800	0.900
C_1^a	0.602	0.601	0.599	0.843
C_1^b	0.571	0.572	0.562	0.843

Notes. The results are based on 5,000 simulations, and the concentration ratios are reported for month $T = 48$. No standard has an initial advantage; $y_0 = (0, 0)$.

^aAll estimated model parameters were obtained for $\beta = 0.9$.

^bPredictions where the model parameters were reestimated for each consumer discount factor, β .

are estimated. To verify the sensitivity of our results to the specific value of β assumed during the estimation stage, we repeat the previous comparative statics exercise by reestimating the model parameters for each of the discount factors: $\beta = 0.6, 0.7, 0.8, 0.9$. Referring to Table 3, we see that the results are very similar to those where we only varied β but not the other model parameters. In particular, the market outcomes are almost symmetric for the discount factors $\beta < 0.9$ and then become very concentrated for $\beta = 0.9$.

Using our estimated demand parameters, we did not see tipping or multiple equilibria arise for consumer discount factors $\beta \leq 0.8$. In principle, multiple equilibria and strong degrees of market concentration can arise even if consumers are myopic. For example, if we set the consumer's discount factor to $\beta = 0$, we find multiple equilibria if we scale the estimated software coefficient by a factor of 11 or higher.

7.3. Measuring Tipping: The General Case

The symmetric case discussed in §7.2 establishes the intuition for the model predictions. We now turn to the measurement of tipping due to indirect network effects in the general case where firms are asymmetric ex ante (but we retain the initial condition $y_0 = (0, 0)$). With heterogeneous competitors, markets can obviously become concentrated even in the absence of indirect network effects. Hence, we measure tipping relative to a specific counterfactual outcome where one or all mechanisms leading to indirect network effects are absent or smaller in magnitude.

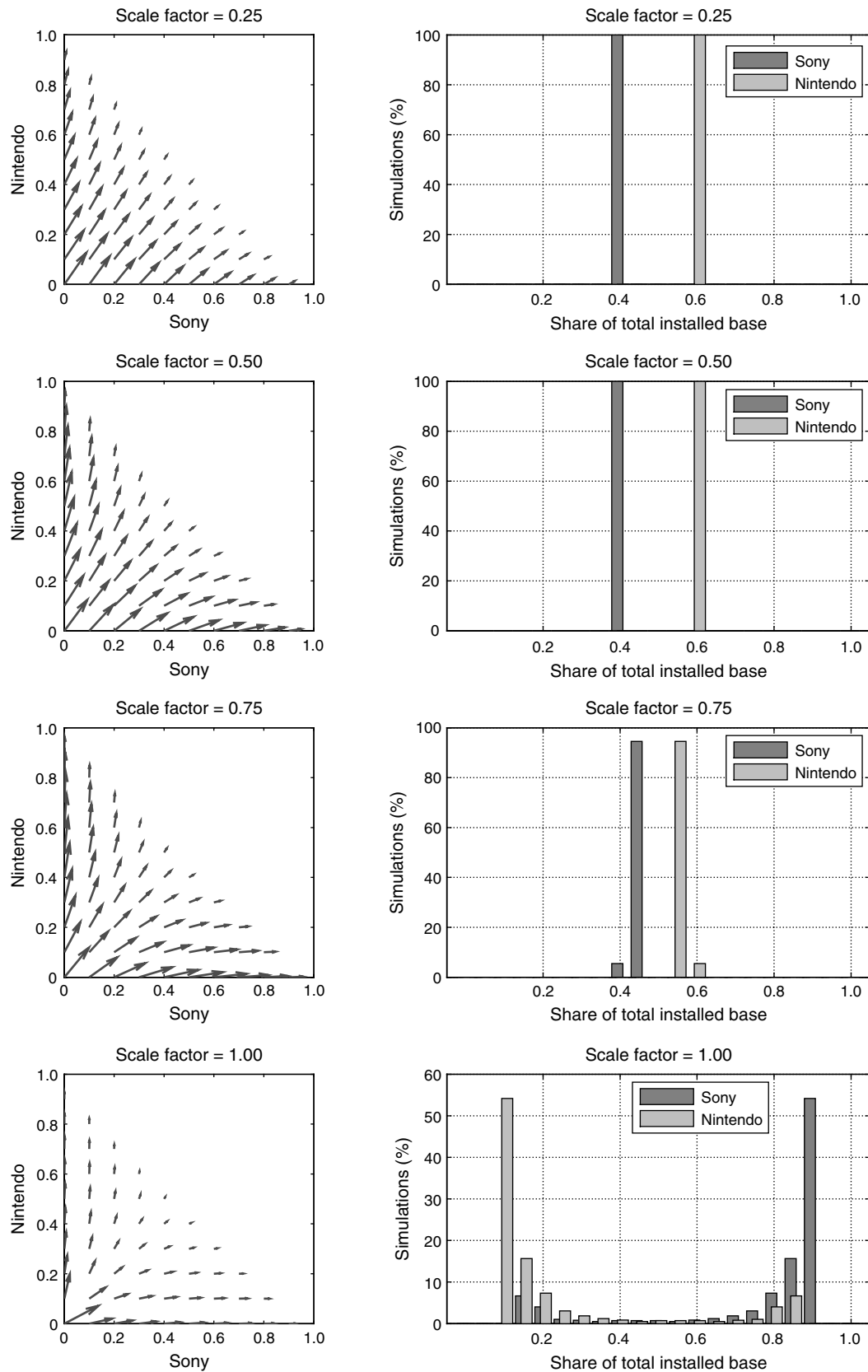
We first focus on the consumers' software utility parameter, γ . As in the symmetric case studied above, we scale this parameter by the factors 0.25, 0.5, 0.75, and 1.²³ Figure 5 shows the expected market evolution and distribution of shares in the installed base for the different scale factors. Unlike in the case of symmetric competition, one standard, Nintendo, has a persistent advantage for all of the smaller scale factors (0.25, 0.5, and 0.75). In all 5,000 model simulations, Nintendo obtains a larger installed base share than Sony by the end of period $T = 48$, and the expected one-firm concentration ratio C_1 ranges from 0.562 to 0.6 (Table 3). At the estimated parameter values (scale factor = 1), however, we once again see a big qualitative and quantitative change in the equilibrium. First, the market becomes significantly more concentrated; $C_1 = 0.843$. Second, Sony is now predicted to obtain a larger installed base share than Nintendo in 85% of all cases. That is, indirect network effects strongly increase the concentration of the market and, in addition, the identity of the larger standard changes. The reason for this difference in outcomes for different magnitudes of the indirect network effect is that, according to our estimates, Sony dominates Nintendo in terms of the quantity of software titles supplied at any given value of the installed base. On the other hand, Nintendo has a lower console production cost (\$122 versus \$147). For small values of the software utility, Nintendo's cost advantage results in lower equilibrium prices and thus a market share advantage over Sony. Once the software utility gives rise to sufficiently large network effects, however, Sony's advantage in the supply of games becomes important and helps it to win the standards war against Nintendo. The same argument also explains why initially, for the scale factors 0.25, 0.50, and 0.75, the concentration ratio C_1 slightly decreases: Sony obtains a larger market share as its relative advantage as a result of indirect network effects becoming more pronounced.

Figure 6 shows the (expected) pricing policies and expected price paths for the different software scale factors. As in the case of symmetric competition, the two standards engage in penetration pricing for a scale factor of 1. Pricing becomes substantially less aggressive for the smaller scale factors, although for a scale factor of 0.75, Nintendo still engages in a few periods of below cost pricing to fight Sony's software advantage. We also see that Nintendo's marginal cost advantage translates into generally lower equilibrium prices.

Next, we examine the market outcomes under different consumer discount factors ($\beta = 0.6, 0.7, 0.8, 0.9$).

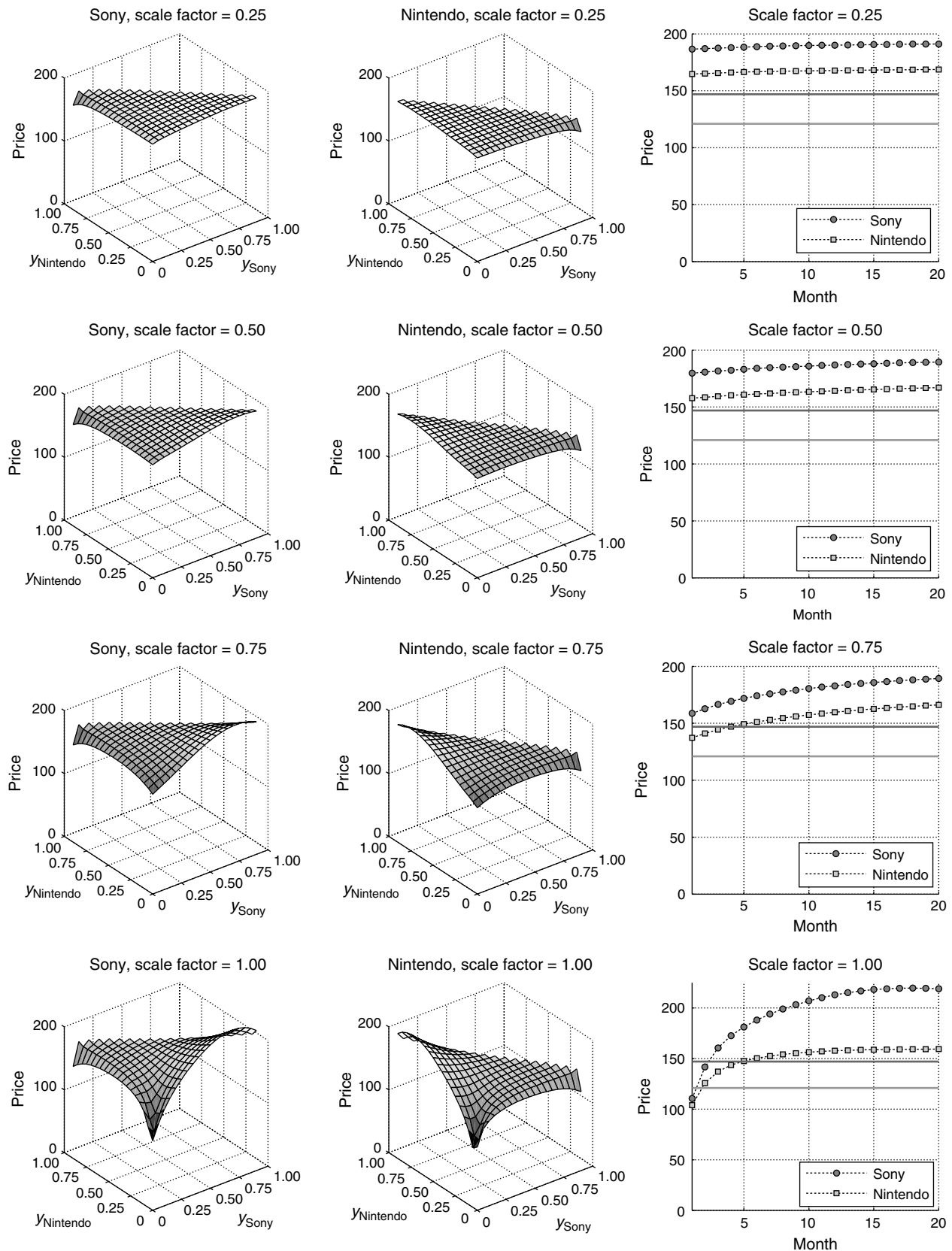
²³ The consumer discount factor is set to $\beta = 0.9$.

Figure 5 Predictions from Estimated Parameter Values: Expected State Evolution and Distribution of Shares in the Installed Base After 48 Months



Note. Consumer's software coefficient is scaled by different factors.

Figure 6 Equilibrium Pricing Policies and Price Paths from Estimated Parameter Values



Notes. Consumer's software utility coefficient is scaled by different factors. Marginal production costs are indicated by horizontal lines.

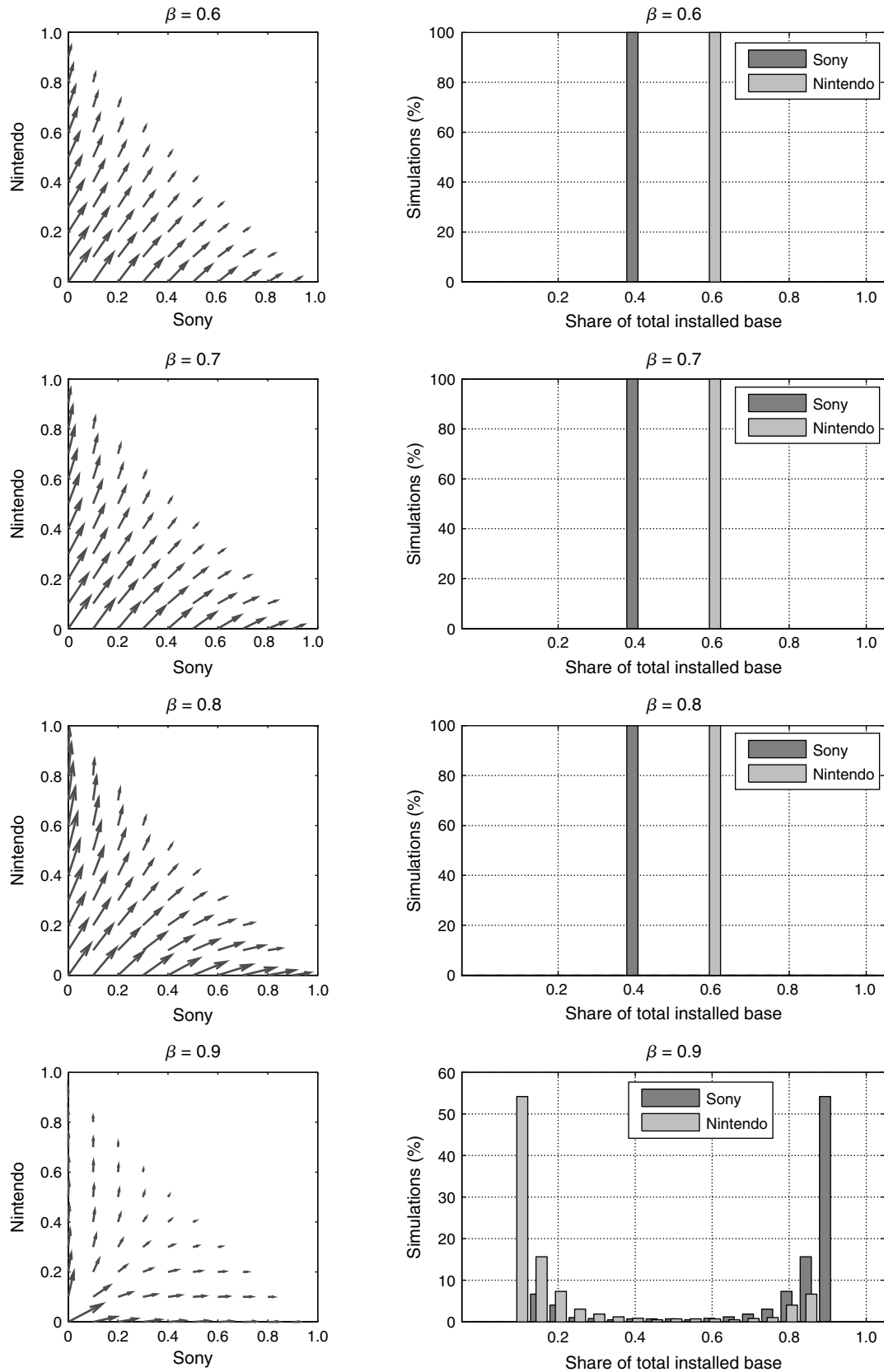
Figure 7 Predictions from Estimated Parameter Values: Expected State Evolution and Distribution of Shares in the Installed Base After 48 Months for Different Consumer Discount Factors (β)

Table 4 Predicted Degree of Tipping at Estimated Parameter Values
($\beta = 0.9$)

Scale factor for γ	0.250	0.500	0.750
C_1	0.243	0.249	0.280
Discount factor (β)	0.600	0.700	0.800
ΔC_1^a	0.241	0.242	0.244
ΔC_1^b	0.272	0.271	0.280

Notes. This table displays the increase in market concentration relative to a specific counterfactual model, where either the marginal utility of software, γ , is scaled or a different consumer discount factor is chosen. The results are based on 5,000 simulations, and the tipping measures are reported for month $T = 48$. No standard has an initial advantage; $y_0 = (0, 0)$.

^aAll estimated model parameters were obtained for $\beta = 0.9$.

^bPredictions where the model parameters were reestimated for each consumer discount factor, β .

We vary β but hold all other parameters constant at their estimated values, which were obtained for a discount factor of 0.9. The results (see Table 3 and Figure 7) show that the market concentration increases from $C_1 \approx 0.6$ for $\beta < 0.9$ to $C_1 = 0.843$ for $\beta = 0.9$. Furthermore, as already discussed, although Sony has a larger share of the installed base than Nintendo in 85% of all cases when $\beta = 0.9$, Nintendo is always predicted to win for the smaller discount factors. These predictions remain quantitatively very similar when we reestimate all parameters for each separate consumer discount factor.

Finally, Table 4 shows our measure of tipping, ΔC_1 , which is the change in concentration associated with model predictions based on the estimated parameter values relative to model predictions based on several counterfactual sets of parameter values that either lower the software utility parameter γ or the consumers' discount factor β . For example, compared to a market where the consumers' flow utility from software is only 25% of the estimated value, indirect network effects are predicted to increase the market concentration by 24.3 percentage points. Relative to a market where consumers discount the future using $\beta = 0.6$, the market concentration increases by 24.1 percentage points. If we look at the counterfactual where all parameters are reestimated for the alternative discount factor $\beta = 0.6$, the increase in concentration is 27.2 percentage points. Hence, in our particular example, we predict a large, quantitatively significant degree of tipping as a result of indirect network effects.

7.4. Tipping in the Absence of Multiple Equilibria: Initial Advantage and Market Concentration

In the previous subsections, tipping was associated with the emergence of multiple asymmetric equilibria. In this section, we show that tipping is not per

se a result of the multiplicity of equilibria. In particular, we show how an initial advantage in the installed base for one firm, possibly due to an earlier launch, can also tip the market in its favor. This initial advantage can create tipping even in the presence of a symmetric, pure strategy equilibrium. Tipping is still moderated by the indirect network effects in that an initial advantage increases the relative utility that consumers gain from adopting the incumbent standard at the time the competitor enters the market, exacerbating its advantage. Although we do not model the source of the initial advantage, it could reflect a firm's strategy of increased research and development (R&D) spending, superior R&D capabilities, or simply a particularly favorable random draw from the R&D production function.

We consider the case where the standard labeled "1" has gained an installed base of y_{10} at the time its incumbent enters the market. We simulate the model using the initial condition $y_0 = (y_{10}, 0)$. As before, we simulate the model 5,000 times and record the expected market outcomes in month $T = 48$.

We could measure market concentration using the expected cumulative one-firm concentration ratio, as we did in the analysis in §§7.2 and 7.3. However, this measure would also incorporate the initial installed base advantage, which would be confounded with the indirect network effect on market concentration. We thus measure the degree of concentration based on the cumulative one-firm concentration ratio in the "remaining market":

$$\tilde{C}_1(\Theta, \mathcal{E}(\Theta)) \equiv \mathbb{E}(\max\{\tilde{\rho}_{1T}, \tilde{\rho}_{2T}\} \mid \Theta, \mathcal{E}(\Theta)),$$

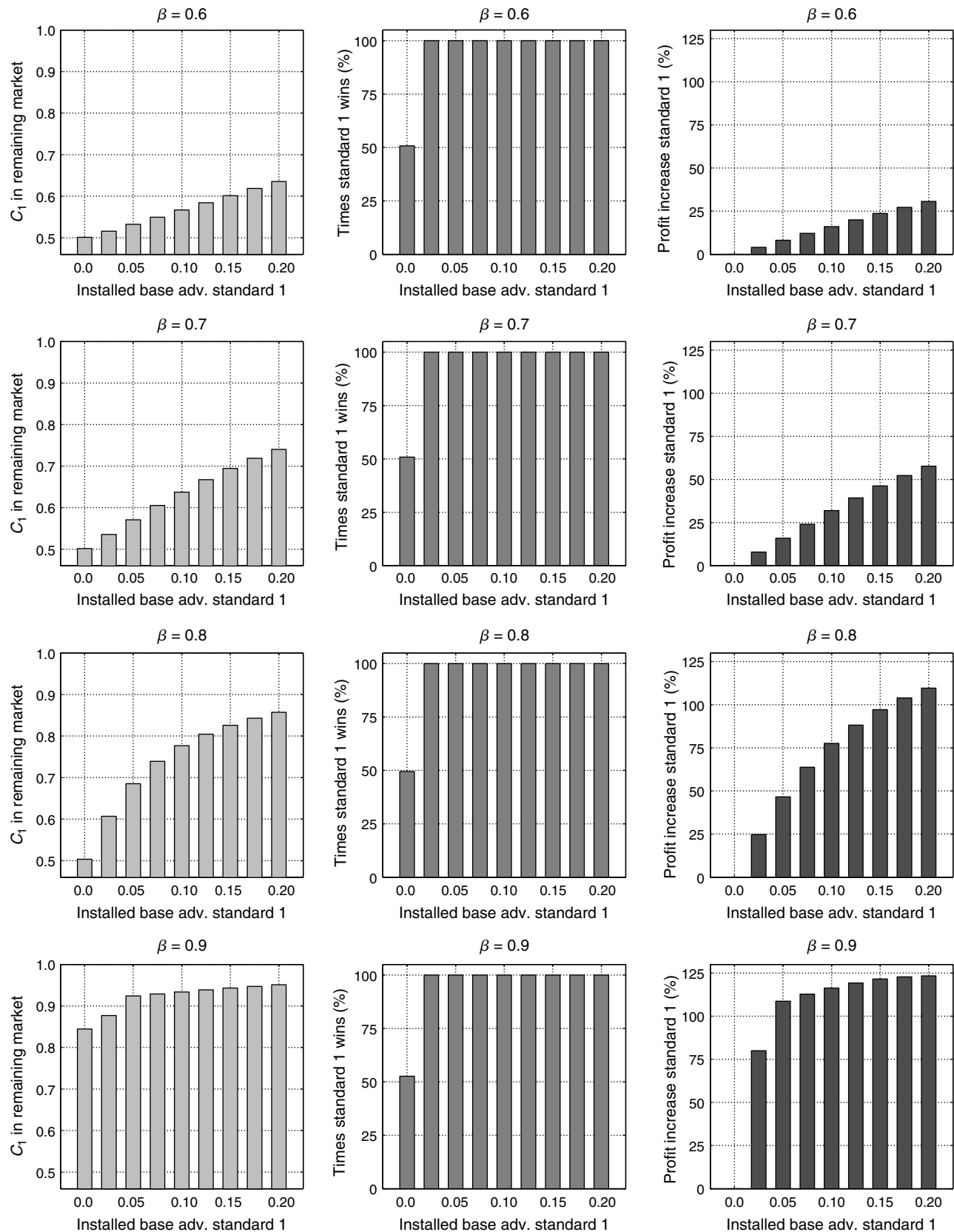
$$\tilde{\rho}_{1T} = \frac{y_{1,t+1} - y_{10}}{(y_{1,t+1} - y_{10}) + y_{2,t+1}},$$

$$\tilde{\rho}_{2T} = \frac{y_{2,t+1}}{(y_{1,t+1} - y_{10}) + y_{2,t+1}}.$$

If indirect network effects had no impact on adoption choices and if both standards were symmetric, we would expect $\tilde{C}_1 \approx 0.5$.

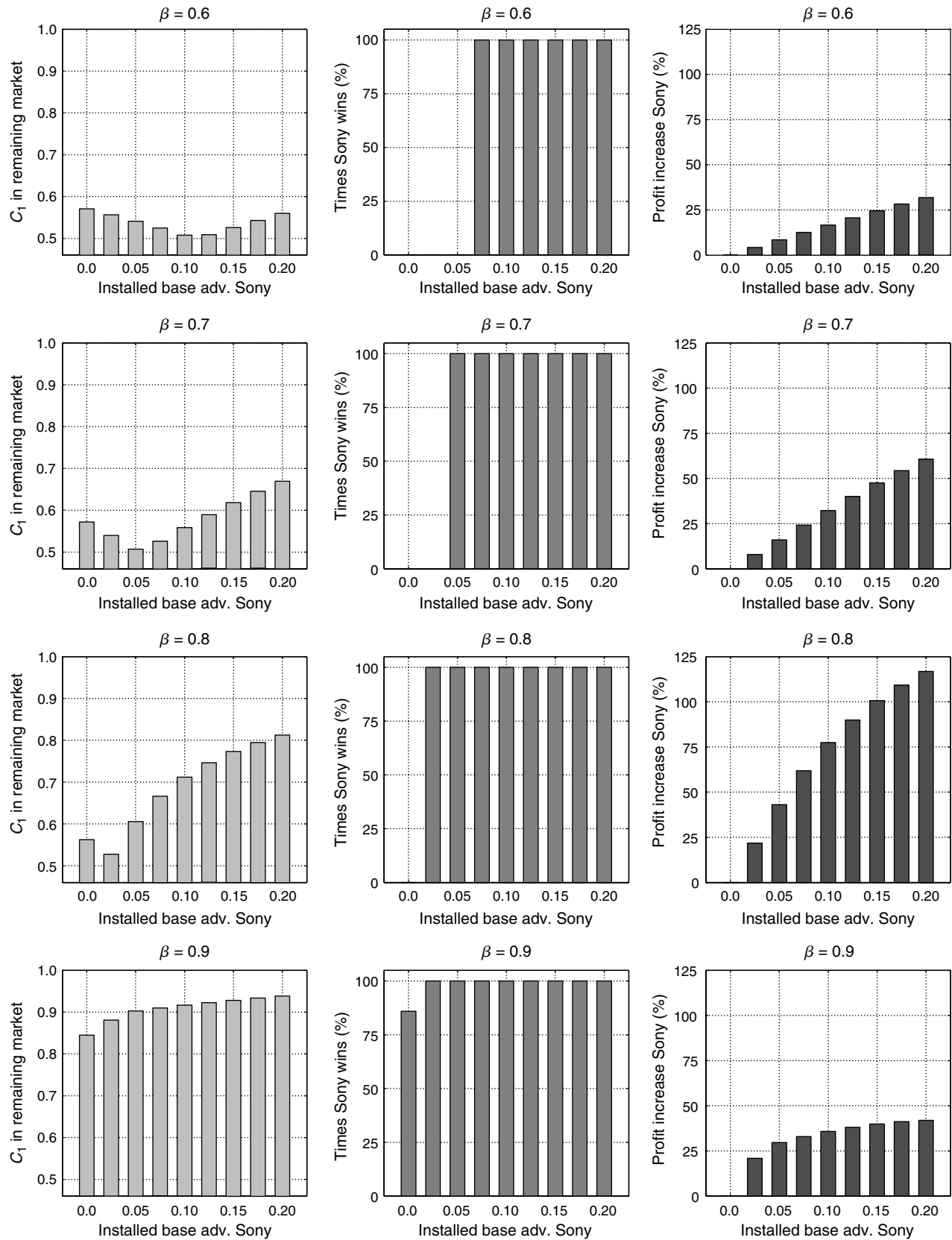
For the case of symmetric competition, Figure 8 displays the effects of an initial installed base advantage for standard 1 on \tilde{C}_1 and on the percentage of simulations in which standard 1 wins, in the sense that $\tilde{\rho}_{1T} > \tilde{\rho}_{2T}$. The results are displayed for different discount factors, β (all parameters were reestimated for each β). The market concentration is increasing in the installed base advantage of standard 1. For example, for an initial advantage of $y_{10} = 0.1$, which corresponds to 10% of the total market, $\tilde{C}_1 = 0.64$ for $\beta = 0.7$ and $\tilde{C}_1 = 0.78$ for $\beta = 0.8$. Also, in all simulations, standard 1 wins across all simulations for any initial advantage $y_{10} \geq 0.025$.

Figure 9 shows the corresponding results for the general case with heterogeneous competitors. Here,

Figure 8 Symmetric Competition with an Installed Base Advantage for Standard 1: Market Concentration in Remaining Market, Percent Times Standard 1 Wins, and Percent Profit Increase for Standard 1

Note. Parameters were reestimated for different consumer discount factors (β).

Figure 9 Competition with an Installed Base Advantage for Sony: Market Concentration in Remaining Market, Percent Times Sony Wins, and Percent Profit Increase for Sony



Note. Parameters were reestimated for different consumer discount factors (β).

we consider an advantage for Sony in terms of its initial installed base. For the three discount factors $\beta = 0.6, 0.7$, and 0.8 , we find a nonmonotonic relationship between the initial installed base share of Sony and the resulting concentration ratio in the remaining market. \bar{C}_1 decreases for small values of the initial advantage and then increases for larger values of the initial advantage. Across all simulations, Nintendo always wins the standards war until Sony's initial advantage reaches a certain threshold, after which Sony always wins. The reason for these findings relates again to the different sources of competitive advantage of Sony and Nintendo. Recall from §6 that for small values of the consumers' discount factor (i.e., $\beta < 0.9$), the present value of software is sufficiently low so that Nintendo "wins" because of its cost advantage if $y_0 = (0, 0)$. However, if Sony gains an initial advantage in the installed base, its platform becomes more attractive to consumers. For small initial advantages, Sony captures some of Nintendo's market share and the market concentration decreases. Once Sony's advantage is sufficiently large, it captures a large fraction of the market and the market concentration increases.

7.5. Quantifying the Value of Tipping

We now quantify the value of tipping for a firm. As we saw in §7.4, tipping can arise from an initial relative installed base advantage. To measure the value of tipping, we vary the magnitude of the initial advantage for a firm and examine how the initial advantage impacts on profits.

Figures 8 and 9 show the percent increase in the expected present discounted value of profits for a given initial installed base advantage. The percent increase is measured relative to the case of no initial advantage and is shown for standard 1 (Figure 8) and Sony (Figure 9). Present discounted profits are calculated over a time horizon of $T = 48$ months after the entry of the competitor and do not include profits (or losses) during the initial period when the incumbent enjoyed a monopoly position.

Even for fairly small initial advantages, the profit increase can be large. For example, in the symmetric case, a five-percentage-point installed base advantage translates into a profit increase of 16% for $\beta = 0.7$ and 47% for $\beta = 0.8$. In the general case, the corresponding profit increase is 16% ($\beta = 0.7$) and 43% ($\beta = 0.8$). For the general case, the predicted absolute increase in profits is shown in Table 5. For example, for an initial advantage of 10 percentage points, Sony's predicted profit increase over a four-year horizon is \$547 million for $\beta = 0.7$ and \$1,317 million for $\beta = 0.8$. These predicted profit numbers are not directly comparable to the actual realized profits of Sony in the U.S. market, as we hold the marginal production cost constant in

Table 5 Profit Increase for Installed Base Advantage

Installed base adv. of Sony	Discount factor (β)			
	0.6	0.7	0.8	0.9
0.025	70	134	370	808
0.050	139	271	732	1,142
0.075	207	410	1,052	1,271
0.100	274	547	1,317	1,381
0.125	339	680	1,529	1,470
0.150	403	807	1,711	1,541
0.175	464	922	1,857	1,589
0.200	523	1,030	1,985	1,617

Notes. This table shows the increase in the expected present discounted value of Sony's profits, measured in millions of dollars, for a given initial installed base advantage. The results are based on 5,000 simulations, and the present discounted value of profits is calculated for a time horizon of 48 months after the competitor (Nintendo) enters the market.

our simulations. Nonetheless, our results clearly indicate how tipping, arising from an initial advantage, can have a large impact on profits.

8. Conclusions

We provide a framework for studying the dynamics of hardware/software markets. The framework enables us to construct an empirically practical definition of tipping: the level of concentration relative to a counterfactual in which indirect network effects are reduced or eliminated. Computational results using this framework also provide several important insights into tipping. Using the demand parameters from the video game industry, we find that consumer expectations play an important role for tipping. In particular, tipping emerges as we strengthen the indirect network either by increasing the utility from software or by increasing the degree of consumer patience. In some instances, this can lead to an increase in market concentration by 24 percentage points or more. Interestingly, tipping is not a necessary outcome even if indirect network effects are present. For discount factors as high as 0.8, we observe market concentration at roughly the level that would emerge in the absence of any indirect network effects, provided that no standard has gained an initial installed base advantage. However, if one standard has gained such an initial advantage, market concentration arises as a result of positive feedback.

Studying other aspects of the equilibrium sheds some interesting managerial insights into the pricing and diffusion. In particular, strengthening the indirect network effect toughens price competition early on during the diffusion, leading firms to engage in penetration pricing (pricing below marginal cost) to invest in the growth of their networks. When tipping arises, the market diffuses relatively quickly. Thus, an interesting finding is that increasing consumer patience to

the point of tipping leads to a more rapid diffusion of consoles.

Our approach to measuring tipping and its role as a source of market concentration should be of interest to antitrust economists, academics, and practitioners. For policy workers, our counterfactual approach provides an important method for assessing damages to “bad acts” in markets with indirect network effects. Our results relating consumer and firm beliefs and patience to tipping should also be of interest to academics studying dynamic oligopoly outcomes in markets with durable goods, in particular with indirect network effects. Finally, the modeling framework constitutes a state-of-the-art quantitative paradigm for practitioners to assess the long-run market share of new durable goods, in particular those exhibiting network effects. Suppose one could estimate demand prior to a product’s launch, perhaps with a conjoint experiment. Our supply-side methodology could serve as a decision-support framework to predict how the market would unfold. The framework could assess the extent to which long-run profits and market shares will arise from marketing (e.g., product differentiation) versus natural market forces (e.g., network effects). It would also help predict whether the strength of differentiation and network effects are such that tipping per se could potentially arise.

Our main goal herein is to study the role of consumer beliefs and expectations for tipping, not to explain the empirical diffusion of video game consoles per se. Therefore, even though we calibrate the model with data from the 32/64-bit video game console market, we abstract from certain aspects of the industry. For instance, we do not account for declining production costs and persistent consumer heterogeneity when we simulate the market outcomes. We caution that our model predictions should not be seen as an attempt to “explain” directly the historical market outcome in the 32/64-bit video game console industry. Nevertheless, studying learning-by-doing on the supply side and consumer segmentation on the demand side are two interesting directions for future research in this area.

Another area for future research is the role of the game content for console adoption. We intentionally chose the 32/64-bit generation of consoles to allow us to work with a simpler model of the game side of the market. However, during subsequent generations, blockbuster games have become crucial for console adoption decisions. A very interesting direction for future research would be to extend the framework we provide herein to study the role of market power and dynamics on the software side of the model.

More recent generations of game consoles have become increasingly targeted (e.g., Nintendo Wii appeals to families, whereas Xbox 360 appeals more

narrowly to adult males), which could increase the incidence of households purchasing multiple consoles. According to Horwitz (2002), “nearly one in 10 current-system owners, and nearly one in five legacy-system owners, has two or more consoles from within their respective generations.” Here, “current-system” refers to Xbox, Nintendo GameCube, and PS2, whereas “legacy-system” refers to all previous generations including the one we use in the paper. Note that these numbers also include ownership of handheld game devices, which we consider to be a separate market from TV-based consoles. The numbers also include households who owned a console from the 16/32-bit generation and then upgraded to a newer-generation console. Therefore, we can think of 20% as a very conservative upper bound for multihoming in our data. Accounting for multihoming is the subject of new research studying later video game console generations (Lee 2009).

9. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://mktsci.pubs.informs.org>.

Acknowledgments

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Appendix A. Equilibrium Provision of Software

In this appendix, we illustrate how we can derive the hardware demand model based on tastes for variety of software. We use a CES model of preferences for software and assume a spot market of monopolistically competitive software suppliers.

After purchasing a hardware platform j , a consumer i purchases an assortment of compatible software each period, $x_{it} = (x_{i1t}, \dots, x_{injt})'$, by maximizing her software utility subject to the budget constraint:

$$\max_{\{x_1, \dots, x_{njt}\}} U_{ij}^{\text{SW}}(x_{i1}, \dots, x_{injt}, z_i)$$

$$\text{s.t. } \sum_{k=1}^{n_{jt}} \rho_k x_{ik} + z_i = I_i - p_{jt} Q_{ijt},$$

where p_{jt} is the price of hardware standard j in period t and Q_{ijt} indicates whether the consumer also purchases hardware that period (it is zero if they adopted hardware at some time period prior to t). The term z_i is a numéraire capturing expenditures on other goods, and ρ_k is the price of software k . We use CES preferences to model the consumer's utility for software:

$$U_{ij}^{\text{SW}}(x_{i1}, \dots, x_{in_{jt}}, z_i) = \left(\sum_{k=1}^{n_{jt}} x_{ik}^{1/b} \right)^{1/a} + \alpha z_i, \quad a \geq 1, b > 1.$$

The corresponding individual demand for software k is

$$x_{kt}^* = (ab\alpha)^{ab/(1-ab)} \rho_k^{b/(1-b)} \left(\sum_{l=1}^{n_{jt}} \rho_l^{1/(1-b)} \right)^{(ab-b)/(1-ab)}.$$

Turning to the software supply side, we assume that consumers derive utility from software for only one period. Hence, a software firm earns profits on a software product for only one period. Let y_{jt+1} represent the installed base of consumers that have adopted hardware standard j prior to period $(t+1)$. This installed base represents the potential demand at time t for a manufacturer of software compatible with hardware standard j . Each software title is treated as a separate firm. The profit function for a software firm k active in period t producing a software title compatible with hardware j is

$$\pi_{kt} = (\rho_k - c)y_{jt+1}x_{kt}^* - F,$$

where F is the fixed development cost and c is the marginal cost. The marginal costs consist of both royalties to the manufacturer and physical production costs (e.g., CDs and cartridges for Sony and Nintendo, respectively). Because software firms are assumed to be ex ante identical, there exists a symmetric price equilibrium in which each firm sets prices as follows:

$$\rho = \alpha c.$$

This symmetry also allows us to simplify the demand function:

$$x_{kt}^* = (ab\alpha\rho)^{ab/(1-ab)} n_{jt}^{(ab-b)/(1-ab)}.$$

Under free entry, the equilibrium number of software firms n_{jt} can be characterized by the installed base as follows:

$$\log(n_{jt}) = \kappa + \lambda \log(y_{jt+1}), \quad (\text{A1})$$

where

$$\begin{aligned} \kappa &= \frac{ab-1}{ab-b} \frac{\log(\rho-c)}{F(ab\alpha\rho)^{ab/(ab-1)}} \\ &= \frac{ab-1}{ab-b} \frac{\log(c(\alpha-1))}{F(ab\alpha^2c)^{ab/(ab-1)}} \quad \text{and} \quad \lambda = \frac{ab-1}{ab-b}. \end{aligned}$$

We now derive the aggregate sales of software for each standard. Total software sales will be important in determining the software royalties that accrue to each hardware

firm. We can substitute (A1) to express individual demand for software k as follows:

$$\begin{aligned} x_{kt}^* &= (ab\alpha\rho)^{ab/(1-ab)} n_{jt}^{(ab-b)/(1-ab)} \\ &= (ab\alpha^2c)^{ab/(1-ab)} (\exp(\kappa)y_{jt+1}^\lambda)^{-1/\lambda} \\ &= (ab\alpha^2c)^{ab/(1-ab)} \left(\exp\left(-\frac{\kappa}{\lambda}\right) \right) y_{jt+1}^{-1}. \end{aligned}$$

We then obtain the corresponding aggregate demand for software k :

$$\begin{aligned} X_{kt}^* &= x_{kt}^* y_{jt+1} \\ &= (ab\alpha^2c)^{ab/(1-ab)} \exp\left(-\frac{\kappa}{\lambda}\right). \end{aligned}$$

Finally, we obtain total software sales for the standard j :

$$\begin{aligned} Q_{jt} &= \sum_{k=1}^{n_{jt}} X_{kt}^* \\ &= n_{jt} \left[(ab\alpha^2c)^{ab/(1-ab)} \exp\left(-\frac{\kappa}{\lambda}\right) \right] \\ &= [\exp(\kappa)y_{jt+1}^\lambda] \left[(ab\alpha^2c)^{ab/(1-ab)} \exp\left(-\frac{\kappa}{\lambda}\right) \right] \\ &= \exp\left(\frac{\kappa(1-\lambda)}{\lambda}\right) (ab\alpha^2c)^{ab/(1-ab)} y_{jt+1}^\lambda. \end{aligned}$$

We can therefore estimate the elasticity of total software sales with respect to the installed base as follows:

$$\log(Q_{jt}) = \phi + \lambda \log(y_{jt+1}).$$

Appendix B. A Two-Step Estimator for the Demand Parameters

Stage 1. In the first stage, we estimate the consumer choice strategies along with the firms' pricing strategies and the software supply function. The supply function of software variety is specified as follows:

$$\log(n_{jt}) = \mathcal{H}_j(y_{j,t+1}; \theta_n) + \eta_{jt}, \quad (\text{A2})$$

where $\eta_{jt} \sim N(0, \sigma_\eta^2)$ captures random measurement error. The pricing strategies are specified as follows:

$$\log(p_{jt}) = \mathcal{P}_j(y_t, z_t^p; \theta_p) + \lambda \xi_{jt}, \quad (\text{A3})$$

where $\xi_{jt} \sim N(0, 1)$. In Equation (A3) we let \mathcal{P}_j be a flexible functional form of the state variables. For the empirical model, we include exogenous state variables z_t^p that are observed by console firms in addition to y_t and ξ_t , the state variables in the model of §3. These additional states are discussed in §4. In Equation (A3), we assume that the video game console manufacturers use only payoff-relevant information to set their prices. However, we do not assume that their pricing strategies are necessarily optimal. This specification has the advantage that it is consistent with the Bayesian Markov perfect equilibrium concept used in our model but does not explicitly impose it.

Conditional on the model parameters, there is a deterministic relationship between the price and installed base

Table B.1 Model Fit for Different Specifications

Model	Log-likelihood	BIC
1. Linear, ξ , 1-comp	−187.88	679.40
2. Linear, time ($t < 60$), 1-comp	−150.11	620.97
3. Quadratic, time ($t < 60$), 1-comp	−79.38	514.01
4. Quadratic, time ($t < 60$), 2-comp	−79.38	522.28
5. Quadratic, time ($t < 60$), 1-comp, PPIs in prices	−63.54	507.69
6. Quadratic, time ($t < 60$), 1-comp, exchange rate in prices	−25.43	422.94
7. Quadratic, time ($t < 60$), 1-comp, exchange rate and PPIs in prices	−7.53	412.79

Table B.2 First-Stage Estimates: Pricing Policies \mathcal{P}_j

	Sony		Nintendo	
	Estimate	Std. err.	Estimate	Std. err.
Intercept	2.653	1.523	−1.592	4.051
y_{Sony}	−3.266	1.785	−0.861	2.490
y_{N64}	1.889	1.499	0.666	0.813
y_{Sony}^2	−0.320	0.218	−0.142	0.282
y_{N64}^2	0.186	0.130	0.077	0.070
Time (<64)	0.005	0.003	0.000	0.002
January	0.102	0.040	−0.054	0.016
February	0.132	0.119	0.005	0.104
March	0.118	0.053	−0.082	0.214
April	0.136	0.057	−0.046	0.046
May	0.045	0.032	−0.055	0.037
June	−0.012	0.003	−0.011	0.028
July	0.000	0.026	−0.052	0.037
August	−0.001	0.012	−0.032	0.021
September	−0.049	0.074	−0.088	0.021
October	0.011	0.048	−0.015	0.039
November	−0.026	0.019	−0.002	0.025
PPI 1	−0.432	0.070	−0.285	0.154
PPI 2	−0.546	0.351	−0.608	0.816
PPI 3	−0.471	0.769	0.227	0.807
Exchange rate (3-month lag)	0.434	0.509	0.397	0.159
Exchange rate (7-month lag)	−4.493	1.118	1.390	8.096
log(λ)	−4.511	0.123	−4.511	0.123

data and the demand unobservable, $\xi_{jt} = \mathcal{X}_j(y_{jt}, p_{jt}, z_t^p)$.²⁴ Then, conditional on y_t and p_t , we can estimate the consumers' optimal choice strategy in log-odds:

$$\begin{aligned}\mu_{jt} &\equiv \log(s_{jt}) - \log(s_{0t}) \\ &= v_j(y_t, \xi_t, p_t, z_t^d) - v_0(y_t, \xi_t, z_t^d) + \zeta_{jt} \\ &= \mathcal{L}_j(y_t, \mathcal{X}(y_t, p_t, z_t^p), z_t^d; \theta_\mu) + \zeta_{jt},\end{aligned}\quad (\text{A4})$$

where $\zeta_{jt} \sim N(0, \sigma_\xi^2)$ is random measurement error and z_t^d denotes exogenous state variables observed by the consumer. By including the control function $\mathcal{X}(y_t, p_t, z_t^p)$ in the

²⁴ We can trivially invert ξ out of the price equation because of the additivity assumption in (A3). This is a stronger condition than in Bajari et al. (2007), but it is analogous to other previous work such as Petrin and Train (2010).

Table B.3 First-Stage Estimates: Log-Odds of Market Shares \mathcal{L}_j

	Sony		Nintendo	
	Estimate	Std. err.	Estimate	Std. err.
Intercept	−13.826	2.539	−1.141	0.744
y_{Sony}	1.456	4.669	0.065	0.012
y_{N64}	−7.670	5.884	−1.740	0.196
y_{Sony}^2	−0.334	0.432	−1.304	0.305
y_{N64}^2	−0.684	0.571	−1.820	0.335
Time (<64)	−0.003	0.008	−2.176	0.469
January	−1.367	0.154	−1.950	0.528
February	−1.345	0.159	−1.625	0.495
March	−1.813	0.475	−1.682	0.525
April	−2.442	0.351	−1.674	0.397
May	−2.596	0.463	−1.268	0.402
June	−1.947	0.395	−1.713	0.215
July	−1.805	0.470	−0.789	0.335
August	−1.871	0.392	−0.288	0.070
September	−1.496	0.426	0.085	0.135
October	−1.644	0.199	−0.406	0.150
November	−0.781	0.120	0.084	0.173
ξ_{Sony}	−27.656	5.216	0.103	0.061
ξ_{N64}	0.844	4.936	−0.220	0.083
ξ_{Sony}^2	−13.383	7.554	0.547	0.107
ξ_{N64}^2	−0.660	0.416	−0.545	0.119

Table B.4 First-Stage Estimates: Equilibrium Game Provision \mathcal{H}_j

	Estimate	Std. err.
Sony intercept	−16.220	2.042
Nintendo intercept	−24.349	1.992
y_{Sony}	1.369	0.126
y_{Nintendo}	1.810	0.126

demand equation, we also resolve any potential endogeneity bias that would arise as a result of the correlation between prices and demand shocks (this is the control function approach used in Petrin and Train 2006, 2010). We assume that a firm's price correlates only with its own demand shock, which is consistent with our modeling assumption that the current demand shock is private information to the firm. The static logit demand estimation literature typically allows for a more general covariance structure between prices and demand shocks. We view our covariance restriction as a reasonable trade-off for the the ability to model forward-looking consumer behavior. The first stage consists then of estimating the vector of parameters $\Theta = (\theta_\mu, \theta_p, \theta_\mu, \lambda)$ via maximum likelihood using the Equations (A2)–(A4).

Stage 2. In the second stage, we estimate the consumers' structural taste parameters, Λ , by constructing a minimum distance procedure that matches the simulated optimal choice rule for the consumers to the observed choices in the data. The idea is to use the estimated consumer choice strategies (A4) and the laws of motion for prices and software variety (A3) and (A2) to forward-simulate the consumers' choice-specific value functions $\mathcal{V}_j(y_t, \xi_t, p_t; \Lambda, \hat{\Theta})$

Figure B.1 In-Sample Fit: Prices

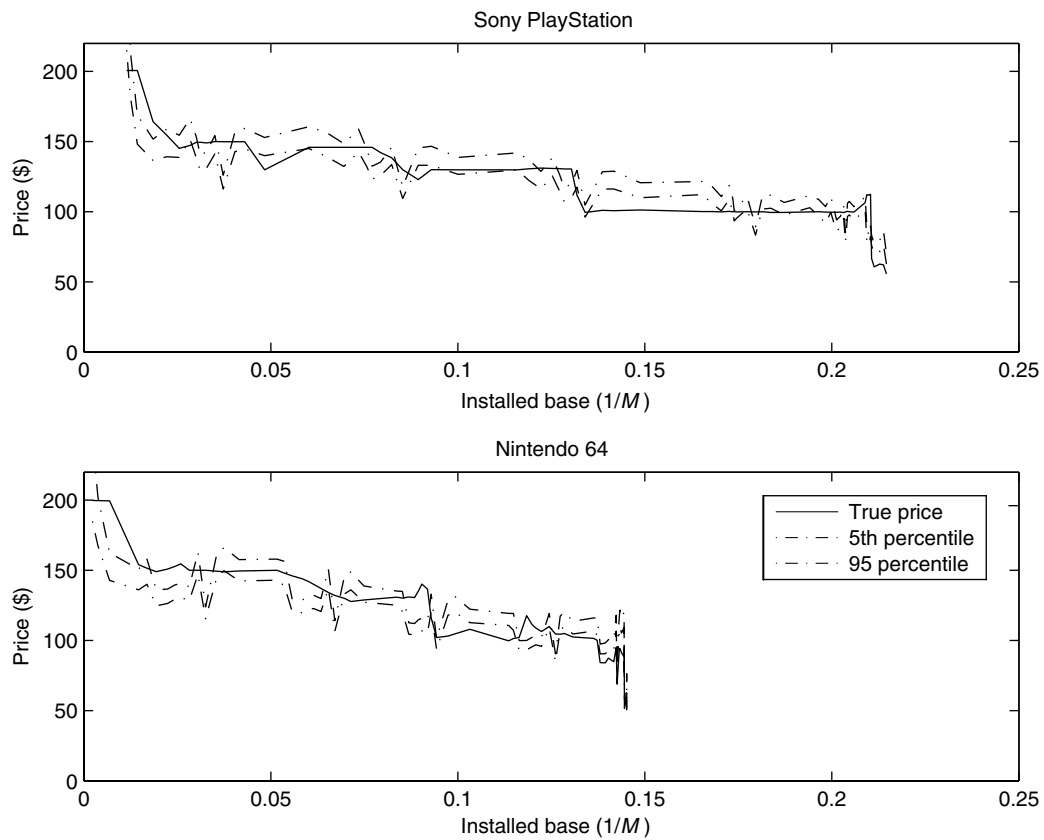


Figure B.2 In-Sample Fit: Log-Odds Ratios

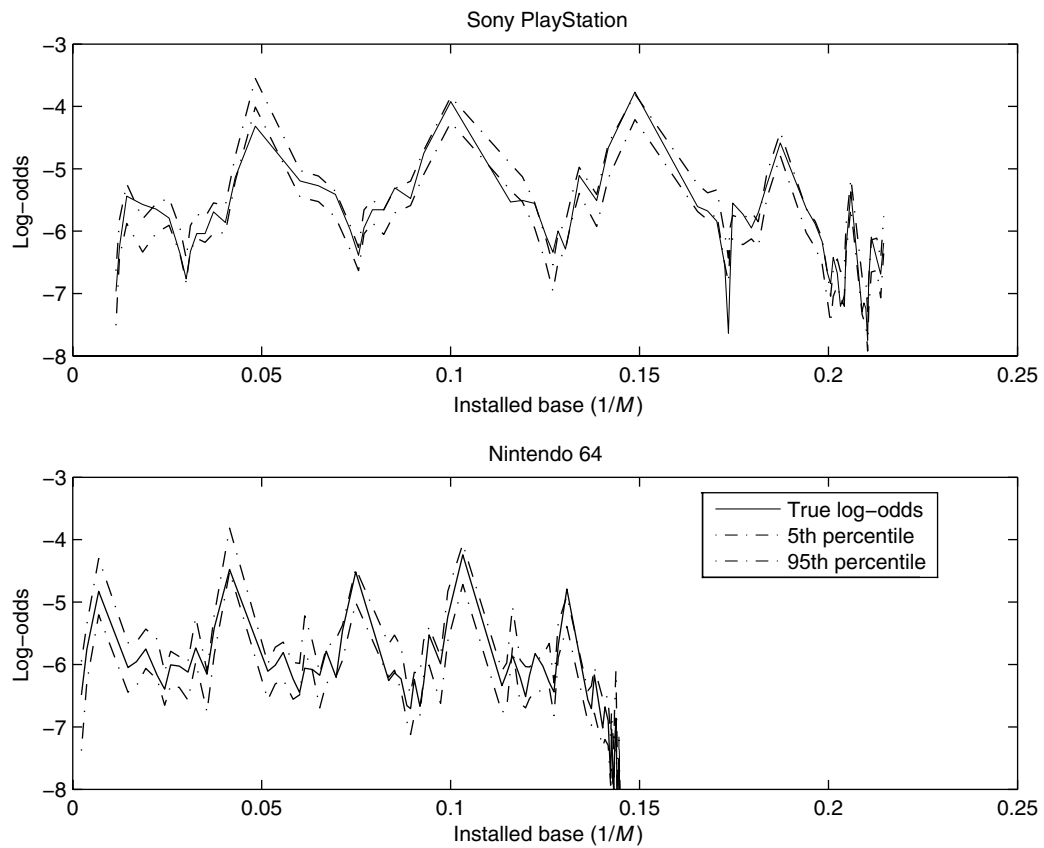
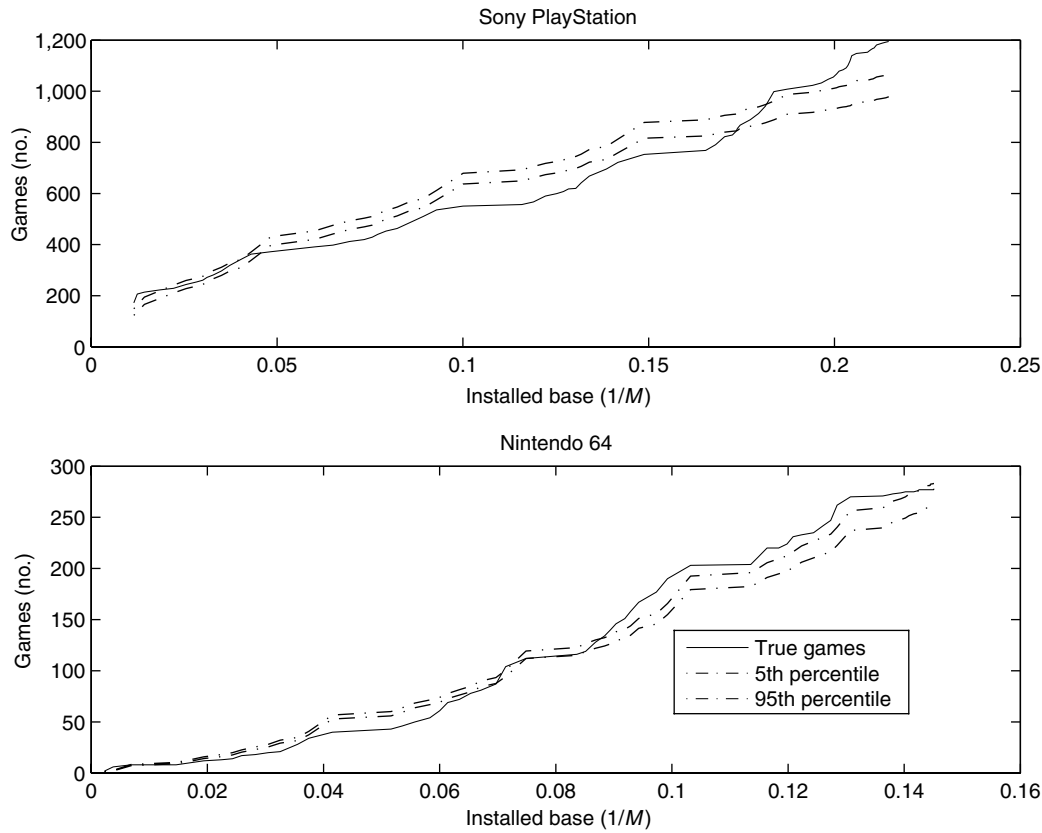


Figure B.3 In-Sample Fit: Provision of Games



and $\mathcal{V}_0(y, \xi; \Lambda, \hat{\Theta})$. The details for the forward simulation are provided in the following subsection. Note that although our two-step approach does not require us to assume that firms play the Markov perfect equilibrium strategies explicitly, we do need to assume that consumers maximize the net present value of their utilities.

The minimum distance procedure forces the following moment condition to hold approximately:

$$Q_{jt}(\Lambda_0, \hat{\Theta}) \equiv \mu_{jt} - (\mathcal{V}_j(y, \xi, p; \Lambda_0, \hat{\Theta}) - \mathcal{V}_0(y, \xi; \Lambda_0, \hat{\Theta})) = 0.$$

That is, at the true parameter values Λ_0 and given a consistent estimate of Θ , the simulated log-odds ratios should be approximately equal to the observed log-odds ratios for each of the observed states in the data. The minimum distance estimator Λ^{MD} is obtained by solving the following minimization problem:

$$\Lambda^{\text{MD}} = \min_{\Lambda} \{Q(\Lambda, \hat{\Theta})' W Q(\Lambda, \hat{\Theta})\},$$

where W is a positive semidefinite weight matrix.²⁵ Wooldridge (2002) shows that the minimum distance estimator has an asymptotically normal distribution with the covariance matrix

$$\begin{aligned} A\text{var}(\Lambda^{\text{MD}}) &= (\nabla_{\Lambda} Q' W \nabla_{\Lambda} Q)^{-1} \\ &\quad \cdot \nabla_{\Lambda} Q' W \nabla_{\Theta} Q \hat{\Omega} \nabla_{\Theta} Q' W \nabla_{\Lambda} Q (\nabla_{\Lambda} Q' W \nabla_{\Lambda} Q)^{-1}, \end{aligned}$$

where $\hat{\Omega} = A\text{var}(\hat{\Theta})$, and $\nabla_{\Lambda} Q$ and $\nabla_{\Theta} Q$ denote gradients of Q with respect to Λ and Θ , respectively.

The approach is closest to Pesendorfer and Schmidt-Dengler (2006, hereafter referred to as PS-D). However, our implementation differs in two ways. First, we examine a model with continuous states; PS-D look at a model with discrete states. Second, we adapt the approach to estimation of aggregate dynamic discrete choice demand, whereas PS-D focus on discrete choice at the individual level.

Forward Simulation of the Consumers' Choice-Specific Value Functions

We outline the procedure for using the first-stage estimates of the consumers' choice strategy (A4), the console firms' pricing strategies (A3), and the software supply (A2) to forward-simulate the consumers' choice-specific value functions.

Conditional on the first-stage estimates and some initial state y_0 , we can simulate histories of all variables affecting the consumers' payoffs. For any period t with beginning-of-period installed base y_t , we draw recursively as follows:

$$\begin{aligned} \xi_{jt} &\sim N(0, 1), \quad (j = 1, \dots, J) \\ p_{jt} | y_t, \xi_t &= \exp(\mathcal{P}_j(y_t; \hat{\theta}_p) + \hat{\lambda} \xi_{jt}), \end{aligned}$$

²⁵ We just set W equal to the identity matrix because it is unclear how to derive the efficient W in closed form for our specific problem.

$$\begin{aligned}\mu_{jt} | y_t, \xi_t &= \mathcal{L}_j(y_t, \xi_t; \hat{\theta}_\mu), \\ s_{jt} | \mu_t &= \frac{\exp(\mu_{jt})}{1 + \sum_{k=1}^J \exp(\mu_{kt})}, \\ y_{j,t+1} | y_t, s_t &= f_j(y_t, \xi_t) = y_{jt} + \left(1 - \sum_{k=1}^J y_{kt}\right) s_{jt}, \\ n_{jt} | y_{j,t+1} &= \exp(\mathcal{H}_j(y_{j,t+1}; \hat{\theta}_n)).\end{aligned}$$

In this manner, we can draw a sequence of states, $\{y_t, \xi_t\}_{t=0}^T$, and corresponding prices, number of software titles, and market shares.

Choice-specific value functions. We first compute the software value functions. We assume the current software utility is given by

$$u_j(y_{j,t+1}) = \gamma \exp(\mathcal{H}_j(y_{j,t+1}; \hat{\theta}_n)) = \gamma n_{jt}.$$

For any initial installed base y_0 , we draw a sequence of states $\{y_t, \xi_t\}_{t=0}^T$ and a sequence of corresponding software titles $\{n_t\}_{t=0}^T$. Repeating this process R times, we calculate the simulated expected present discounted value of software at state $y = y_0^{(r)}$:

$$\mathcal{W}_j(y; \Lambda, \hat{\Theta}) = \frac{1}{R} \sum_{r=1}^R \left(\sum_{t=0}^T \beta^t \gamma n_{jt}^{(r)} \right).$$

The consumers' choice-specific value functions from adopting standard j can then be calculated as

$$\mathcal{V}_j(y, \xi, p; \Lambda, \hat{\Theta}) = \delta_j + \mathcal{W}_j(f(y, \xi); \Lambda, \hat{\Theta}) - \alpha p_j + \psi \xi_j.$$

Here, $\Lambda = (\delta, \alpha, \gamma, \psi)$ is a vector containing all the stage 2 preference parameters to be estimated. Note that T needs to be chosen large enough such that β^T is sufficiently small.

Value of waiting. First, we define the expected per-period utility of a consumer who has not adopted at the beginning of period t , conditional on y_t, p_t , and ξ_t :

$$\begin{aligned}\mathcal{U}(y_t, \xi_t) &= s_{0t} \mathbb{E}(\epsilon_{0t} | 0) \\ &+ \sum_{j=1}^J s_{jt} (\delta_j + \gamma n_{jt} - \alpha p_{jt} + \xi_{jt} + \mathbb{E}(\epsilon_{jt} | j)).\end{aligned}$$

In this equation, s_t, p_t , and n_t are the choice probabilities, prices, and number of software titles as implied by the first-stage estimates, conditional on the current states y_t and ξ_t . Furthermore, $\mathbb{E}(\epsilon_{jt} | j) = -\log(s_{jt})$ is the expected value of the type I extreme value random utility component, given that choice j is optimal.

Next, we define m_{0t} as the probability that a consumer has not adopted one of the hardware standards prior to period t . Note that $m_{01} = 1$, because we want to calculate the value of waiting in period $t = 0$. Thereafter ($t > 1$), m_{0t} evolves according to

$$m_{0t} = s_{0,t-1} m_{0,t-1}.$$

m_{jt} denotes the probability that a consumer has adopted standard j prior to period t . $m_{j1} = 0$, and for $t > 1$,

$$m_{jt} = m_{j,t-1} + s_{j,t-1} m_{0,t-1}.$$

We now draw some sequence of states, $\{y_t^{(r)}, \xi_t^{(r)}\}_{t=0}^T$, with initial conditions $(y, \xi) = (y_0^{(r)}, \xi_0^{(r)})$. Given a corresponding sequence of $m_{0t}^{(r)}$ and $m_{jt}^{(r)}$, define

$$\mathcal{F}^{(r)} = \sum_{t=1}^T \beta^t \left(m_{0t}^{(r)} \mathcal{U}(y_t^{(r)}, \xi_t^{(r)}) + \sum_{j=1}^J m_{jt}^{(r)} (\gamma n_{jt}^{(r)}) \right).$$

$\mathcal{F}^{(r)}$ is the expected present discounted value from waiting, given that the market evolves according to $\{y_t^{(r)}, \xi_t^{(r)}\}_{t=0}^T$. Averaging over R draws, we obtain the expected value from waiting, conditional on $(y, \xi) = (y_0^{(r)}, \xi_0^{(r)})$:

$$\mathcal{V}_0(y, \xi; \Lambda, \hat{\Theta}) = \frac{1}{R} \sum_{r=1}^R \mathcal{F}^{(r)}.$$

Appendix C. Computational Details

To solve the model, we need to find choice-specific value functions, v_0, v_1, \dots, v_J , that satisfy the consumer optimality conditions (5) and (6). These choice-specific value functions depend on the consumers' expectations about the evolution of the state vector $f^e(y, \xi)$ and the firms' pricing policies $\sigma_j^e(y, \xi_j)$. We also need to find value functions for the firms, V_1, \dots, V_J , that satisfy the Bellman equations (9) and corresponding pricing strategies, $\sigma_j(y, \xi_j)$, $j = 1, \dots, J$. These pricing strategies depend on the firms' expectations about the consumers' adoption decisions, which are characterized by the choice-specific value functions, v_0, \dots, v_J , and on the firms' expectations of the pricing strategies of their competitors. In a Markov perfect Bayesian equilibrium, the decisions of the consumers and firms need to be optimal, and the expectations need to be mutually consistent.

To solve for an equilibrium, we adapt a policy iteration algorithm to the case of multiple decision makers (see Judd 1998 for a discussion of policy iteration and the survey by Doraszelski and Pakes 2007 for a discussion of numerical solution techniques for games). We start with some initial guess of the choice-specific value functions $v_0^{(k)}, \dots, v_J^{(k)}$, expectations about the state evolution $f^{(k)}(y, \xi)$, and price expectations $\sigma_j^{(k)}(y, \xi_j)$.

Step 1. Given $f^{(k)}(y, \xi)$, calculate the corresponding present discounted value of software for each standard j , $\omega_j^{(k)}(y)$, by iterating on the contraction mapping defined by Equation (4).

Step 2. Calculate the choice-specific value functions $v_1^{(k+1)}, \dots, v_J^{(k+1)}$ using Equation (5) and calculate the value of waiting $v_0^{(k+1)}$ from the right-hand side of the Bellman equation (6).

Step 3. Find new pricing policies, $\sigma_j^{(k+1)}$, by maximizing the right-hand side of the firms' Bellman equations (9). Here, we use the state transitions implied by Equation (8), which are based on the updated consumer value functions $v_0^{(k+1)}, \dots, v_J^{(k+1)}$.

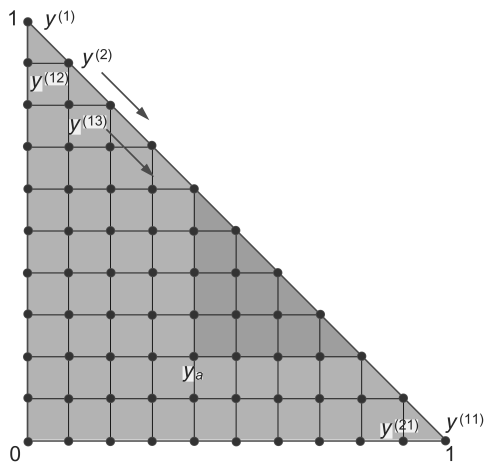
Step 4. Check if convergence has occurred:

$$\|v_j^{(k+1)} - v_j^{(k)}\| < \epsilon_v \quad \text{and} \quad \|\sigma_j^{(k+1)} - \sigma_j^{(k)}\| < \epsilon_\sigma.$$

If so, stop. Otherwise, proceed to the next step.

Step 5. Calculate new consumer expectations, $f^{(k+1)}(y, \xi)$, by substituting $p = \sigma^{(k+1)}(y, \xi)$ into the state transition Equation (8). Then return to 1.

Figure C.1 Illustration of the Model Solution Algorithm



We discretize the installed base part of the state space using a uniformly spaced grid, $\mathcal{Y} = \{y^{(i)}: 1 \leq i \leq N\}$, where $y_j^{(i)} \geq 0$ and $\sum_{j=1}^J y_j^{(i)} \leq 1$. All integrals describing the consumers' and firms' Bellman equations are numerically evaluated using Gauss-Hermite quadrature (see Judd 1998 for a discussion of numerical integration). Gauss-Hermite quadrature is based on a weighted average of the integrand evaluated at the quadrature nodes $\xi^{(1)}, \dots, \xi^{(K)}$. Correspondingly, we discretize the part of the state space corresponding to the demand shocks ξ using the set of quadrature nodes, $\mathcal{X} = \{\xi^{(1)}, \dots, \xi^{(K)}\}$. Outside the grid, the value functions and pricing policies are evaluated using interpolation in the y dimension. However, when we evaluate the integrals in the Bellman equations (6) and (9), we do not need to interpolate in the ξ dimension, because we directly represent the value functions and pricing policies on the quadrature nodes on which they need to be evaluated.

For some parameter values in the case of symmetric competition, as discussed in §7, there are asymmetric multiple equilibria. To calculate these equilibria, we first solve a slightly changed, asymmetric version of the game. In particular, we set the demand intercept for standard 2 to $\delta_2 - \Delta$ for some value $\Delta > 0$ and solve the corresponding game. We then use the equilibrium value functions and pricing policies of the asymmetric game as starting values for the symmetric game, which is solved using the original demand intercept δ_2 .

Unfortunately, unlike in the standard single-agent case, policy iteration algorithms do not necessarily converge in the case of games. Correspondingly, when updating the value functions, we often found oscillations. In these cases, we used a dampening scheme,

$$\sigma^{(k+1)}(y, \xi) \leftarrow (1 - \lambda)\sigma^{(k+1)}(y, \xi) + \lambda\sigma^{(k)}(y, \xi),$$

to achieve convergence, where λ was set very close to one. This problem was particularly pronounced in the presence of multiple equilibria. Future work using similar models may benefit from the use of homotopy methods (see Doraszelski and Pakes 2007). Given current software availability and computing speed, however, we do not believe that homotopy methods are yet practical for a model as complex as the one in our paper.

The evolution of the state vector in our model allows for a particular time-saving approach to solving for the equilibrium. Note that $y_t \leq y_{t+1}$. Hence, for example, starting at state y_a in Figure C.1, the value functions and pricing policies will only depend on the future value functions and pricing policies in the darkly shaded area of the state space. We can thus solve the game backwards. First, we move along the grid points on the diagonal boundary ($y_1 + y_2 = 1$) of the state space, corresponding to points $y^{(1)}, \dots, y^{(11)}$ in Figure C.1. Then, we move to the second diagonal of grid points and solve the game for points $y^{(12)}, \dots, y^{(21)}$, etc.

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