Empirical IO: Problem Set 4

Due date: Nov 13, 2020

Your answers should be produced in LaTeX, and should include all relevant graph and code. Code should be in the appropriate verbatim environment and properly documented.

Part 1: Computing the HZ Model

- Find (or write!) a underflow safe function that handles $\log sum \exp(\cdot)$ called **logsumexp**. A useful approximation is that $\log(\sum_i e^{x_i}) \approx \log(\sum_i e^{x_i - A}) + A$. A good choice for A is the maximum value $\max_i x_i = A$.
- Calculate (analytically) the gradient of the log-likelihood function in Rust with respect to the parameters of the model and write down the analytic results.

Part 2: Estimation MLE and MPEC

- Estimate the model using the NFXP approach of Rust. You will want to use the gradient.
  1. Compute the transition probabilities in a separate first stage – you should have 5 of them.
  2. Compute $EV(x, \theta)$ for a given guess of the parameters via the fixed point.
  3. Construct the CCP given your $EV(x, \theta)$
  4. Construct the likelihood and its gradient with respect to θ
- Estimate the model using the MPEC method of Su and Judd.
- Compare the results in a table, including the nonparametric answers below and discuss the results.
- Plot the $EV(\cdot)$ you have obtained for both estimators.

Part 3: The Stata Estimator

This is taken from Han Hong's problem set at Stanford, the idea is that we can use the arguments in Hotz-Miller (1993), or Pesendorfer Schmidt-Dengler (2008) to construct an optimization free method to recover the utility parameters in the Rust problem.

We began by defining the choice specific value function with ϵ_{it} i.i.d. and EV.

$$v(x,d) = u(x,d) + \beta \int \log \left(\sum_{d' \in D} \exp(v(x',d')) \right) p(x'|x,d) dx'$$

$$v(x,d) = u(x,d) + \beta \int \log \left(\sum_{d' \in D} \exp(v(x',d') - v(x',1)) \right) p(x'|x,d) dx' + \beta \int v(x,1) p(x'|x,1) dx'$$

1. Estimate $p(x'|x,d)$ non parametrically or parametrically (for example as a set of multinomial with n outcomes or an exponential distribution). Call your estimate $\hat{p}(x'|x,d)$.
2. Estimate $p(d|x)$ (the CCP) non-parametrically. You can use the binomial logit model with a basis function (increasing number of terms) or you can use a kernel such as **ksdensity** or **ecdf**.

3. Now use the Hotz-Miller inversion to estimate: $\hat{v}(x,d) - \hat{v}(x,1) = \log \hat{p}(d|x) - \log \hat{p}(1|x)$
4. Normalize $u(x,1) = 0$ and so for $= 1$ we have that

$$\begin{aligned} v(x,1) &= \beta \int v(x',1)p(x'|x,1)dx' + \beta \int \log\left(\sum_{d' \in D} \exp(\hat{v}(x',d') - \hat{v}(x',1))\right) \hat{p}(x'|x,1)dx' \\ &= \beta \int v(x',1)p(x'|x,1)dx' - \beta \int \log\left(\hat{p}(1|x')\right) \hat{p}(x'|x,1)dx' \end{aligned}$$

   This defines a fixed point that we can iterate on to obtain a nonparametric estimate of $\hat{v}(x,1)$. Add this to $\hat{v}(x,d) - \hat{v}(x,1)$ to recover the choice specific value functions for $d = 1, \ldots, D$.
5. Once we know $\hat{v}(x,d)$ for all $d \in D$ we can recover the nonparametric estimate of $u(x,d)$ for $d \geq 2$ by

$$\hat{u}(x,d) = \hat{v}(x,d) - \beta \int \log\left(\exp(\hat{v}(x',d'))\right) \hat{p}(x'|x,d)dx'$$

This estimator should be very simple to implement (and only requires one fixed point) so we could do inference via the bootstrap if we wanted to.