Empirical IO, Problem Set 2:
Logit, Nested Logit, and Random-Coefficient Logit

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Instructions As in your first problem set: when asked to describe an estimation algorithm, please provide enough detail so that an RA who knows Python/Matlab/R but has never taken any IO could use your description to write the estimation program. Actual estimation can be done in groups if you prefer. Please provide individual write-ups of your work, and note the members of your group. Attach a printout of your programs to your solutions.

Data Download "ps2_data.txt" from the course's web page. It is a simulated panel dataset that contains the following information:

Column	1	2	3	4	5	6	7	8	9	10
Variable	car id	year	firm id	price	quantity	weight	hp	ac	nest3	nest4

Market size Assume throughout that the market size M is equal to 100 million in each year, which is approximately the number of households in the US.

Outside good Let the share of the outside good vary across the three years, but keep M fixed throughout.

Normalization and instruments Estimation procedures can have problems when they contain variables that differ by several orders of magnitude. To avoid problems you should normalize the covariates you use. You will also need to create instruments. For all estimation procedures in this problem set, create the instruments from the original values, and then normalize the continuous covariates before performing the regression. Do not normalize dummy variables, constants, the log of within group share (in the nested logit problem), the left-hand side variables (functions of deltas or shares), or the instruments that you create. In other words, you only need to normalize weight, horsepower, and price.

Now suppose that the model is a logit model. The difference between the vertical model and this logit model is that the utility error is now i.i.d. extreme value and $\alpha_i = \alpha$ for all i, i.e.:

$$\boldsymbol{u_{ijt} = \delta^*_{jt} + \epsilon_{ijt}} \quad \text{and} \quad \boldsymbol{\delta^*_{jt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt}} \quad \boldsymbol{\forall\, t \in T}$$

1. Beginning with the utility function, derive market share as a function of δ^*s. Then invert the equation to solve for δ^* as a function of shares. Allow each year to be separately estimated.
2. If instead, I asked you to pool the data and estimate a single model, what would change in your derivation?
3. Estimate the demand system parameters using GMM with just the demand-side moment conditions. Since price is endogenous, you will need to use at least one instrument. Construct the BLP instruments (characteristics of competing products **in a given year**) analogously to the vertical demand problem set.
4. Explain why own and cross price elasticites from this logit model may be unrealistic.

Now use a nested logit model with a single level of nesting.

5. (a) Write down market share for a single product as a function of the vector $\boldsymbol{\delta}^*$ and the nesting parameter σ. Use the σ and group notation used by Berry (1994 Rand), not the λ notation used by Train and McFadden.

   (b) Invert that equation to solve for δ^*_j as a function of market shares, within group shares, and σ.

   (c) Finally, use this second equation to write a regression equation with an observed quantity on the left-hand-side and observed variables and coefficients (including σ) on the right-hand-side.

6. (a) Estimate the demand system parameters using just the demand-side moment conditions for this nested-logit model. Instrument for price using the average of product characteristics (i.e. weight, hp, ac) of products produced by other firms **in a given year**. Similarly, construct the instrumental variables by using the average of product characteristics of other products (including those of the same firm) within the same group in a given year. For your nests, use two different nesting structures: one that groups products based on the "nest3" variable, and one that groups products based on the "nest4" variable. (Implicitly, the outside good is in its own additional group in each year.) Calculate 2SLS estimates, allowing all coefficients to vary across the three years of data, and use them as starting values for the optimization in your GMM routine. Report your results for $n = 3$ (inside) groups and $n = 4$ (inside) groups.

   (b) Suppose instead that we estimated a version of a nested-logit model that pooled all three years' worth of data. What assumption on intertemporal substitution patterns is implicit in this choice?

   (c) Are your estimates of sigma sensitive to the number of groups? Can you give an explanation for this result?

(d) How does your estimate of α change across the three years' of data?

7. (a) How is nested logit an improvement over plain logit?

   (b) Think of another nesting structure you would use, and explain what additional data you would need to estimate it.

   (c) Why might all forms of nested logit be problematic?

8. Suppose we were interested in improving the substitution patterns.

   (a) Would the Multinomial Probit model be appealing in this setting? Why, or why not?

   (b) Would the Pure Characteristics model be appealing in this setting? Why, or why not?

Now assume that the model is a logit model but each individual has a different price coefficient, i.e. $u_{ijt} = x_{jt}\beta - \alpha_i p_{jt} + \xi_{jt} + \epsilon_{ijt}$

9. (a) Suppose $\alpha_i = 1/y_i$ and y_i is distributed lognormally. Write out the moment conditions and estimation algorithm you would use to estimate this model.

   (b) Now suppose $\alpha_i = \alpha_1 + \alpha_2/y_i$ and y_i is still distributed lognormally. How exactly would this change the estimation? Write out the moment conditions and estimation algorithm you would use to estimate β in this model. Are the parameters of the lognormal distribution, α_1 and α_2 identified by the data provided to you?

10. (a) Among the other parameters you would have estimated in question 9a are the mean and the variance of the lognormal distribution. Now assume that you knew that the mean of income was $\$35,000$ and that the standard deviation is $\$45,000$. Using only the demand system, estimate the β parameters under these assumptions. Continue to use the moment conditions involving the excluded instruments you used for plain logit.

    (b) Are the own and cross price elasticities from this system more realistic than those in the plain and nested logit models, and if so why? One way to evaluate this is to produce the J matrix of diversion ratios, as well as reporting the median own price elasticity.

    (c) Harder: Can you estimate a model with $\alpha_i = \alpha_0 + \frac{\alpha_1}{y_i} + \alpha_2 \nu_i$ where ν_i is standard normal? I recommend using a small number of quadrature points for ν_i.