Investigating Experimental Data Using Linear Regression

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Research Question for Today

- Does class size reduction benefit students educational attainment?

- Observational data: not controlled by researcher
    - Grade 3 classroom sizes and corresponding EQAO scores
    - Correlation between class size and test score not causal

- Experimental data: component(s) manipulated by researcher
    - Randomly assign students to varying class sizes

Data Description

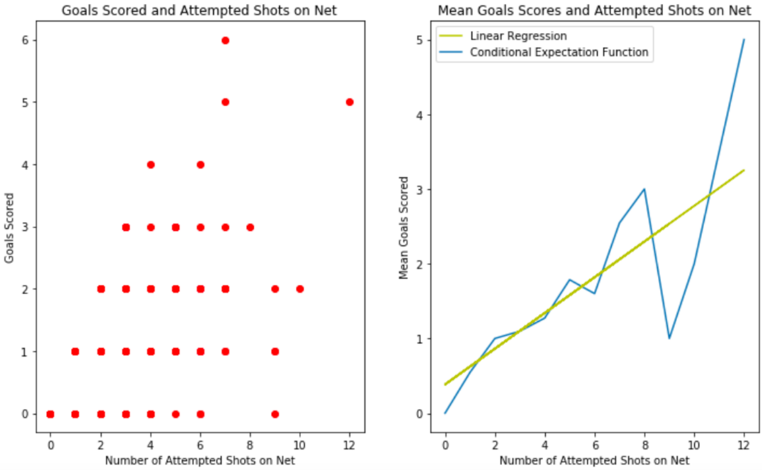
- Primary outcomes are math and reading test scores
  - $Score_i$ = test score (out of 100) for student i
- Tennessee 1985: Student/Teacher Achievement Ratio (STAR)
- About 6000 students in 70 schools were randomly assigned into small (13-17) and large (22-25) classes in kindergarten
- Roughly 350 teachers were also randomly assigned to either small or large classes

Introduction to Hypothesis Testing

- Parameters describe features about the population of interest
  - Mean parameter denoted by μ (typically unknown)
- Samples from the population used to infer parameters
  - Sample mean $\overline{Y}$ used to infer μ
- Hypothesis is a statement about population parameters
  - $H_0 : \mu_{small} - \mu_{big} = 0$ (Null), $H_1 : \mu_{small} - \mu_{big} \neq 0$ (Alternate)
  - pvalue < 0.05 is evidence beyond reasonable doubt to reject H_0

Introduction to Linear Regression

- Regression estimates the impact of the variation in X (features) on the central tendency of Y (outcome)
- Linear regression: $E(Y_i|X_i) = \beta_0 + \beta_1 X_i$



Class Size and Achievement

Main Effect

$$SmallClass_i = \text{I(student i in small class)} =$$

$$\begin{cases} 1 & \text{student i in small class} \\ 0 & \text{student i in big class} \end{cases}$$

- $E(Score_i | SmallClass_i) = \beta_0 + \beta_1 SmallClass_i$
  - $E(Score_i | SmallClass_i = 0) = \beta_0$
  - $E(Score_i | SmallClass_i = 1) = \beta_0 + \beta_1$
- $\beta_1 = E(Score_i | SmallClass_i = 1) - E(Score_i | SmallClass_i = 0)$

Class Size and Math Achievement Results

Main Effect

- $E(Score_i | SmallClass_i) = \beta_0 + \beta_1 SmallClass_i$
  - $\widehat{\beta_0} = \overline{Score}_{SmallClass=0} = 72.2$ (pvalue ≈ 0)
  - $\widehat{\beta_1} = \overline{Score}_{SmallClass=1} - \overline{Score}_{SmallClass=0} = 4.4$ (pvalue ≈ 0)
- Students in small classrooms obtain 4.4 percentage points higher math score on average relative to the larger classrooms

Teacher Experience and Achievement

Main Effect

- $ExpTeacher_i = \mathrm{I}(\text{student } i\text{'s teacher experience} > \text{median})$
  - Median teacher experience in data is 9 years

- $E(Score_i | ExpTeacher_i) = \alpha_0 + \alpha_1 ExpTeacher_i$
  - $E(Score_i | ExpTeacher_i = 0) = \alpha_0$
  - $E(Score_i | ExpTeacher_i = 1) = \alpha_0 + \alpha_1$

- $\alpha_1 = E(Score_i | ExpTeacher_i = 1) - E(Score_i | ExpTeacher_i = 0)$

Teacher Experience and Math Achievement Result

Main Effect

- $E(Score_i | ExpTeacher_i) = \alpha_0 + \alpha_1 ExpTeacher_i$
  - $\widehat{\alpha_0} = \overline{Score}_{ExpTeacher=0} = 71.6$ (pvalue ≈ 0)
  - $\widehat{\alpha_1} = \overline{Score}_{ExpTeacher=1} - \overline{Score}_{ExpTeacher=0} = 3.6$ (pvalue ≈ 0)
- Assigned to an experienced teacher raises students test score on average by 3.6 percentage points relative to newer teachers

Class Size Effects Depends on Teacher Experience

Interaction Effect

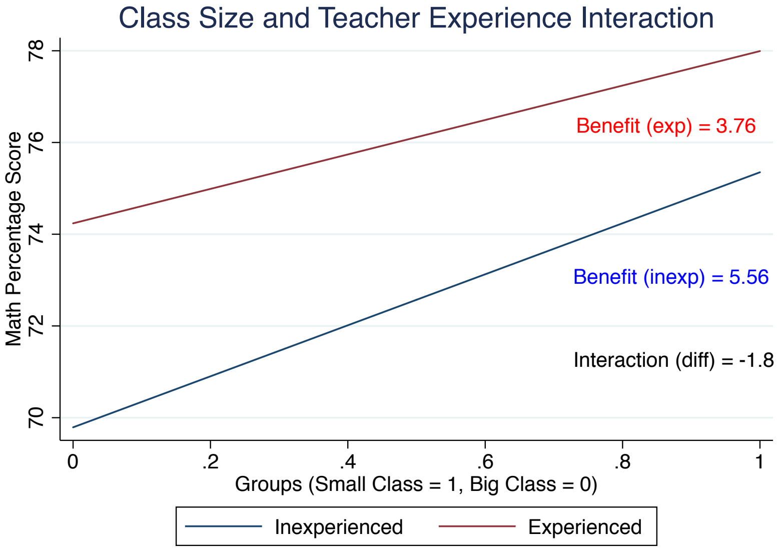
- $E(Score_i|SmallClass_i, ExpTeacher_i) = \theta_0 + \theta_1 SmallClass_i + \theta_2 ExpTeacher_i + \theta_3 SmallClass_i \times ExpTeacher_i$
    1. $E(Score_i|SmallClass_i = 1, ExpTeacher_i = 1) = \theta_0 + \theta_1 + \theta_2 + \theta_3$
    2. $E(Score_i|SmallClass_i = 0, ExpTeacher_i = 1) = \theta_0 + \theta_2$
    3. $E(Score_i|SmallClass_i = 1, ExpTeacher_i = 0) = \theta_0 + \theta_1$
    4. $E(Score_i|SmallClass_i = 0, ExpTeacher_i = 0) = \theta_0$
- Interaction effect: $\theta_3 = [(1) - (2)] - [(3) - (4)]$

Class Size Effects Depends on Teacher Experience Results

Interaction Effect

- $E(Score_i | SmallClass_i, ExpTeacher_i) = \theta_0 + \theta_1 SmallClass_i + \theta_2 ExpTeacher_i + \theta_3 SmallClass_i \times ExpTeacher_i$
- $\widehat{\theta}_3 = \left[\overline{Y}_{S=1,ET=1} - \overline{Y}_{S=0,ET=1}\right] - \left[\overline{Y}_{S=1,ET=0} - \overline{Y}_{S=0,ET=0}\right]$
- $\widehat{\theta}_3 = -1.8$ (pvalue ≈ 0)
- Less experienced teachers have an 1.8 percentage point higher benefit on average from having a smaller class relative to experienced teachers

Visualizing The Interaction Effect



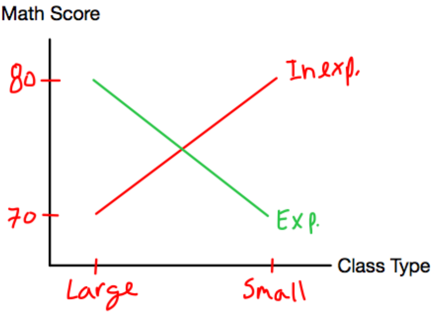
Summary of Regression Results

Effects of Class Size Reduction and Teacher Experience

	(1) Math	(2) Math	(3) Math
I(small class)	4.396***		5.561***
I(experienced teacher)		3.607***	4.447***
I(small class)×I(experienced teacher)			-1.806***
Adjusted R^2	.06	.048	.117
No. observations	5871	5850	5850

*** (pvalue $<$ 0.01), ** (pvalue $<$ 0.05), and * (pvalue $<$ 0.1)

Interaction Effects Practice



Predict the sign of the parameter estimates:

- $E(Score_i | SmallClass_i) = \beta_0 + \beta_1 SmallClass_i$ (Ans. B0 > 0, B1 = 0)
- $E(Score_i | ExpTeacher_i) = \alpha_0 + \alpha_1 ExpTeacher_i$ (Ans. a0 > 0, a1 = 0)
- $E(Score_i | SmallClass_i, ExpTeacher_i) = \theta_0 + \theta_1 SmallClass_i + \theta_2 ExpTeacher_i + \theta_3 SmallClass_i \times ExpTeacher_i$ (Ans. T0 > 0, T3 < 0)

Extensions To Consider

Homework: See jupyter notebook

- Heterogeneous class size effects (depend on context):
  - $E(Score_i | SmallClass_i, Male_i) = \tau_0 + \tau_1 SmallClass_i + \tau_2 Male_i + \tau_3 SmallClass_i \times Male_i$
- Three class size groups $\implies$ 3x2 factorial design
  - Class: {small, big, big + teacher aide}, experience: {below median, above median}
- Use non-cognitive outcomes
  - Motivation and self-concept