Midterm

MACS 40000, Dr. Evans
Wenesday, Oct. 25, 1:30pm

1. **2-period-lived agent model (15 points).** In this problem, I want you to solve the characterizing equations of the 2-period-lived agent model with endogenous labor, which is just a small version of the model in Chapter 7. Let the lifetime optimization problem of each individual in the economy be to choose labor supply $\{n_{1,t}, n_{2,t+1}\}$ and savings $b_{2,t+1}$ to maximize lifetime utility subject to a budget constraint in every period.

$$\max_{n_1,n_2,b_2} U\bigl(c_{1,t}, n_{1,t}\bigr) + \beta U\bigl(c_{2,t+1}, n_{2,t+1}\bigr)$$
$$\text{s.t.} \quad c_{1,t} + b_{2,t+1} = w_t n_{1,t}$$
$$\text{and} \quad c_{2,t+1} = (1 + r_{t+1})b_{2,t+1} + w_{t+1}n_{2,t+1}$$

And let the utility function in every period be the following,

$$U(c_{s,t}, n_{s,t}) = \frac{\bigl(c_{s,t}\bigr)^{1-\sigma} - 1}{1-\sigma} + \chi_s \frac{\bigl(1 - n_{s,t}\bigr)^{1-\sigma} - 1}{1-\sigma}$$

where $\sigma > 1$ is the coefficient of relative risk aversion and $\chi_s > 0$ is an age-specific level parameter influencing the relative disutility of labor.
Assume that a representative firm in this economy produced output Y_t every period using aggregate capital K_t and aggregate labor L_t according to the following Cobb-Douglas production function.

$$Y_t = A\bigl(K_t\bigr)^\alpha \bigl(L_t\bigr)^{1-\alpha} \quad \text{for} \quad A > 0 \quad \text{and} \quad \alpha \in (0, 1)$$

Let the profits of a representative firm be given by revenues minus costs each period,

$$PR_t = Y_t - (r_t + \delta)K_t - w_t L_t$$

where $\delta \in [0, 1]$ is the rate of depreciation, r_t is the interest rate, and w_t is the wage.

   (a) Solve for, show your work, and write down the three Euler equations that characterize the three lifetime decisions of an agent in this problem. Write these functions in terms of labor supply, savings, interest rates, wages, and parameters.

   (b) If the representative firm's production and profit functions are given above, solve for, show your work, and write down the two first order conditions that characterize the optimal capital stock that a firm wants to rent and the optimal labor that the firm wants to hire.

   (c) Write down the market clearing condition for capital and the market clearing condition for labor in this market. (Ignore goods market clearing.)

2. **Feasibility (5 points).** In the two-period-lived agent problem above, suppose that the wage in every period is $w_t = 1.5$ and the interest rate in every period is $r_t = 0.1$ for all t and the time endowment to each agent each period is $\tilde{l} = 1$. Is the following household decision—$n_{1,t} = 0.2$, $n_{2,t+1} = 0.15$, and $b_{2,t+1} = 0.31$—for a household born in period t feasible? If not, what constraints does it break? (HINT: Make sure to check individual constraints on consumption and labor supply as well as any aggregate constraints. This is not a steady-state problem.)

3. **Root finder in Python (10 points).** Assume that an individual in the third period $s = 3$ of their 10-period life $S = 10$in period-t has CRRA utility of leisure and must decide how much to work $n_{3,t}$. [Note that I have made a slight adjustment to the exponent of the marginal utility of leisure so that it is not the same as the exponent in the marginal utility of consumption.] The Euler equation governing this decision is the following.

$$w_t\Big([1+r_t]b_{3,t} + w_t n_{3,t} - b_{4,t+1}\Big)^{-\sigma} = \chi_3\big(1 - n_{3,t}\big)^{-\sigma-0.1}$$

   Solve for the optimal labor supply $n_{3,t}$ when $w_t = 1.0$, $r_t = 0.1$, $b_{3,t} = 1.0$, $b_{4,t+1} = 1.1$, $\sigma = 2.2$, and $\chi_3 = 2.0$. Report both your solution for $n_{3,t}$ and the value of the Euler error (in the difference form, not ratio form).

4. **Minimizer in Python (10 points).** Let the marginal disutility of labor $v'(n)$ for the constant relative risk aversion functional form be given by the following expression.

$$v'_{crra}(n) = (1-n)^{-\sigma}, \quad \text{for} \quad \sigma \geq 1 \quad \text{and} \quad n \in [0,1)$$

   Let the marginal disutility of labor $v'(n)$ for the elliptical functional form be given by the following expression.

$$v'_{elp}(n) = b(n)^{\upsilon-1}\left[1 - n^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}, \quad \text{for} \quad b,\upsilon > 0 \quad \text{and} \quad n \in [0,1)$$

   (a) Assume that the coefficient of constant relative risk aversion in $v'_{crra}(n)$ is $\sigma = 2.2$. Using 1,000 evenly spaced points from the support of labor between $n = 0.05$ and $n = 0.95$, estimate the two elliptical disutility of labor parameters $b, \upsilon > 0$ that minimize the sum of squared deviations between the two marginal disutility of labor functions $v'_{crra}(n)$ and $v'_{elp}(n)$ for those 1,000 points. Report your estimates. [HINT: You may want to plot your estimated $v'_{elp}(n)$ function against the $v'_{crra}(n)$ function to make sure you have a good fit.]

5. **Plotting in Python (10 points).** Let the marginal utility of consumption for a constant relative risk aversion utility function be given by $u'(c) = c^{-2.2}$. A line that intersects the marginal utility curve at $c = 0.5$ and has the same slope as that curve is given by the equation $\tilde{MU} = -20.217c + 14.7033$, where $\tilde{MU}$ represents an alternative linear marginal utility of consumption. Plot the CRRA marginal utility $u'(c) = c^{-2.2}$ for 100 evenly spaced points of consumption between $c = 0.3$ and $c = 3.0$. Also plot in the same figure the alternative linear marginal utility of consumption $\tilde{MU} = -20.217c + 14.7033$ for 100 evenly spaced points between $c = -0.1$ and $c = 0.5$. Make sure that consumption c is on the x-axis, marginal utility is on the y-axis, your axes are labeled correctly, your figure includes a legend for the two curves, and your figure has a title.