Problem Set #9

MACS 40000, Dr. Evans
Due Wednesday, Nov. 29 at 1:30pm

1. **Tax functions of total income (5 points).** For these exercises, you will use the `tax_data.pkl` file. This file is a Python pickle of a Pandas DataFrame of 3,013 observations of 8 variables. You can load this file into memory by using the following commands.

```
import pickle

tax_data = pickle.load(open('tax_data.pkl', 'rb'))
```

   Each observation represents a synthetic tax filer with 8 tax rate, income and population weight characteristics.

   (a) Create a scatterplot of `Total Income` on the x-axis and `Effective Tax Rate` on the y-axis. Use a weighted least squares criterion to estimate a linear model of ETR as a function of $TotInc$ (Use the `Weight` variable from `tax_data`).

$$ETR_i = \beta_0 + \beta_1 TotInc_i + \varepsilon_i$$

   Plot your predicted ETRs as a line on your scatterplot of the data. Label the line in a legend.

   (b) Use a weighted least squares criterion to estimate a Gouveia and Strauss (1994) functional form for the ETR as a function of $TotInc$.

$$ETR_i = \left(\frac{\psi_0}{TotInc_i}\right)\left(TotInc_i - \left[(TotInc_i)^{-\psi_1} + \psi_2\right]^{-\frac{1}{\psi_1}}\right) + \varepsilon_i$$

   Plot your predicted ETRs as a curve on your scatterplot of the data and including your linear predictions from part (a). Label the curves in a legend.

   (c) Estimate a DeBacker, et al (2017) functional form for the ETR as a function of $TotInc$.

$$ETR_i = (max - min)\left(\frac{A(TotInc_i)^2 + B(TotInc_i)}{A(TotInc_i)^2 + B(TotInc_i) + 1}\right) + min + \varepsilon_i$$

   Estimate max and min as the respective maximum and minimum values of the ETR_i data. Then estimate parameters A and B given max and min using a weighted least squares criterion. Plot your predicted ETRs as a curve on your scatterplot of the data and including your predictions from parts (a) and (b). Label the curves in a legend.

2. **Tax rates as functions of labor income and capital income separately (5 points).**

   (a) Create a 3D scatterplot of `Total Labor Income` on the x-axis, `Total Capital Income` on the y-axis, and `Effective Tax Rate` on the z-axis.

   (b) Estimate a DeBacker, et al (2017) functional form for the ETR as a function of labor income x and capital income y.

$$
\begin{aligned}
ETR_i = &\Big[\tau(x_i) + shift_x\Big]^{\phi}\Big[\tau(y_i) + shift_y\Big]^{1-\phi} + shift \\
\text{where} \quad &\tau(x_i) \equiv (max_x - min_x)\left(\frac{Ax_i^2 + Bx_i}{Ax_i^2 + Bx_i + 1}\right) + min_x \\
\text{and} \quad &\tau(y_i) \equiv (max_y - min_y)\left(\frac{Cy_i^2 + Dy_i}{Cy_i^2 + Dy_i + 1}\right) + min_y \\
\text{where} \quad &A, B, C, D, max_x, max_y, shift_x, shift_y > 0 \quad \text{and} \quad \phi \in [0, 1] \\
\text{and} \quad &max_x > min_x \quad \text{and} \quad max_y > min_y
\end{aligned}
$$

   Estimate max_x and min_x as the respective maximum and minimum values of the ETR_i when capital income is less than $y_i < \$30,000$. Estimate max_y and min_y as the respective maximum and minimum values of the ETR_i when labor income is less than $x_i < \$30,000$. Estimate $shift_x = -min_x + 0.001$, $shift_y = -min_y + 0.001$, and $shift$ equal the minimum ETR_i in the data. Then estimate parameters A, B, C, D, and ϕ given max_x, min_x, max_y, min_y, $shift_x$, $shift_y$ and $shift$ using a weighted least squares criterion. Plot your predicted ETRs as a 3D surface curve on your scatterplot of the data.