

# Part 8: Program Evaluation (c)

## Local Average Treatment Effects

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Applied Econometrics

## What about IV?

Recall the simple IV estimator with  $\beta_i = \beta$ :

$$Y_i(0) = \beta_0 + u_i(0)$$

$$Y_i(1) = \beta_0 + \beta_1 T_i + u_i(1)$$

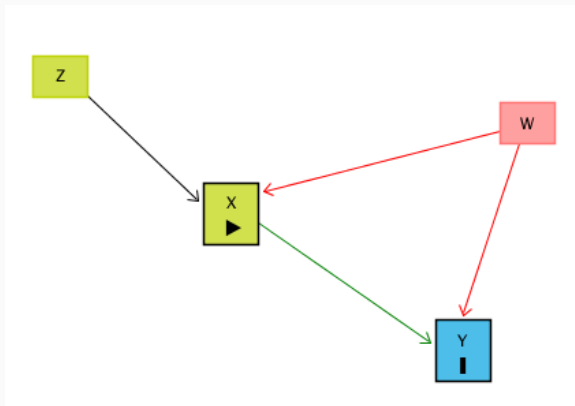
$$Y_i = Y_{i0} + \beta_1 T_i + \underbrace{[u_i(1) - u_i(0)]}_{\eta_i}$$

We are interested in the **treatment effect**  $\beta_1 = Y_i(1) - Y_i(0)$

- But  $E[\eta_i|T_i] \neq 0$  because of **selection problem**
- IV  $Z_i$  gives us  $\beta_1 = \frac{Cov(Y_i, Z_i)}{Cov(T_i, Z_i)}$  if  $Z_i$  is “excluded”...
- But what does that mean?

## What about IV?

- Note the absence of direct path between  $z \rightarrow y$ . This is the **exclusion restriction**.
- The fact that  $z \rightarrow x$  is the **relevance** condition.



## So what does IV do?

Now let's think about **heterogeneous treatment effects**  $\beta_i = Y_i(1) - Y_i(0)$

- Let's assume a binary instrument  $Z_i = 1$
- $Y_i(1), Y_i(0)$  depends on the value of  $T_i$
- But now we allow  $T_i(1), T_i(0)$  where the argument is the value of  $Z_i$
- We observe  $\{Z_i, T_i = T_i(Z_i), Y_i = Y_i(T_i(Z_i))\}$ .
- "Reduced form" is regression of  $y_i \sim z_i$  or  $E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]$   
(Nothing about  $T_i$ !)

## IV Assumptions

**Independence:**  $Z_i \perp Y_i(1), Y_i(0), T_i(1), T_i(0)$ . Instrument is as if randomly assigned and does not directly affect  $Y_i$

- $T_i = T_i(0) + (T_i(1) - T_i(0))Z_i$
- Change in treatment status  $(T_i(1) - T_i(0))$  is the causal effect of  $Z_i$  on  $T_i$
- Under independence, the **first stage** is the average causal effect of  $Z_i$  on  $T_i$

$$\begin{aligned} E[T_i|Z_i = 1] - E[T_i|Z_i = 0] &= E[T_i(1)|Z_i = 1] - E[T_i(0)|Z_i = 0] \\ &= E[T_i(1) - T_i(0)] \end{aligned}$$

- This is not implied by random assignment. In that case there would be four potential outcomes  $Y_i(z, t)$ .

## IV Assumptions

**Independence**  $Z_i \perp Y_i(1), Y_i(0), T_i(1), T_i(0)$ .

Instrument is as if randomly assigned and does not directly affect  $Y_i$

**Random Assignment**  $Z_i \perp Y_i(0, 0), Y_i(0, 1), Y_i(1, 0), Y_i(1, 1), T_i(1), T_i(0)$ .

**Exclusion Restriction**  $Y_i(z, t) = Y_i(z', t)$  for all  $z, z', t$ .

We require **both RA and ER** to guarantee **Independence**.

The second assumption is a substantive one.

## IV Assumptions

- Consider four possible pairs of  $(t_i, z_i)$
- Let  $T_i(z_i)$  denote how treatment status responds to the instrument:

$T_i(1)$	$T_i(0)$	
	0	1
0	never-taker	defier
1	complier	always-taker

## IV Assumptions

We are stuck without further assumptions, so we assume:

**Monotonicity/No Defiers**  $T_i(1) \geq T_i(0)$

- Works in many applications (classical drug compliance).
- Implied by many latent index models with constant coefficients
- Works as long as sign of  $\pi_{1,i}$  doesn't change

$$T_i(z) = 1[\pi_0 + \pi_1 z + \varepsilon_i > 0]$$



## IV Assumptions

Monotonicity rules out defiers

$T_i(1)$	$T_i(0)$	
	0	1
0	never-taker	defier
1	complier	always-taker

The compliers are the only group we learn about with the IV estimator.

## IV Assumptions (Updated)

Compliance Types by Treatment and Instrument ( $t_i, z_i$ )

Monotonicity rules out defiers

$T_i$	$Z_i$	
	0	1
0	complier/never-taker	never-taker/defier
1	always-taker/defier	complier/always-taker

$$\{\text{treated}\} = \{\text{always-takers}\} + \{\text{compliers assigned } z_i = 1\}$$

$$\{\text{control}\} = \{\text{never-takers}\} + \{\text{compliers assigned } z_i = 0\}$$

- We can derive the expression for  $\beta_{IV}$  as the **Wald Estimator**:

$$\beta_{IV} = \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[T_i|Z_i = 1] - E[T_i|Z_i = 0]} = E[Y_i(1) - Y_i(0)|\text{complier}]$$

- We can derive the expression for  $\pi_c$  (the fraction of compliers):

$$\pi_c = E[T_i|Z_i = 1] - E[T_i|Z_i = 0]$$

- Proof see Angrist and Imbens

# Is this what we wanted?

- We learn about the average treatment effect for the group of compliers
- Different people comply with different instruments
  - Imagine two ways to increase college attendance: merit scholarships and financial aid
  - Different instruments trace out different groups of compliers
- Is this what we want? Probably not.
- Maybe it isn't that far away...?

## How Close to ATE?

Angrist and Imbens give some idea how close to the ATE the LATE is:

- $E[Y_i(0)|\text{never-taker}]$  and  $E[Y_i(1)|\text{always-taker}]$  can be estimated from the data
- Compare these to their respective compliers  $E[Y_i(0)|\text{complier}]$ ,  $E[Y_i(1)|\text{complier}]$ .
- When these are close then possibly  $ATE \approx LATE$ .

## How Close to ATE?

Angrist and Imbens give some idea how close to the ATE the LATE is:

$$\widehat{\beta}_1^{TSLS} \rightarrow^p \frac{E[\beta_{1i}\pi_{1i}]}{E[\pi_{1i}]} = LATE$$
$$LATE = ATE + \frac{Cov(\beta_{1i}, \pi_{1i})}{E[\pi_{1i}]}$$

- Weighted average for people with large  $\pi_{1i}$ .
- Late is treatment effect for those whose probability of treatment is most influenced by  $Z_i$ .
- If you always (never) get treated you don't show up in LATE.

## How Close to ATE?

- With different instruments you get different  $\pi_{1i}$  and TSLS estimators!
- Even with two valid  $Z_1, Z_2$ 
  - Can be influential for different members of the population.
  - Using  $Z_1$ , TSLS will estimate the treatment effect for people whose probability of treatment  $X$  is most influenced by  $Z_1$
  - The LATE for  $Z_1$  might differ from the LATE for  $Z_2$
  - A J-statistic might reject even if both  $Z_1$  and  $Z_2$  are exogenous! (Why?).

## Example: Cardiac Catheterization

- $Y_i$  = survival time (days) for AMI patients
- $X_i$  = whether patient received cardiac catheterization (or not) (intensive treatment)
- $Z_i$  = differential distance to CC hospital

$$SurvivalDays_i = \beta_0 + \beta_{1i}CardCath_i + u_i$$

$$CardCath_i = \pi_0 + \pi_{1i}Distance_i + v_i$$

- For whom does distance have the great effect on probability of treatment?
- For those patients what is their  $\beta_{1i}$ ?



## Example: Cardiac Catheterization

- IV estimates causal effect for patients whose value of  $X_i$  is most heavily influenced by  $Z_i$ 
  - Patients with small positive benefit from CC in the expert judgement of EMT will receive CC if trip to CC hospital is short (**compliers**)
  - Patients that need CC to survive will always get it (**always-takers**)
  - Patients for which CC would be unnecessarily risky or harmful will not receive it (**never-takers**)
  - Patients for who would have gotten CC if they lived further from CC hospital (hopefully don't see) (**defiers**)
- We mostly weight towards the people with small positive benefits.

## Local Average Treatment Effect

So how is this useful?

- It shows why IV can be meaningless when effects are heterogeneous.
- It shows that if the monotonicity assumption can be justified, IV estimates the effect for a particular subset of the population.
- In general the estimates are specific to that instrument and are not generalisable to other contexts.
- As an example consider two alternative policies that can increase participation in higher education.
  - Free tuition is randomly allocated to young people to attend college ( $Z_1 = 1$  means that the subsidy is available).
  - The possibility of a competitive scholarship is available for free tuition ( $Z_1 = 1$  means that the individual is allowed to compete for the scholarship).

## Local Average Treatment Effect

- Suppose the aim is to use these two policies to estimate the returns to college education. In this case, the pair  $\{Y^1, Y^0\}$  are log earnings, the treatment is going to college, and the instrument is one of the two randomly allocated programs.
- First, we need to assume that no one who intended to go to college will be discouraged from doing so as a result of the policy (monotonicity).
- This could fail as a result of a General Equilibrium response of the policy; for example, if it is perceived that the returns to college decline as a result of the increased supply, those with better outside opportunities may drop out.

## Local Average Treatment Effect

- Now compare the two instruments.
- The subsidy is likely to draw poorer liquidity constrained students into college but not necessarily those with the highest returns.
- The scholarship is likely to draw in the best students, who may also have higher returns.
- It is not a priori possible to believe that the two policies will identify the same parameter, or that one experiment will allow us to learn about the returns for a broader/different group of individuals.

## A Classic Example

- What is the effect of prices on quantity demanded?
- But a regression of  $Q_t = \beta_0 + \beta_1 P_T + u_t$  is going to be flawed.
- For one thing, how do we know that relationship represents supply or demand?
- Imagine an instrument  $Z_t \in \{0, 1\}$  that reduces supply but does not effect demand.
- What about?
  - Monotonicity?
  - Heterogeneity in treatemnt effects?
  - Exclusion Restriction?

# LATE at the Fulton Fish Market (Graddy 1995)

Table 2

**Ordinary Least Squares and Instrumental Variable Estimates of Demand Functions with Stormy Weather as an Instrument**

Variable	Ordinary least squares (dependent variable: log quantity)		Instrumental variable	
	(1)	(2)	(3)	(4)
Log price	-0.54 (0.18)	-0.54 (0.18)	-1.08 (0.48)	-1.22 (0.55)
Monday		0.03 (0.21)		-0.03 (0.17)
Tuesday		-0.49 (0.20)		-0.53 (0.18)
Wednesday		-0.54 (0.21)		0.58 (0.20)
Thursday		0.09 (0.20)		0.12 (0.18)
Weather on shore		-0.06 (0.13)		0.07 (0.16)
Rain on shore		0.07 (0.18)		0.07 (0.16)
$R^2$	0.08	0.23		
No. of Obs.	111	111	111	111

Source: The data used in these regressions are available by contacting the author.

Note: Standard errors are reported in parentheses.

## Local Average Treatment Effect

Finally, we need to understand what monotonicity means in terms of restrictions on economic theory.

- To quote from Vytlacil (2002) Econometrica:  
“The LATE assumptions are not weaker than the assumptions of a latent index model, but instead impose the same restrictions on the counterfactual data as the classical selection model if one does not impose parametric functional form or distributional assumptions on the latter.”
- This is important because it shows that the LATE assumptions are equivalent to whatever economic modeling assumptions are required to justify the standard Heckman selection model and has no claim to greater generality.
- On the other hand there are no magical solutions to identifying effects when endogeneity/selection is present; this problem is exacerbated when the effects are heterogeneous and individuals select into treatment on the basis of the returns.