Econometrics 2

Exam

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- Answer **all** of the following exercises in either German or English.
- Explain your answers and derivations. All your computations and intermediate steps need to be verifiable and understandable.
- Formulas which we covered in the lecture and class need not to be derived again.
- If you prefer a notation different from the one used in the course, define it.
- Always use significance level $a = 5\%$.
- Please report 3 decimal places in numerical answers.
- If not otherwise stated, assume the validity of the assumption A, B and C given in the lecture.
- Permissible aids:
  - non-programmable pocket calculator
  - cheat sheet: one-sided A4 white sheet of paper with annotations, formulas, texts, sketches, etc.

1 Understanding

(a) The standard output of an OLS estimation yields a Durbin-Watson statistic of 0.32. What does this imply?

(b) The estimation of a dynamic model yields

$$y_t = 0.6 y_{t-1} + 0.3 x_t + \hat{u}_t$$

where y_t and x_t are **both measured in logs**. Compute the dynamic effect (multiplier) of a 1% increase in x on y (i) in the same period (ii) in the next period and (iii) in the long run.

(c) List the assumptions on the error term one needs to show that the covariance matrix of the OLS estimator $\hat{\beta}$ is given by $\sigma^2(X'X)^{-1}$?

2 Violation of B1

Assume that the true model is given by

$$y_t^* = \alpha + \beta x_t + u_t$$

where $u_t \sim N(0, \sigma_u^2)$ with $\sigma_u^2 > 0$. However, y_t^* is only observable with a measurement error

$$y_t^* = y_t + v_t$$

where $v_t \sim N(0, \sigma_v^2)$ with $\sigma_v^2 > 0$ and $cov(u_t, v_s) = 0$ for all t and s. The observed model is hence given by

$$y_t = \alpha + \beta x_t + \underbrace{u_t - v_t}_{\varepsilon_t}$$

(a) Compute the (i) expectation, (ii) variance and (iii) autocorrelation function of ε_t.

(b) Compute the bias of the OLS estimators for α and β.

(c) What happens to the standard errors of the OLS estimators for α and β? Are hypothesis tests and interval estimators still valid?

(d) Provide intuition whether or not the OLS estimator remains consistent. A formal proof is not necessary.

3 Testing violations

The demand for money is determined using the regression model

$$m_t = \alpha + \beta_1 y_t + \beta_2 i_t + u_t$$

where

$$\begin{aligned} m_t &: \text{real money stock in logs} \\ y_t &: \text{real income in logs} \\ i_t &: \text{interest rate in \%} \end{aligned}$$

Given quarterly data for the period 1970-1996 ($T = 108$), a least squares estimation shows that

$$\sum_{t=1}^{T} \hat{u}_t^2 = 108$$

A separate estimation for the period 1970-1989 (80 observations) yields an unbiased estimate for σ^2 of 0.02, whereas considering the period 1990-1996 (28 observations) yields an unbiased estimate for σ^2 of 2. Test the following null hypotheses:

(a) The variance σ^2 did not change after the reunification 1990.

(b) The coefficients of the regression model did not change after the reunification 1990.

4 Instrument variables

The aim is to investigate the influence of education, *educ*, on a person's salary, *wage*, by using the following regression model

$$\log(wage_t) = \beta_0 + \beta_1\, educ_t + u_t$$

Since it is assumed that *educ* is an endogenous variable, the geographical distance of the person's place of residence to the nearest university, $nearc4$, and the number of years of the father's education, $fatheduc$, are used as instrumental variables. The **IV estimation** yields the following result

Model: IV Estimation, observations 1–2320
Endogenous Variable: log(wage)
Instruments: const nearc4 fatheduc
Instruments used for: educ

	Coefficient	Std. error	t statistic	p-value
const	5,3155	0,0966	55,0198	0,0000
educ	0,0715	0,0071	10,0722	0,0000

(a) What is an endogenous regressor? What is a relevant and valid instrument?

(b) An OLS estimation of

$$educ_t = \pi_0 + \pi_1 nearc4_t + \pi_2 fatheduc_t + v_t$$

yields

Model: OLS, observations 1–2320
Endogenous Variable: educ

	Coefficient	Std. error	t-statistic	p-value
const	10,0549	0,1465	68,6545	0,0000
nearc4	0,3388	0,1038	3,2653	0,0011
fatheduc	0,3269	0,0129	25,3125	0,0000

$F(2, 2317) = 344{,}2389$, p-value(F) = 1,3e–131

Are the instruments $nearc4$ and $fatheduc$ relevant?

5 Stochastic convergence

Suppose that the infinite sequence $Y_1, Y_2, \ldots$ is an $AR(1)$ process, i.e.

$$Y_t - \mu = \rho\,(Y_{t-1} - \mu) + \varepsilon_t$$

where $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$ and $|\rho| < 1$. Furthermore, let $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} Y_t$ denote the sample mean.
Hint: From the lecture we know that Y_t is stationary, so that $E(Y_t) = \mu$ and $Var(Y_t) = \sigma_\varepsilon^2/(1-\rho^2)$ for all $t = 1, ..., T$.

(a) Show that $\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \varepsilon_t \xrightarrow{d} U_\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

(b) Show that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \varepsilon_t = \sqrt{T}\left[(1-\rho)\,(\hat{\mu} - \mu) + \rho\left(\frac{Y_T - Y_0}{T}\right)\right]$$

Hint: $\frac{1}{T}\sum_{t=1}^{T} Y_{t-1} = \left(\frac{1}{T}\sum_{t=1}^{T} Y_t\right) - \frac{1}{T}(Y_T - Y_0) = \hat{\mu} - \frac{Y_T - Y_0}{T}$.

(c) Assume that

$$\underset{T\to\infty}{plim}\left[\frac{\rho}{1-\rho}\left(\frac{Y_T - Y_0}{\sqrt{T}}\right)\right] = 0$$

Put the results of (b) and (c) together and show that

$$Z_T = \sqrt{T}\frac{\hat{\mu} - \mu}{\sigma_Z} \xrightarrow{d} U \sim N(0,1) \tag{1}$$

for $\sigma_Z = \sqrt{\sigma_\varepsilon^2/(1-\rho)^2}$.
Hint: Start with (c) and divide by $(1-\rho)$, then use (b) to derive the asymptotic distribution.

(d) Briefly describe the intuition and result of the usual Central Limit Theorem for the sample mean of iid random variables. Why does it not hold for the $AR(1)$ process?
*Hint: Note that $\sigma_Z = \sqrt{\sigma_\varepsilon^2/(1-\rho)^2}$ **is not equal to** $\sqrt{Var(Y_t)} = \sqrt{\sigma_\varepsilon^2/(1-\rho^2)}$.*

Table of the quantiles of the F_{ν_1,ν_2}-distribution, given are the 0.95 -quantiles (i.e. $a = 0.05$)

	ν_1										
ν_2	1	2	3	4	5	10	15	20	25	50	∞
1	161.45	199.50	215.71	224.58	230.16	241.88	245.95	248.01	249.26	251.77	254
2	18.51	19.00	19.16	19.25	19.30	19.40	19.43	19.45	19.46	19.48	19.5
3	10.13	9.55	9.28	9.12	9.01	8.79	8.70	8.66	8.63	8.58	8.53
4	7.71	6.94	6.59	6.39	6.26	5.96	5.86	5.80	5.77	5.70	5.63
5	6.61	5.79	5.41	5.19	5.05	4.74	4.62	4.56	4.52	4.44	4.37
6	5.99	5.14	4.76	4.53	4.39	4.06	3.94	3.87	3.83	3.75	3.67
7	5.59	4.74	4.35	4.12	3.97	3.64	3.51	3.44	3.40	3.32	3.23
8	5.32	4.46	4.07	3.84	3.69	3.35	3.22	3.15	3.11	3.02	2.93
9	5.12	4.26	3.86	3.63	3.48	3.14	3.01	2.94	2.89	2.80	2.71
10	4.96	4.10	3.71	3.48	3.33	2.98	2.85	2.77	2.73	2.64	2.54
15	4.54	3.68	3.29	3.06	2.90	2.54	2.40	2.33	2.28	2.18	2.07
20	4.35	3.49	3.10	2.87	2.71	2.35	2.20	2.12	2.07	1.97	1.84
25	4.24	3.39	2.99	2.76	2.60	2.24	2.09	2.01	1.96	1.84	1.71
30	4.17	3.32	2.92	2.69	2.53	2.16	2.01	1.93	1.88	1.76	1.62
35	4.12	3.27	2.87	2.64	2.49	2.11	1.96	1.88	1.82	1.70	1.56
40	4.08	3.23	2.84	2.61	2.45	2.08	1.92	1.84	1.78	1.66	1.51
45	4.06	3.20	2.81	2.58	2.42	2.05	1.89	1.81	1.75	1.63	1.47
50	4.03	3.18	2.79	2.56	2.40	2.03	1.87	1.78	1.73	1.60	1.44
60	4.00	3.15	2.76	2.53	2.37	1.99	1.84	1.75	1.69	1.56	1.39
70	3.98	3.13	2.74	2.50	2.35	1.97	1.81	1.72	1.66	1.53	1.35
80	3.96	3.11	2.72	2.49	2.33	1.95	1.79	1.70	1.64	1.51	1.32
90	3.95	3.10	2.71	2.47	2.32	1.94	1.78	1.69	1.63	1.49	1.30
100	3.94	3.09	2.70	2.46	2.31	1.93	1.77	1.68	1.62	1.48	1.28
∞	3.84	3.00	2.60	2.37	2.21	1.83	1.67	1.57	1.49	1.35	1.01