

*E*conometrics  
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Philipp Eisenhauer

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# Introduction to the Econometrics of Policy Evaluation

Philipp Eisenhauer

# **Introduction**

**J. Heckman (2008) defines three policy evaluation tasks:**

- ▶ Evaluating the impact of historical interventions on outcomes including their impact in terms of well-being of the treated and the society at large.
- ▶ Forecasting the impact of historical interventions implemented in one environment in other environments, including their impact in terms of well-being.
- ▶ Forecasting the impacts of interventions never historically experienced to various environments, including their impact on well-being.

## **Econometrics of policy evaluation**

- ▶ is important
- ▶ is complicated
- ▶ is multifaceted

## **Numerous applications**

- ▶ labor economics
- ▶ development economics
- ▶ industrial economics
- ▶ health economics

## **Numerous effects**

- ▶ conventional average effects
- ▶ policy-relevant average effects
- ▶ marginal effects
- ▶ distributional effects
- ▶ effects on distributions

## **Numerous estimation strategies**

- ▶ instrumental variables
- ▶ (quasi-)experimental methods
- ▶ matching

## Generalized Roy model

Potential Outcomes      Observed Outcome

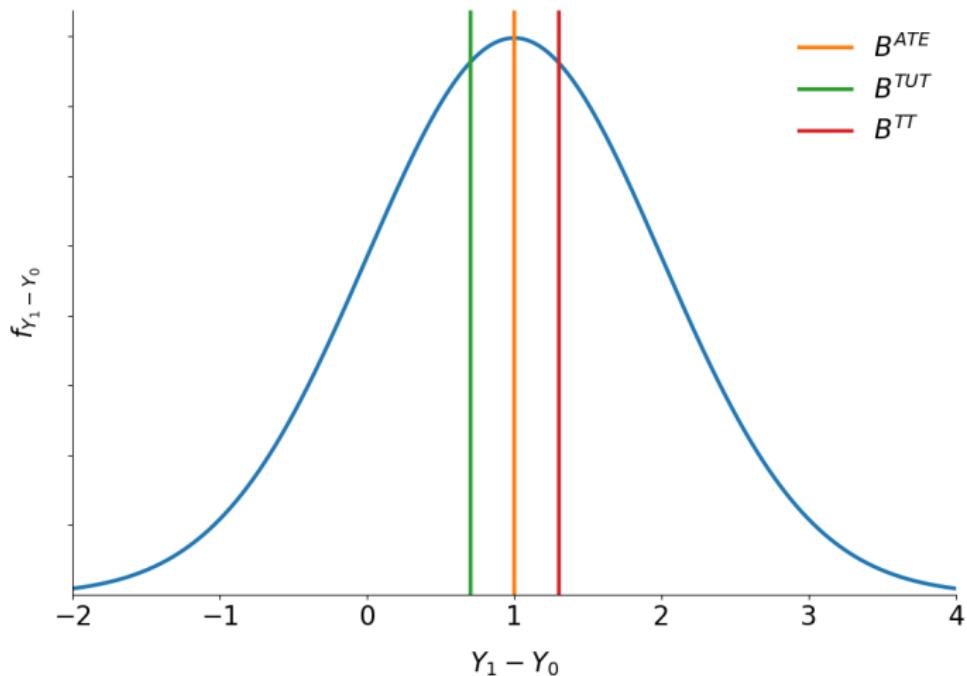
$$Y_1 = \mu_1(X) + U_1 \qquad Y = DY_1 + (1 - D)Y_0$$

$$Y_0 = \mu_0(X) + U_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

Figure: Treatment Effects



# Teaching Tool

The screenshot shows the homepage of the grmpy documentation. At the top left is the GitHub logo with the text "grmpy" and "latest". Below it is a search bar labeled "Search docs". To the right is a navigation menu with links to "Economics", "Installation", "Tutorial", "Reliability", "Software Engineering", "Contributing", "Contact and Credits", "Changes", and "Bibliography". In the center is a large button with the Elastic logo and the text "Try the official hosted Elasticsearch. Latest version, feature-loaded, always.". At the bottom left are links to "Read the Docs" and "v: latest".

Docs » Welcome to grmpy's documentation!

[Edit on GitHub](#)

## Welcome to grmpy's documentation!

[PyPI](#) | [GitHub](#) | [Issues](#)

**grmpy** is an open-source Python package for the simulation and estimation of generalized Roy Model (Heckman & Vytlacil, 2005 [11]). Its main purpose is to serve as a teaching tool to promote the conceptual framework provided by the generalized Roy model which allows to illustrate a variety of issues in the econometrics of policy evaluation.

license: [MIT License](#)

### Contents:

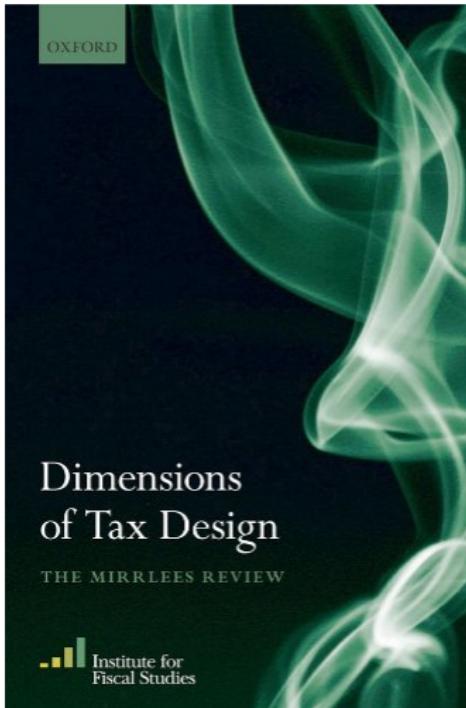
- [Economics](#)
- [Installation](#)
- [Tutorial](#)
- [Reliability](#)
- [Software Engineering](#)
- [Contributing](#)
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- [Changes](#)
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## **The Mirrlees Review**

*The Mirrlees Review brought together a high-profile group of international experts and early career researchers to identify the characteristics of a good tax system for any open developed economy in the 21st century, assess the extent to which the UK tax system conforms to these ideals, and recommend how it might realistically be reformed in that direction.*

Figure: The Mirrlees Review



## ► **Taxation of Earnings**

- A single integrated benefit should be introduced to replace all or most of the current multiplicity of benefits, rationalising the way in which total support varies with income and other characteristics.

## ► **Indirect Taxes**

- VAT should be extended to nearly all spending. This would reduce complexity and avoid costly distortions to consumption choices.

## ► **Environmental Taxes**

- We should work towards a comprehensive system of congestion charging on the roads, replacing most of fuel duty.

## ► **Taxes on Saving**

- The risk-free return to saving should not be taxed, so that saving is not discouraged.

## ► **Business Taxes**

- The tax treatment of employment, self-employment and corporate source income should be aligned.

# **Appendix**

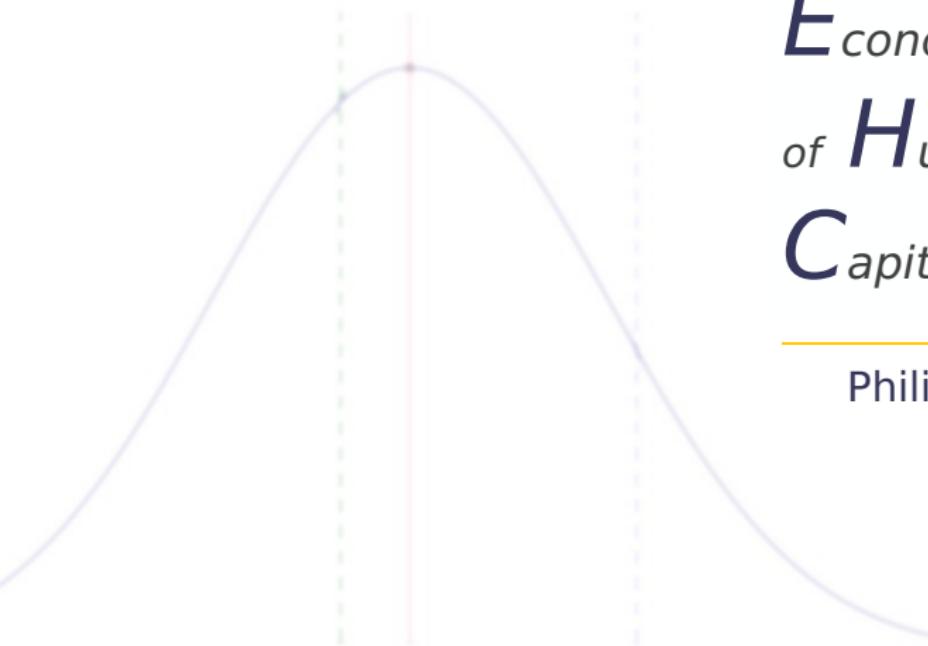
## *References*

- Adam, S., Besley, T., Blundell, R., Bond, S., Chote, R., Gammie, M., ... Poterba, J. (2010). *Dimensions of tax design: The mirrlees review* (O. U. Press, Ed.). Institute for Fiscal Studies (ifs).
- Cho, S., & Rust, J. (2010). The flat rental puzzle. *Review of Economic Studies*, 77(2), 560–594.
- Finkelstein, A., Taubman, S., Wright, B., Bernstein, M., Gruber, J., Newhouse, J., ... Group, O. H. S. (2012). The Oregon health insurance experiment: Evidence from the first year. *The Quarterly Journal of Economics*, 127(3), 1057-1106. Retrieved from

<http://dx.doi.org/10.1093/qje/qjs020> doi:  
doi: 10.1093/qje/qjs020

- Gordon, R. (2011). Commentary on tax by design: The Mirrlees Review. *Fiscal Studies*, 32(3), 395-414.
- Heckman, J. (2008). Schools, skills, and synapses. *Economic Inquiry*, 46, 289–324.
- Heckman, J. J., Moon, S. H., Pinto, R., Savelyev, P., & Yavitz, A. (2010). The rate of return of the High-Scope Perry Preschool Program. *Journal of Public Economics*, 94(1), 114–128.

- Mirrlees, J., Adam, S., Besley, T., Bond, S., Chote, R., Gammie, M., ... Poterba, J. (2011). *Tax by design* (O. U. Press, Ed.). Institute for Fiscal Studies (ifs) and Mirrlees, J.
- Todd, P. E., & Wolpin, K. I. (2006). Assessing the impact of a school subsidy program in Mexico: Using a social experiment to validate a dynamic behavioral model of child schooling and fertility. *American Economic Review*, 96(5), 1384–1417.



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# Generalized Roy Model

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## **Rising wage inequality**

- ▶ changes in distribution of skills
- ▶ changes in relative prices of skills, prices identical across sectors
- ▶ comparative advantage, different skills priced different across sectors  $\implies$  Roy models

- ▶ Does the pursuit of comparative advantage increase or decrease earnings inequality within sectors and in the overall economy?
- ▶ Do the people with the highest  $i$  skill actually work in sector  $i$ ?
- ▶ As people enter a sector in response to an increase in the demand for its services, does the average skill level employed there rise or fall?

## Roy (1951) Model

- ▶ Individuals are income maximizing, act under perfect information, and possess skills  $S_1$  and  $S_2$ .
- ▶ The economy offers two employment opportunities associated with skill prices  $\pi_1$  and  $\pi_2$  and skill  $i$  is only useful in sector  $i$ .

An individual chooses sector one if earnings are greater there:

$$w_1 > w_2 \iff \pi_1 S_1 > \pi_2 S_2$$



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Some Thoughts on the Distribution of Earnings

Author(s): A. D. Roy

Source: *Oxford Economic Papers*, New Series, Vol. 3, No. 2 (Jun., 1951), pp. 135-146

Published by: Oxford University Press

Stable URL: <http://www.jstor.org/stable/2662082>

Accessed: 07/10/2009 02:09

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## Econometric Problems

- ▶ **Evaluation Problem** We only observe an individual's wage in the sector they are working in.
- ▶ **Selection Problem** As individuals pursue their comparative advantage, we only observe selected samples from the latent skill distribution in either sector.

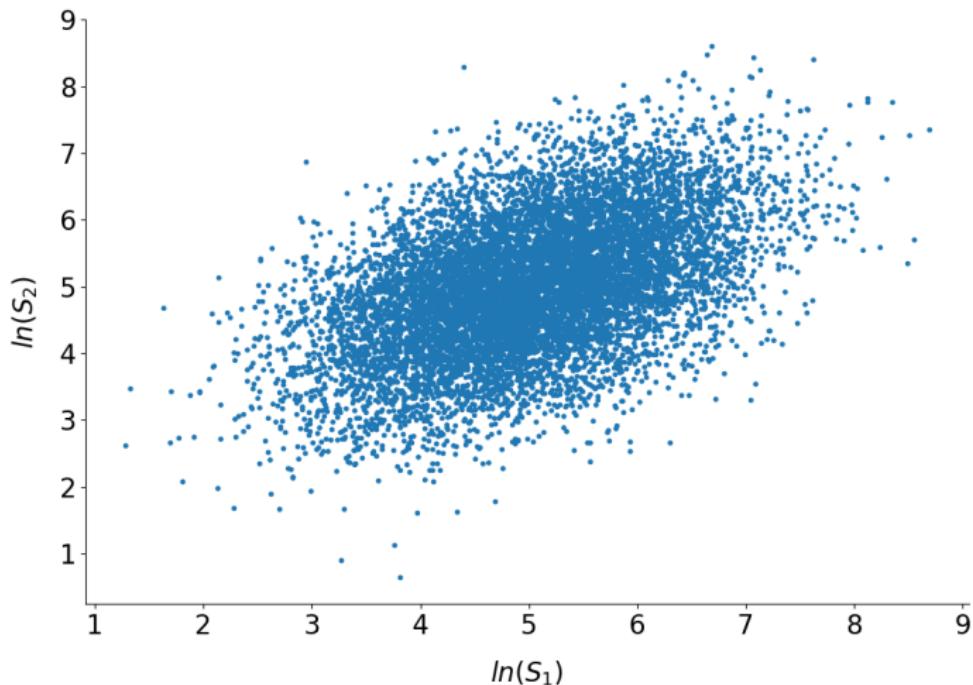
## **Key Questions**

- ▶ What economic concepts are accounted for, which are not?
- ▶ What does the individual, what does the econometrician know?
- ▶ What gives rise to heterogeneity in skills?

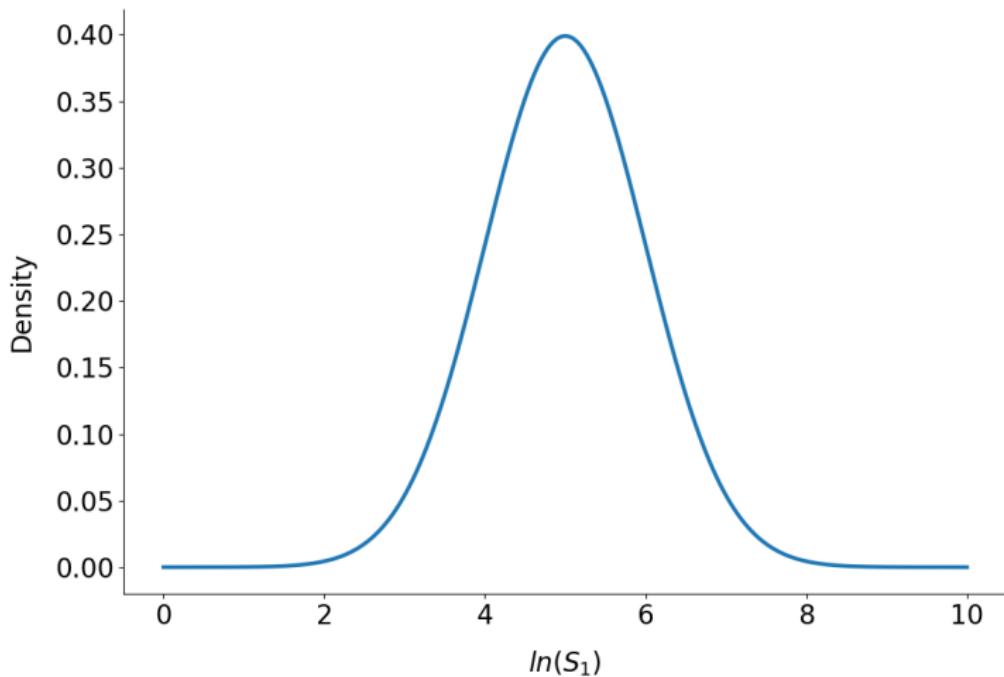
- ▶ Skills follow a bivariate normal distribution denoted by  $F(s_1, s_2)$ .

$$\begin{pmatrix} \ln S_1 \\ \ln S_2 \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \right)$$

**Figure:** Joint Distribution of Skills



**Figure:** Marginal Distribution of Skill



The proportion of the population working in sector one  
 $P_1$

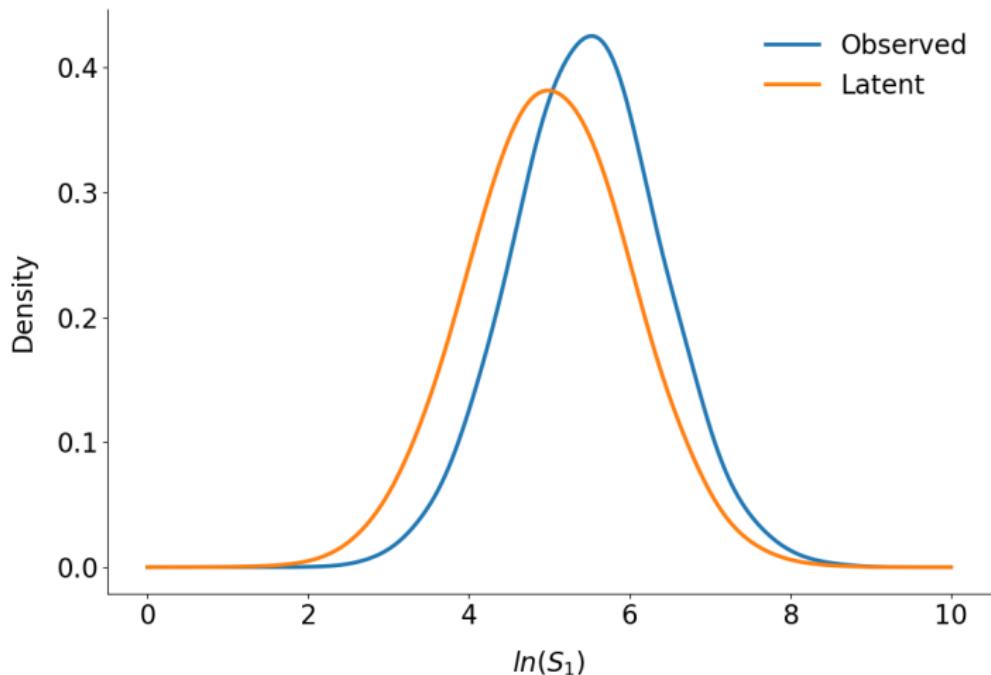
$$P_1 = \int_0^\infty \int_0^{\pi_1 s_1 / \pi_2} f(s_1, s_s) ds_1 ds_2$$

The density of skills employed in sector one differs from the population density of skills.

$$f(s_1) = \int_0^\infty f(s_1, s_2) ds_2$$
$$g_1(s_1 | \pi_1 S_1 > \pi_2 S_2) = \frac{1}{P_1} \int_0^{\pi_1 s_1 / \pi_2} f(s_1, s_2) ds_2$$

The distribution of skills employed in sector 1 differs from the population distribution of skills due to comparative advantage.

**Figure: Latent and Observed Distribution of Skill**



# **Truncation and Censoring**

## Setup

$$\begin{pmatrix} Z \\ I \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1.0 & \rho \\ \rho & 1.0 \end{pmatrix} \right)$$

Figure: Density of truncated standard Normal distribution

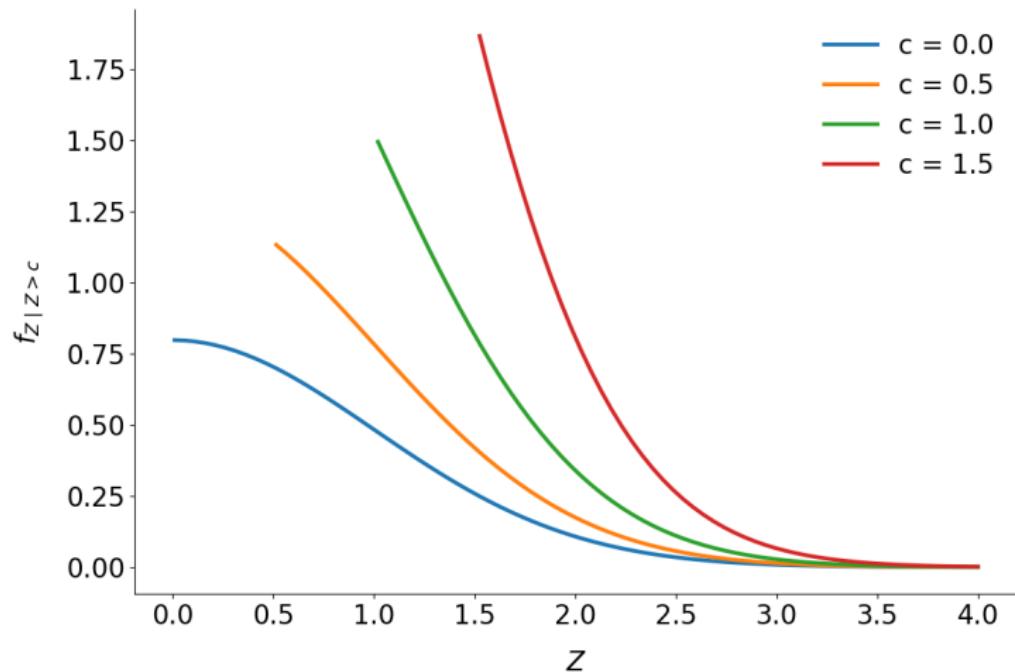


Figure: Expectation of truncated standard Normal distribution

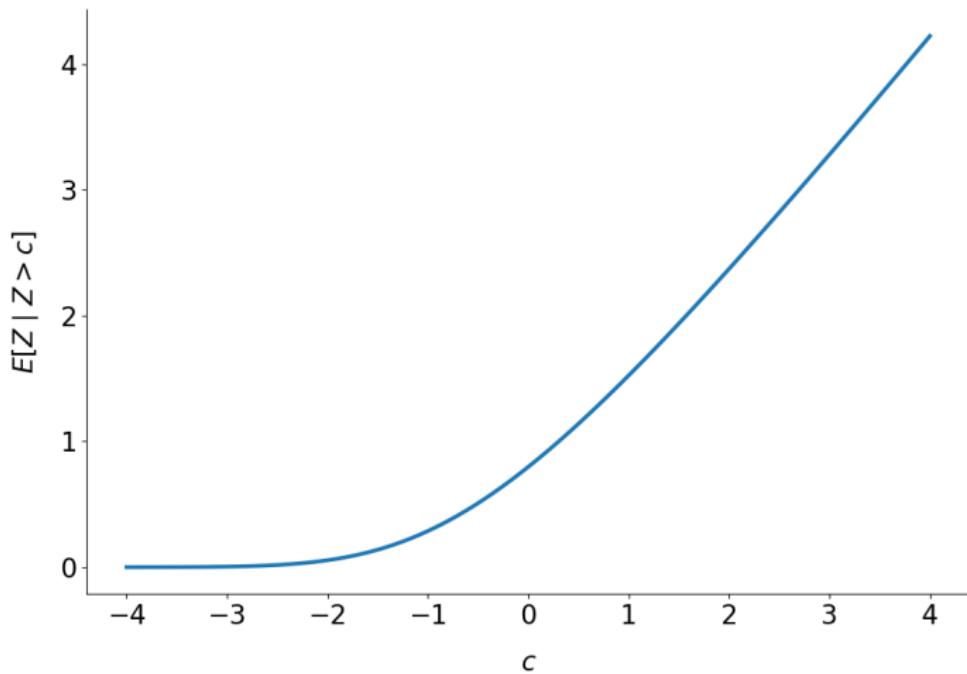


Figure: Variance of truncated standard Normal distribution

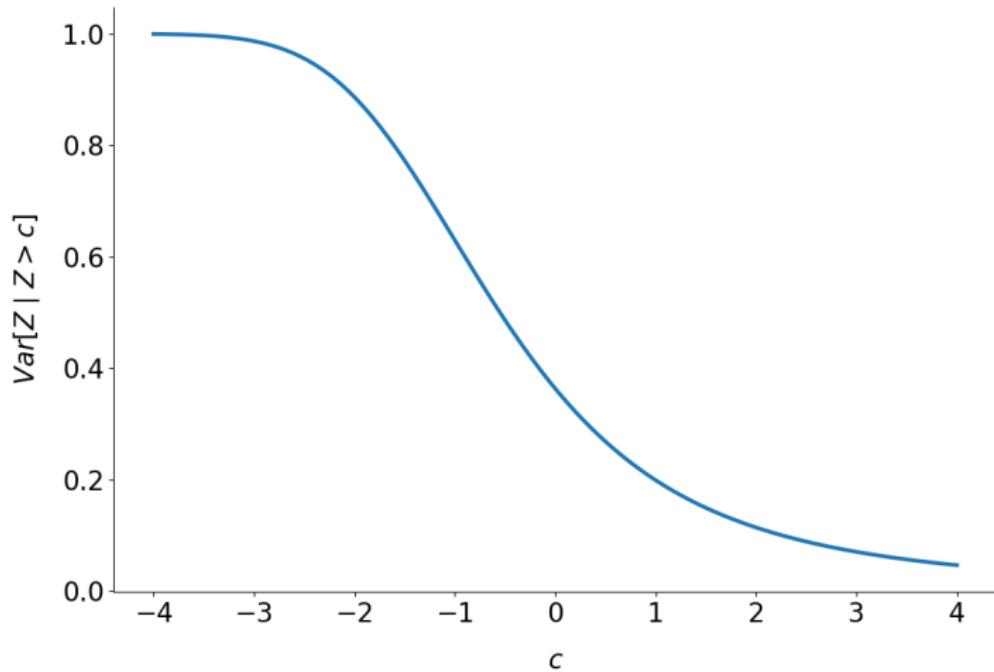
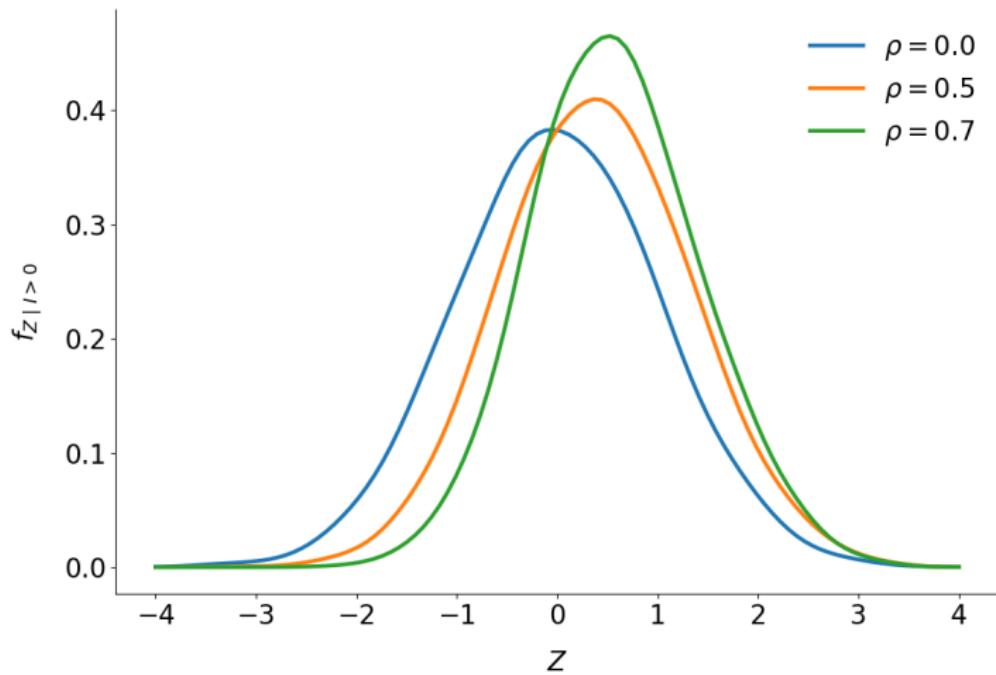


Figure: Density of censored standard Normal distribution



# **Sorting and selection**

## **Wage Equations**

$$\ln W_1 = \ln \pi_1 + \mu_1 + U_1$$

$$\ln W_2 = \ln \pi_2 + \mu_2 + U_2,$$

where  $U_i = \ln S_i - \mu_i$ .

## Some notation

$$\begin{aligned}\sigma^* &= \sigma_{U_1 - U_0} \\ &= \sqrt{(\sigma_{11} - \sigma_{12}) + (\sigma_{22} - \sigma_{12})}\end{aligned}$$

$$c_1^* = (\ln(\pi_1/\pi_2) + \mu_1 - \mu_2)/\sigma^*$$

$$L = U_1 - U_0$$

## Selection bias

$$\begin{aligned} E[\ln W_1 \mid \ln W_1 > \ln W_2] &= \ln \pi_1 + \mu_1 + E[U_1 \mid L > -c_1^*] \\ &= \dots + E[U_1 \mid U_1 - U_0 > -c_1^*] \end{aligned}$$

- ▶ What about identification at infinity arguments?

## Sorting

$$E[\ln S_1 \mid \ln W_1 > \ln W_2] = \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{\sigma^*} \lambda(-c_1)$$

$$E[\ln S_2 \mid \ln W_2 > \ln W_1] = \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*} \lambda(-c_2)$$

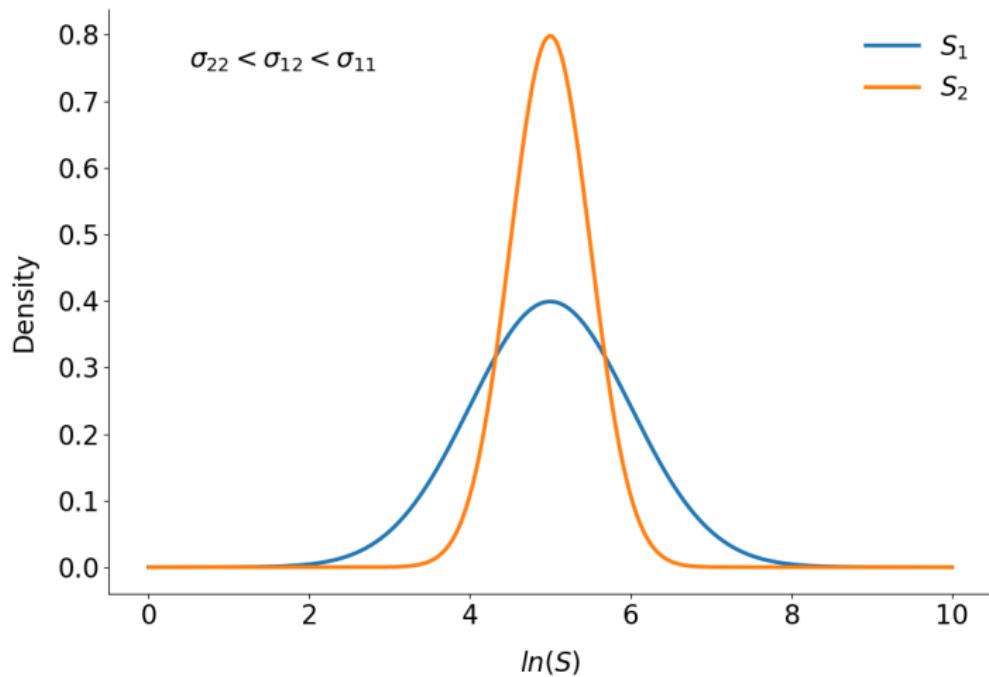
We know the following:

$$\sigma^* = (\sigma_{11} - \sigma_{12}) + (\sigma_{22} - \sigma_{12}) > 0$$

$$\lambda, \lambda' > 0$$

- ▶ There must be positive selection into one of the occupations and there can be positive selection into both.

Figure: Marginal Distributions of Skills



## **What do we know?**

- ▶ There is positive selection in Sector 1 as  $\sigma_{11} > \sigma_{12}$ .
- ▶ There is negative selection in Sector 2 as  $\sigma_{22} < \sigma_{12}$ .

We gain further insights into the effect of self-selection on the distribution of earnings for workers in sector 1 by looking at the distribution of  $\ln S_1$  conditional on  $\ln S_2$ .

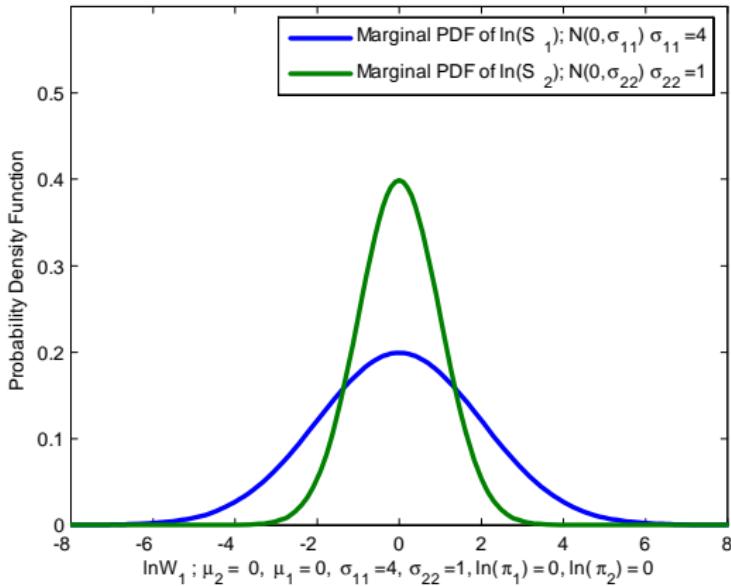
$$\ln S_1 | \ln S_2 \sim \mathbb{N}(\mu, \sigma),$$

where

$$\mu = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} \left( \ln S_2 - \mu_2 \right) \quad \text{and} \quad \sigma = \sigma_{11} \left( 1 - \left( \frac{\sigma_{12}}{\sigma_1 \sigma_2} \right)^2 \right)$$

# **Heckman Productions**

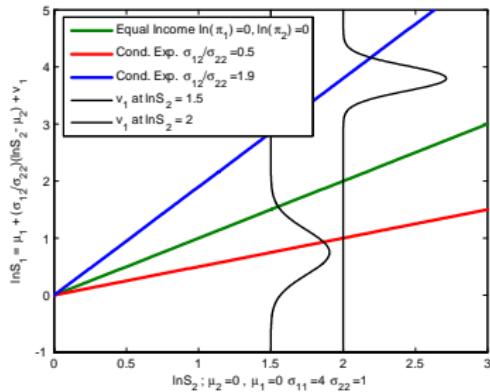
Marginal Probability Density Function (PDF) of  $\ln S_1$ ,  $\ln S_2$



$$\ln S_1 = \ln(\mu_1) + U_1; \quad \ln S_2 = \ln(\mu_2) + U_2;$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix} \right); \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Graph of  $\ln S_1 = f(\ln S_2)$

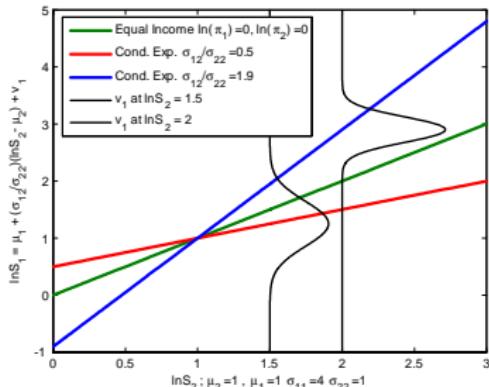


$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

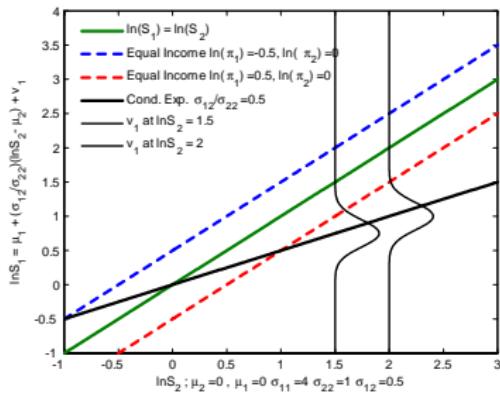
Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \sigma_{12} = 0.5, 1.9;$$

Graph of  $\ln S_1 = f(\ln S_2)$



Graph of  $\ln S_1 = f(\ln S_2)$



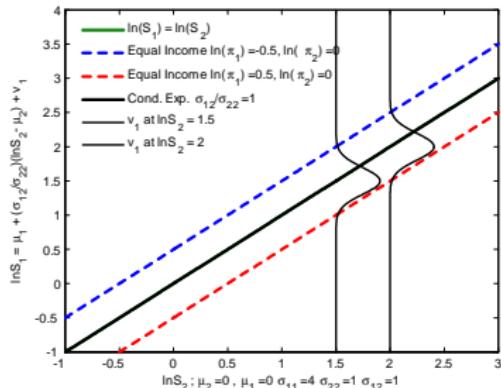
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$$\ln(\pi_1) = -0.5 \text{ and } \ln(\pi_2) = +0.5$$

Parameters:

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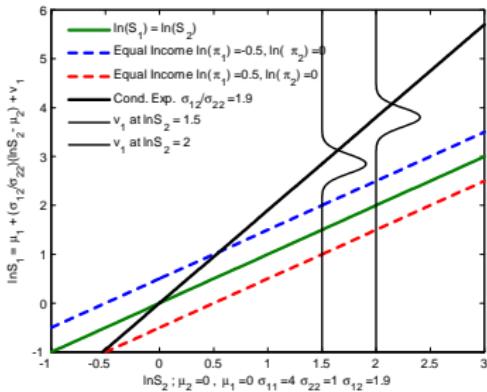


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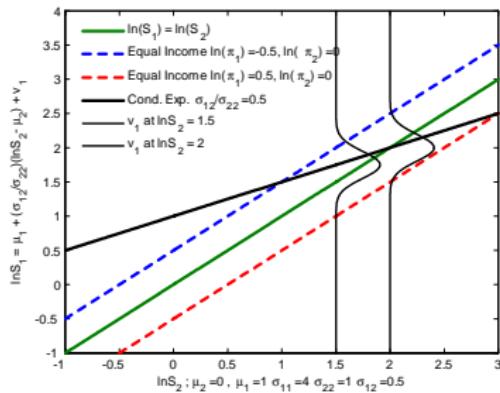
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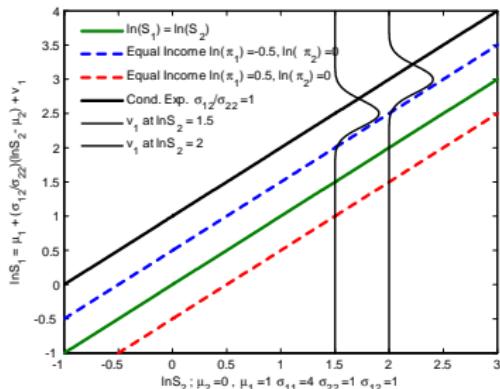
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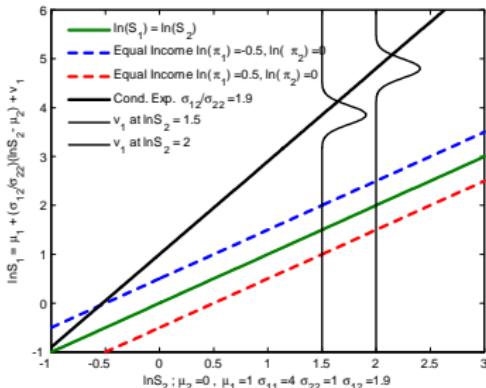
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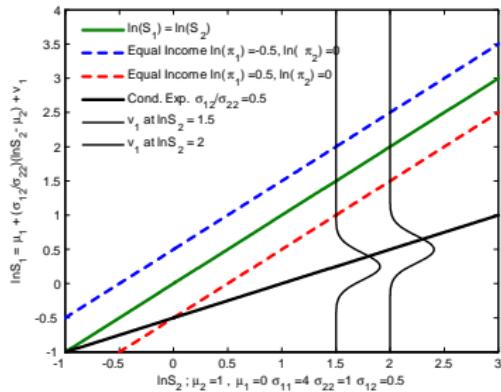
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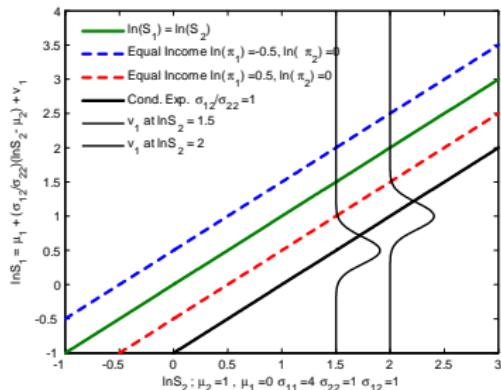
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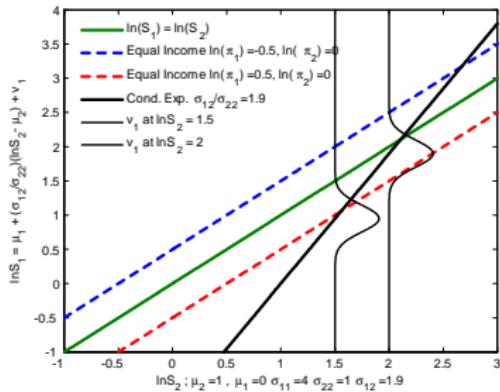


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Parameters:

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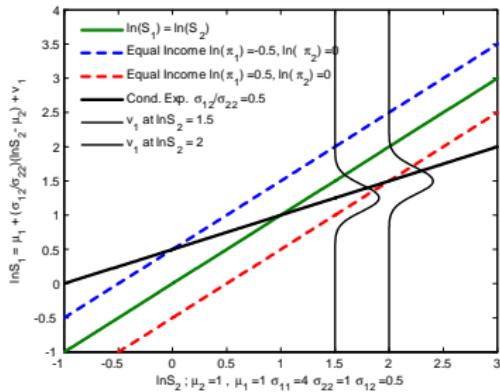
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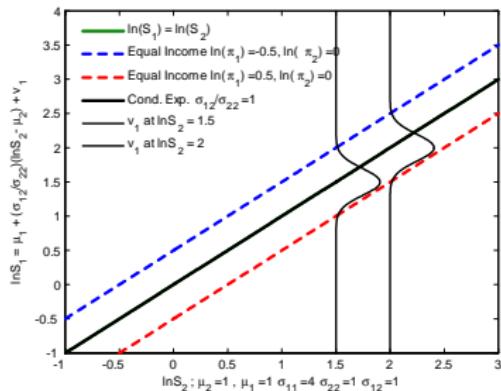
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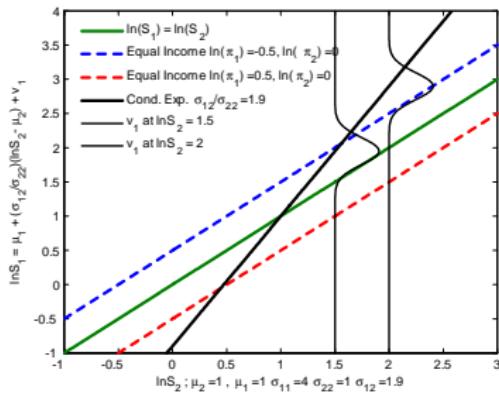
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Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

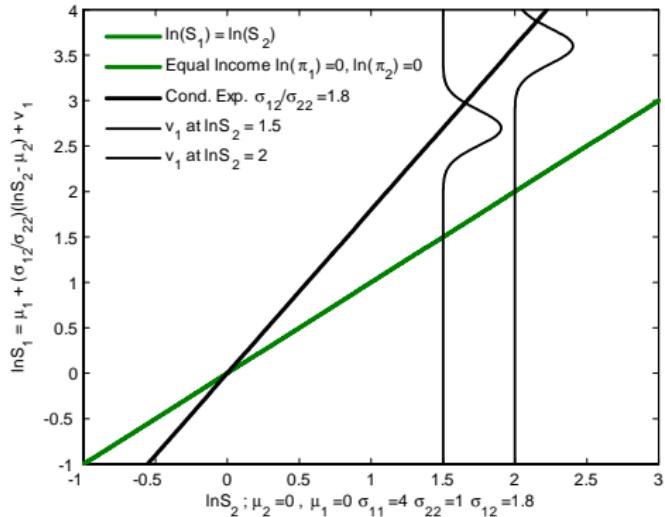


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \ln(\pi_1) &= -0.5 \text{ and } \ln(\pi_1) = +0.5 \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1 & 1.9 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$



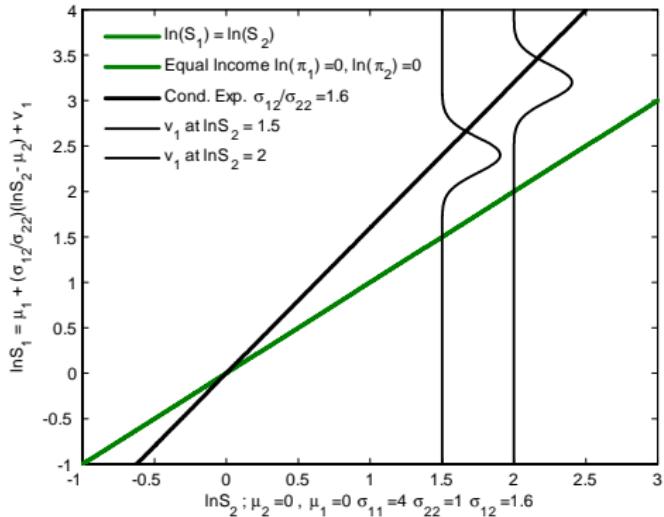
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.8 \\ 1.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



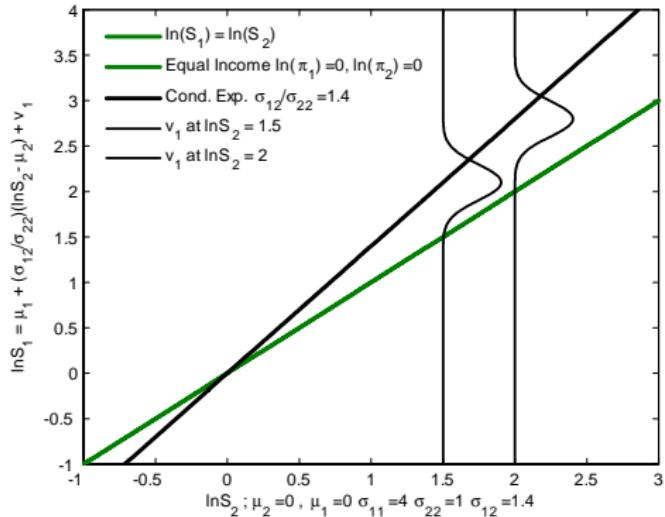
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.6 \\ 1.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



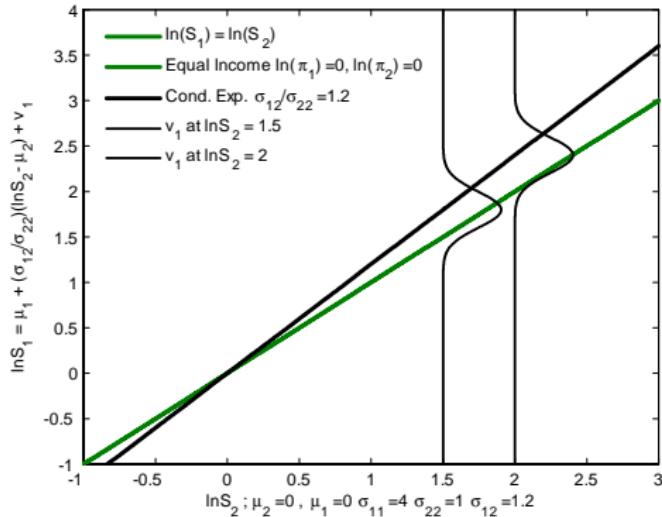
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.4 \\ 1.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



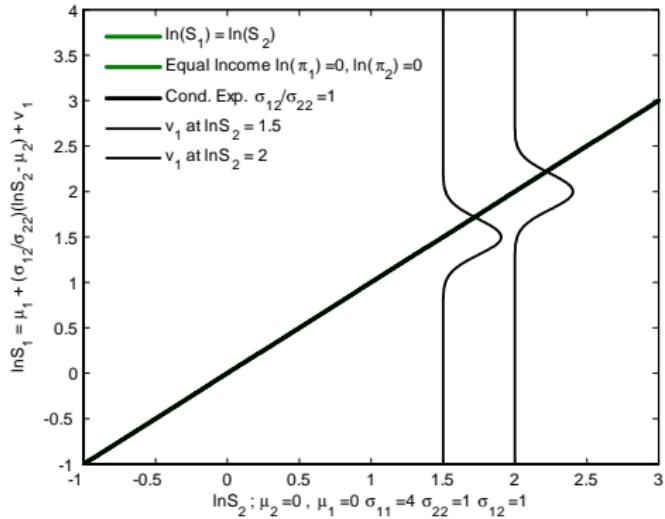
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.2 \\ 1.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



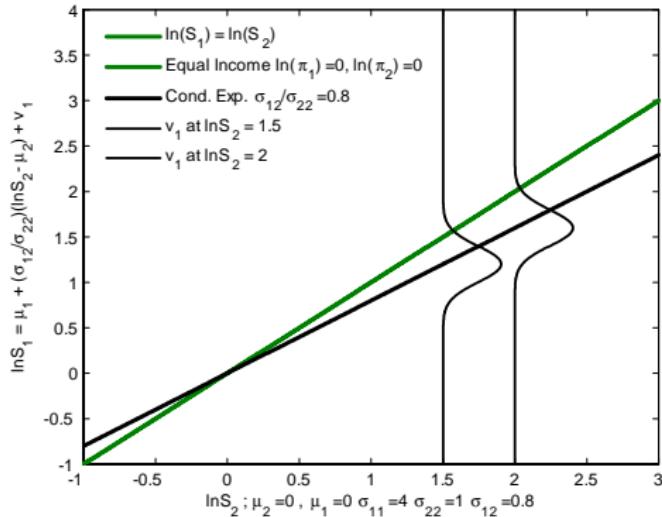
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.0 \\ 1.0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



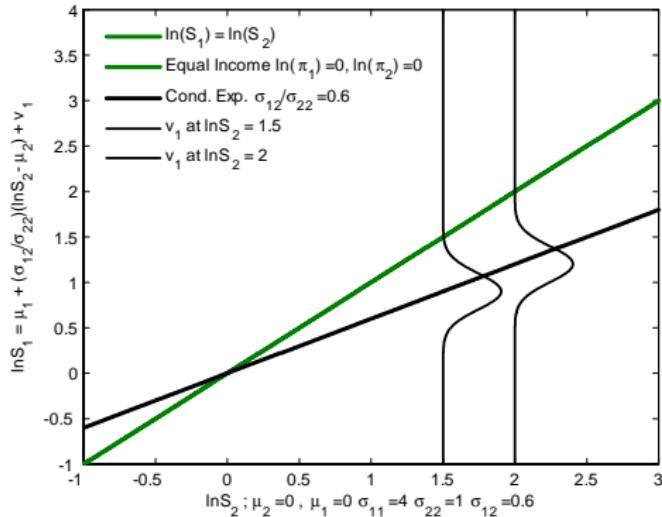
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



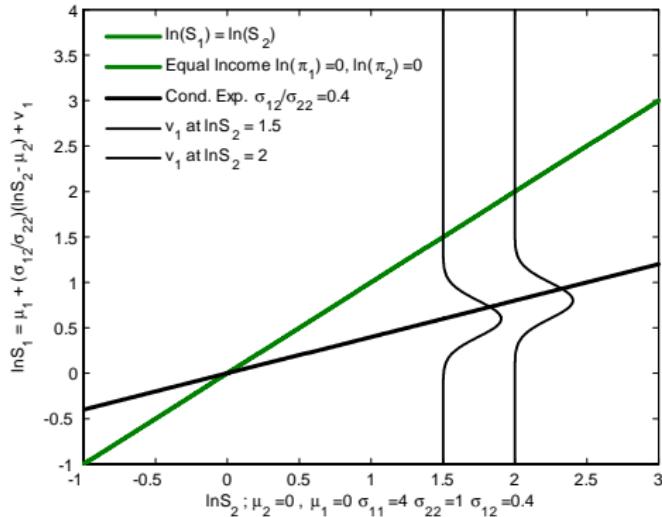
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.6 \\ 0.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



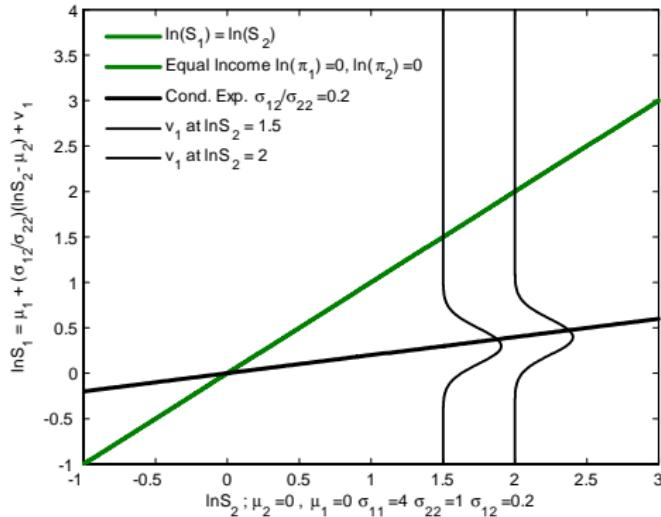
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.4 \\ 0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



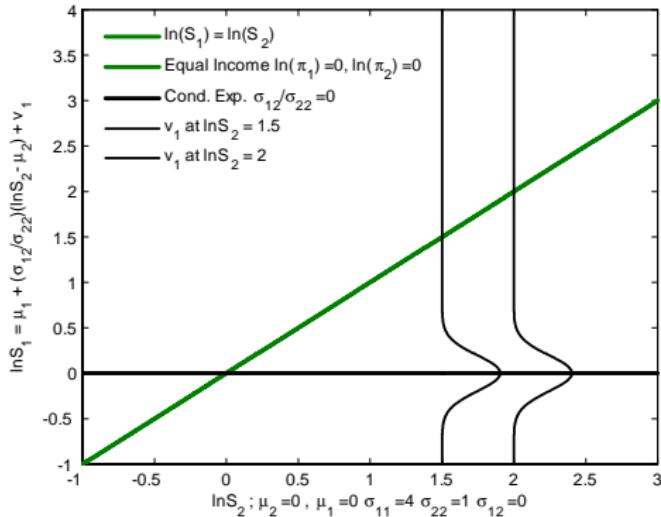
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.2 \\ 0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



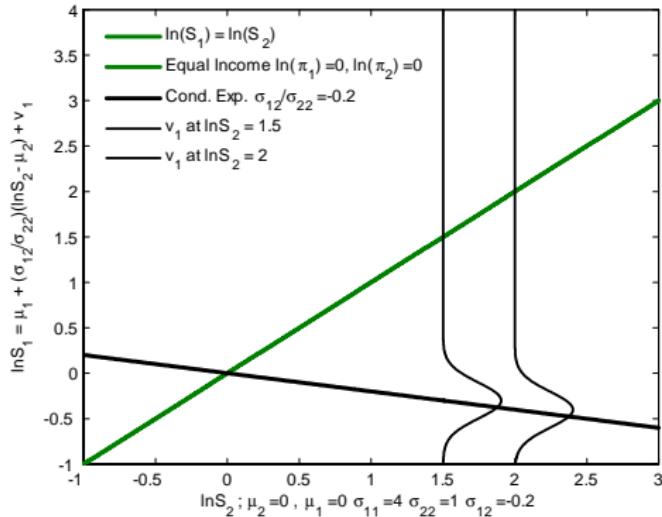
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



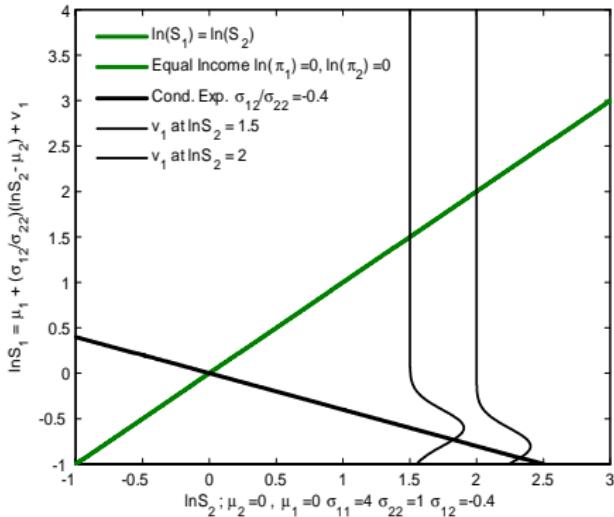
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.2 \\ -0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



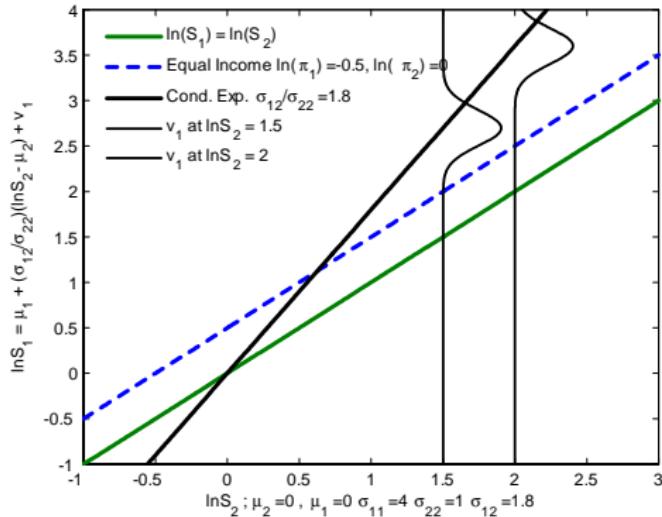
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.4 \\ -0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



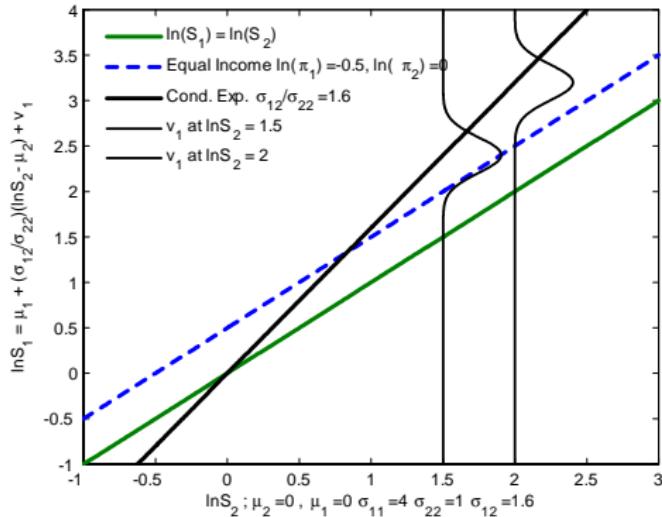
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.8 \\ 1.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



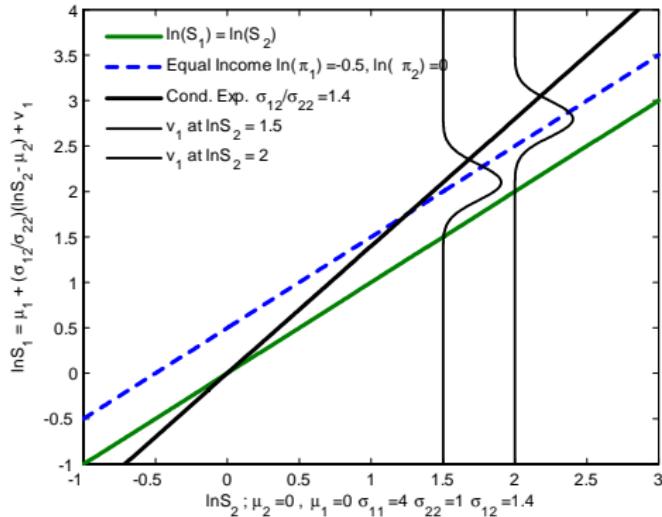
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.6 \\ 1.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



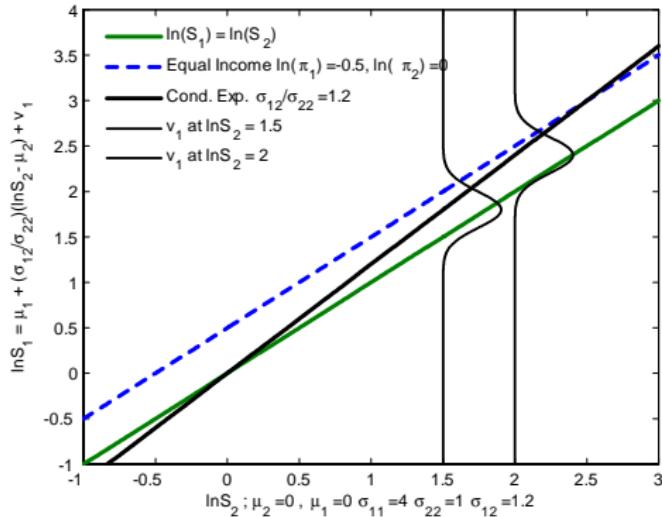
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.4 \\ 1.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



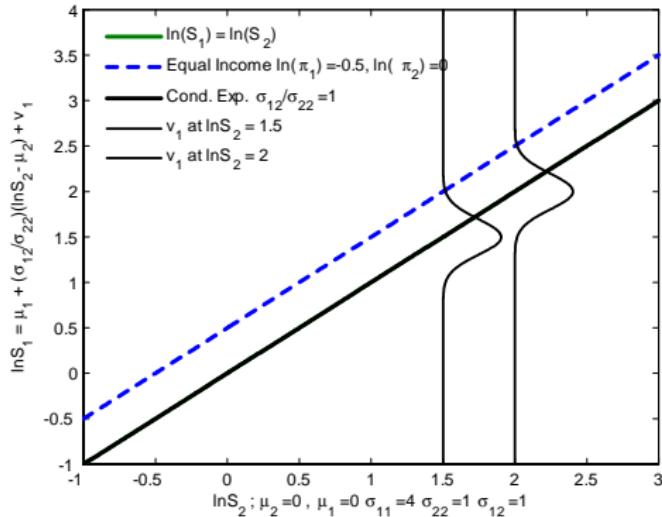
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.2 \\ 1.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$

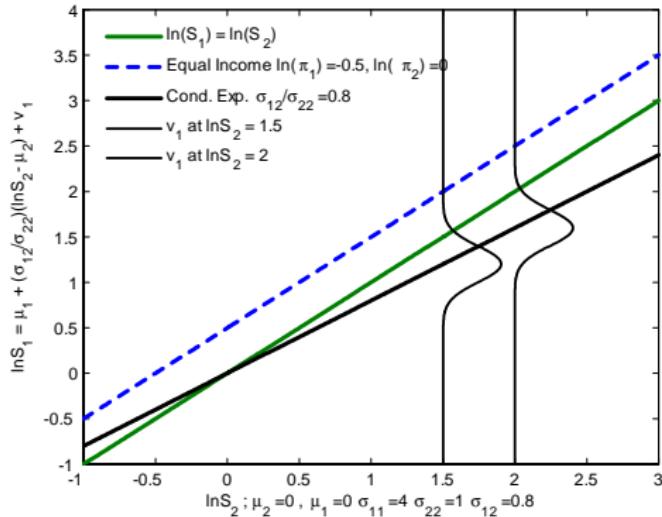


Parameters:

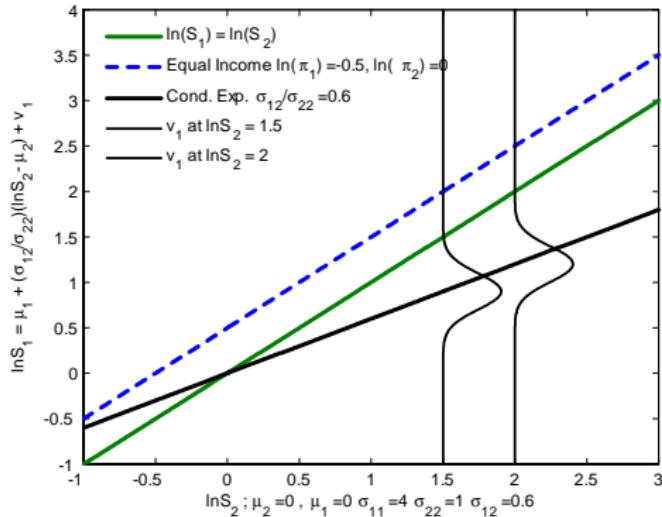
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.0 \\ 1.0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



Graph of  $\ln S_1 = f(\ln S_2)$

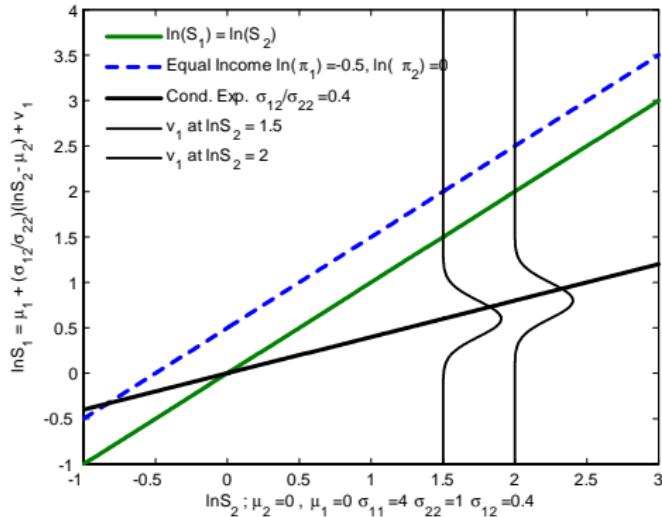


Parameters:

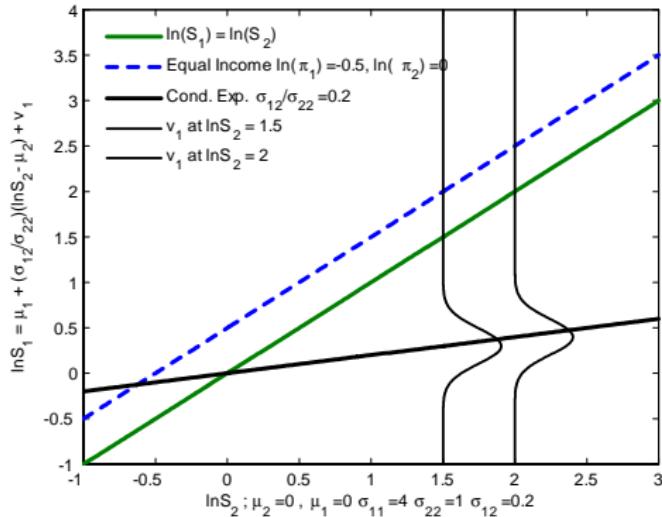
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.6 \\ 0.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



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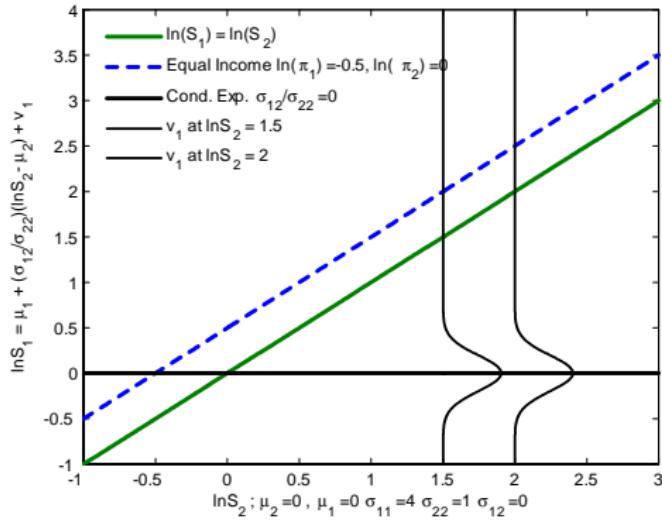
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.2 \\ 0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



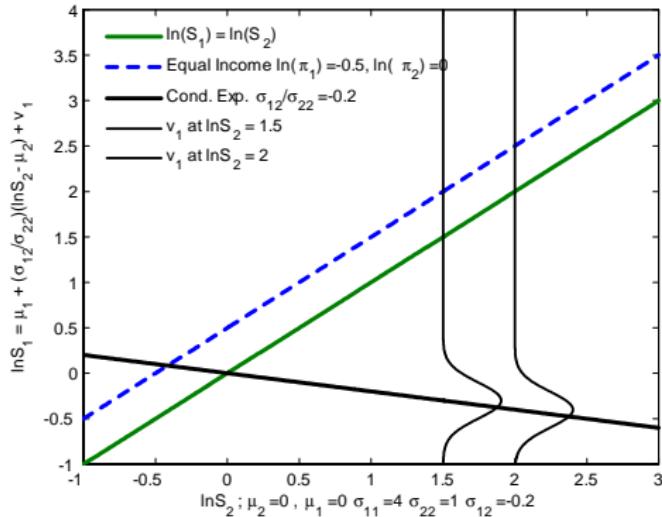
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



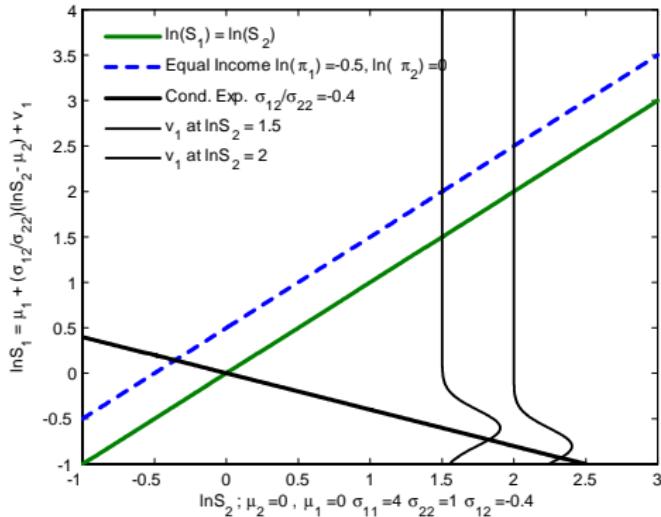
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

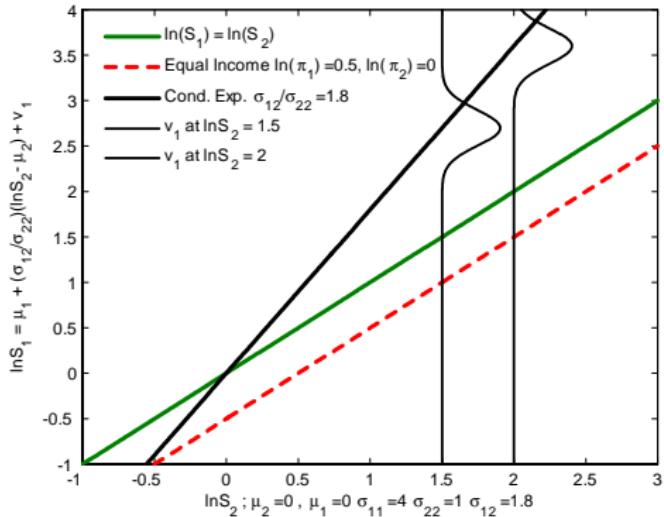
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.2 \\ -0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



Graph of  $\ln S_1 = f(\ln S_2)$



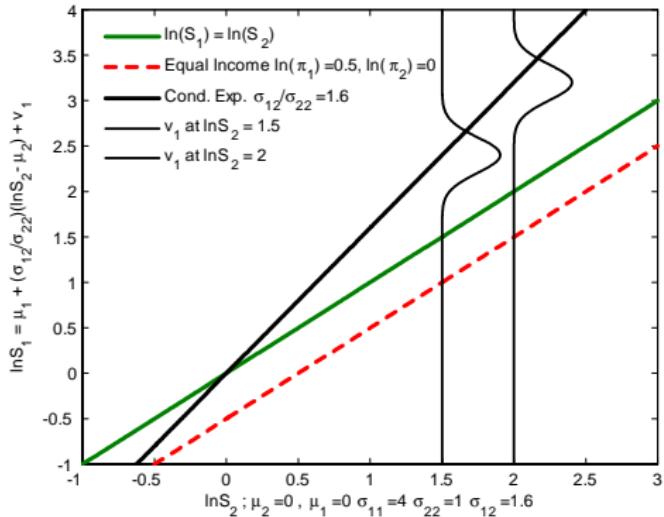
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.8 \\ 1.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



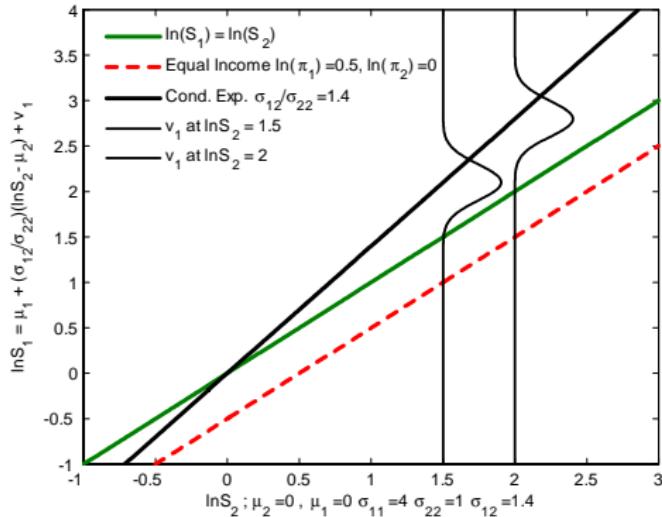
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

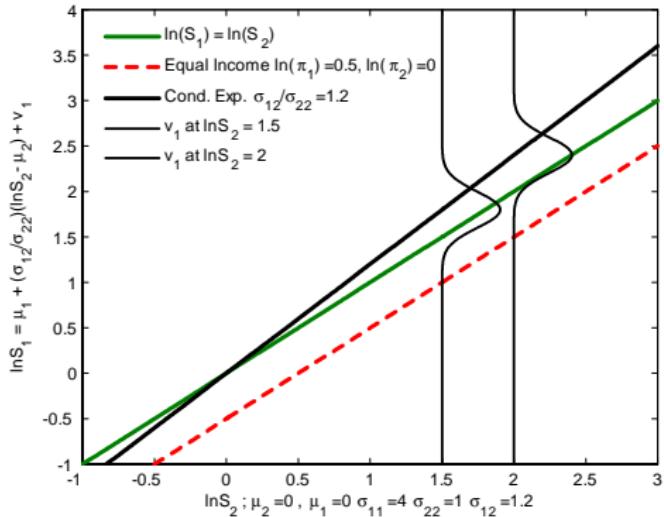
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.6 \\ 1.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



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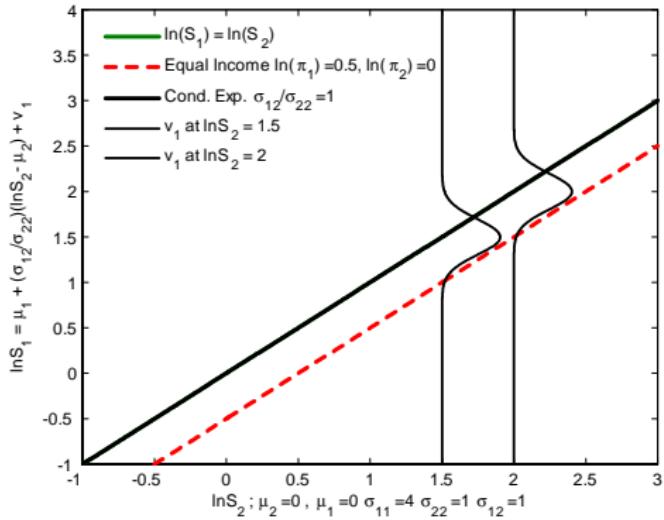
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.2 \\ 1.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$

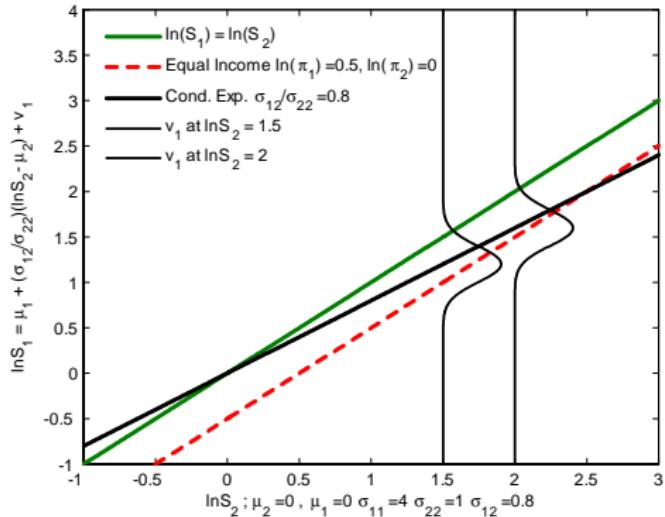


Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.0 \\ 1.0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



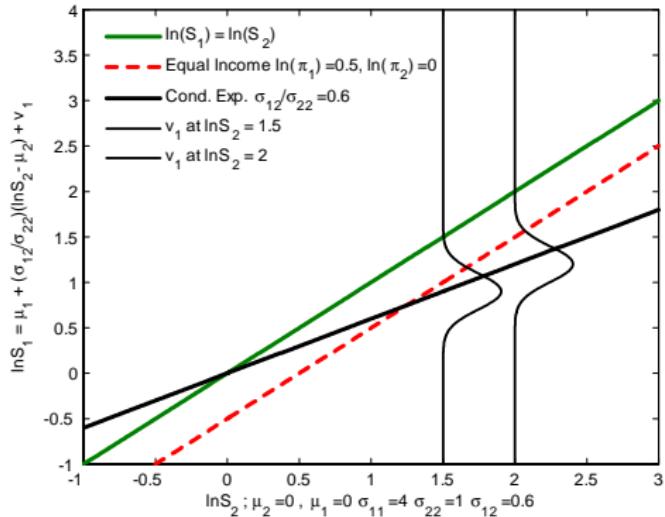
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



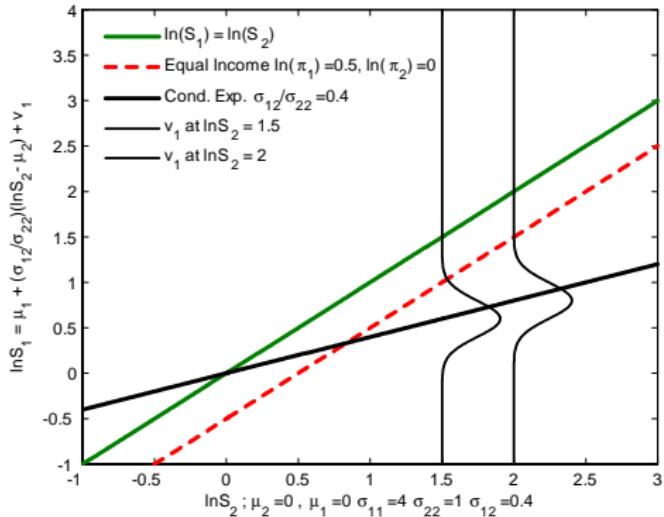
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.6 \\ 0.6 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



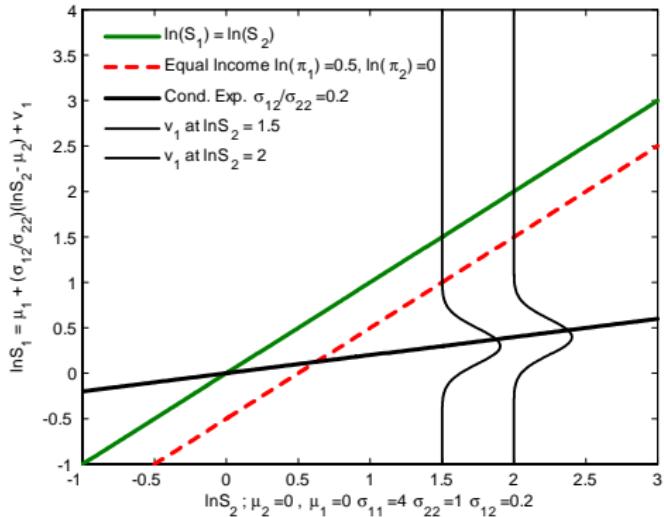
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.4 \\ 0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



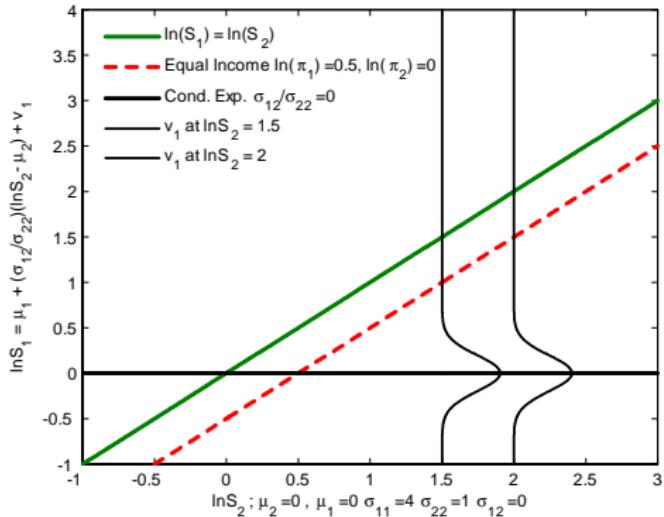
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.2 \\ 0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



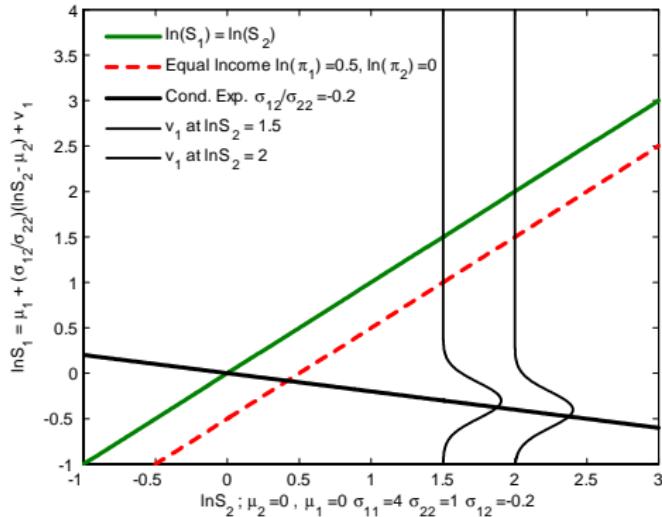
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



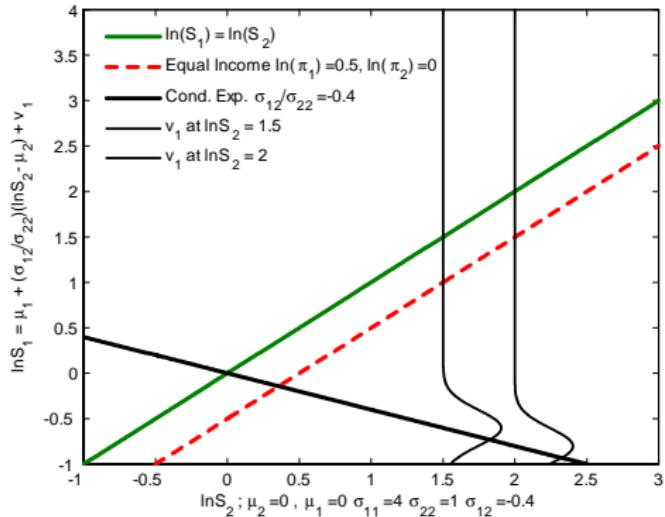
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.2 \\ -0.2 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$



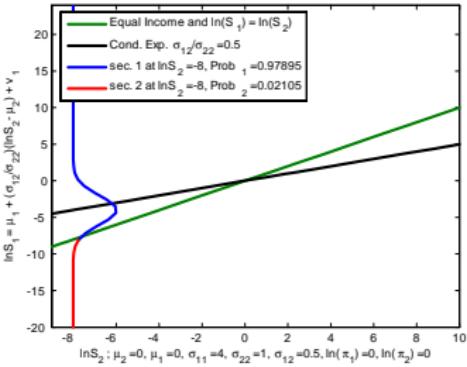
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & -0.4 \\ -0.4 & 1 \end{bmatrix}, \begin{bmatrix} \ln \pi_1 \\ \ln \pi_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix};$$

$$\mu_1 = \mu_2 = 0.$$

Graph of  $\ln S_1 = f(\ln S_2)$

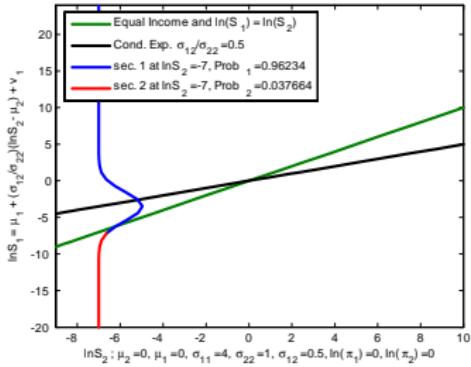


$\ln S_1 = \mu_1 + \frac{\sigma_{11}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -8) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

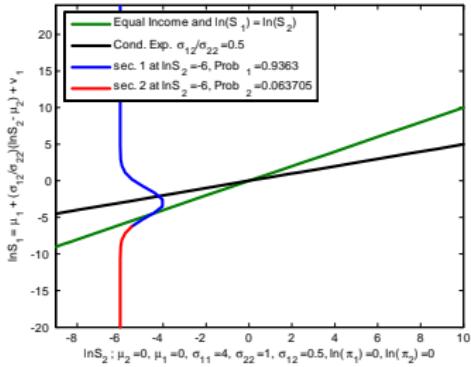


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -7$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -7$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

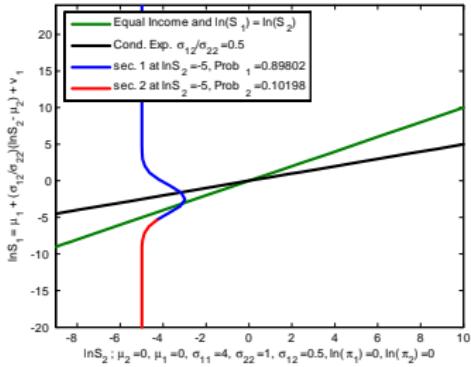


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -6) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$



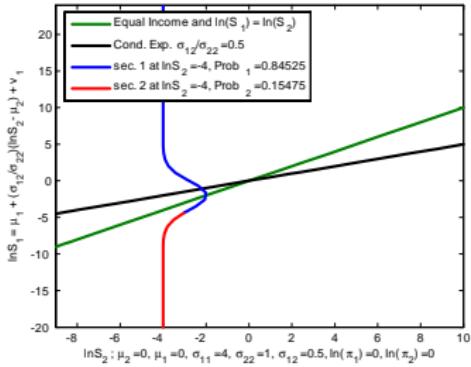
$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

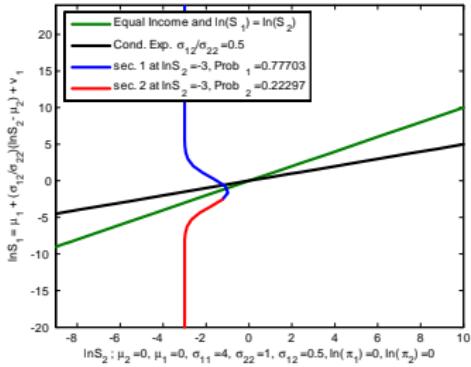


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -4) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

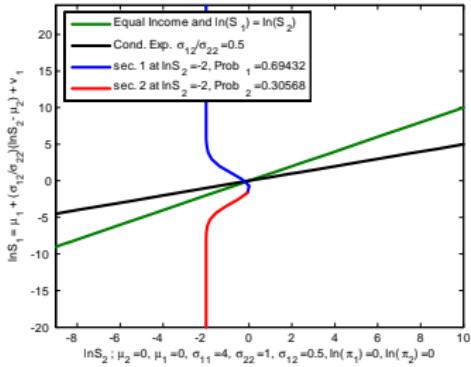


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{11}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

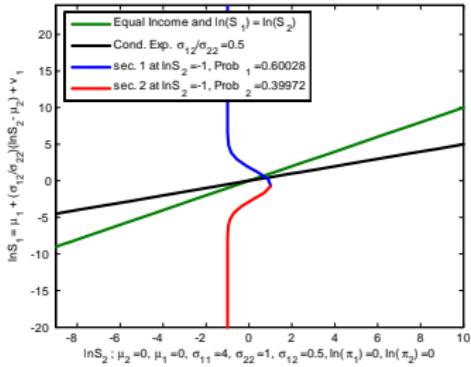


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -2) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$



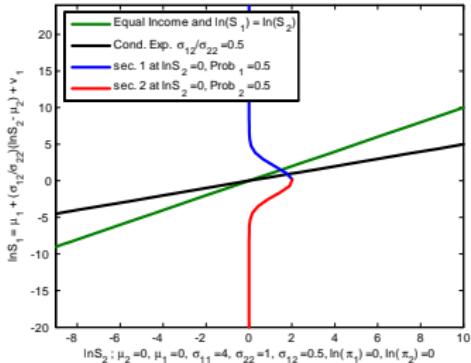
$$\ln S_1 = \mu_1 + (\sigma_{12} \sigma_{22})^{-1} (\ln S_2 - \mu_2) + v_1$$

$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

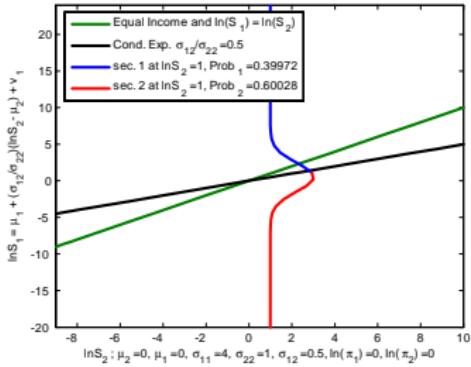


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1$$

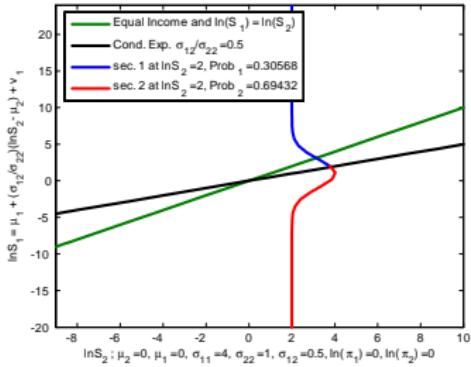
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1}$

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

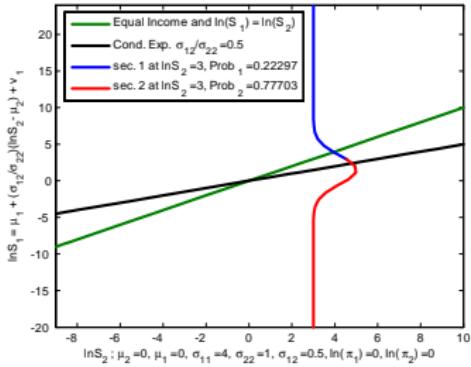


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

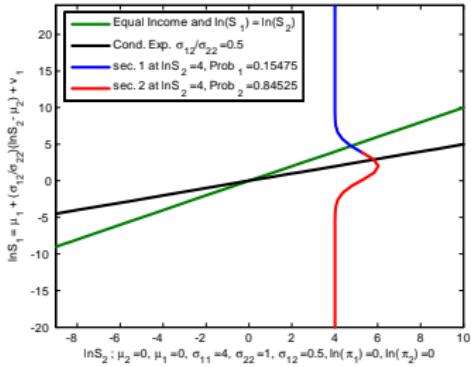


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

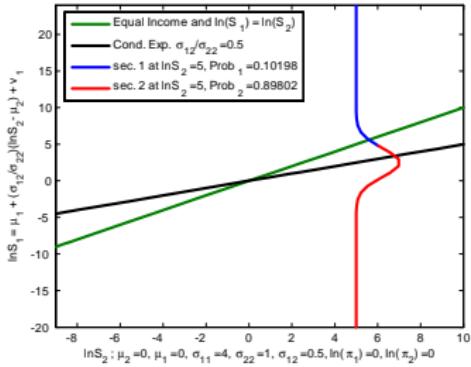


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

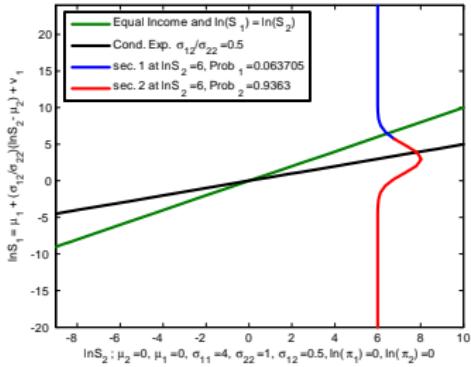
Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$



Graph of  $\ln S_1 = f(\ln S_2)$

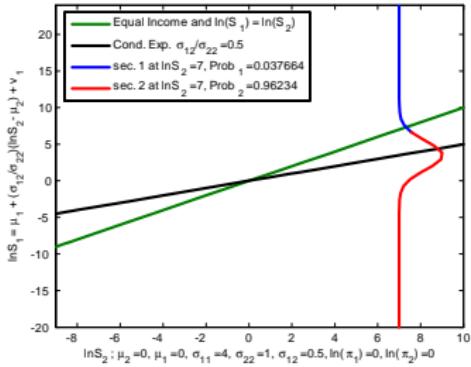


$$\begin{aligned}\ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 2}\end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

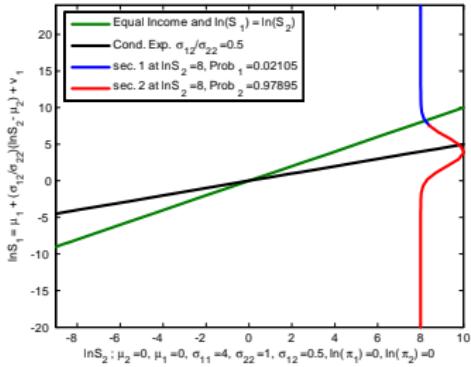


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

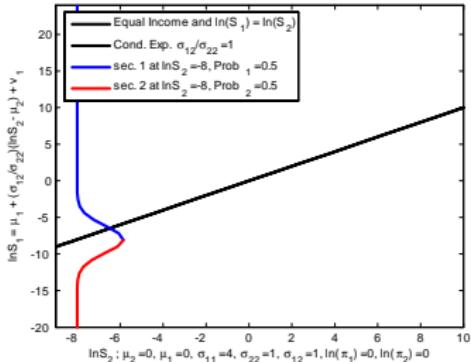


$$\begin{aligned}\ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 8) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 8) \Rightarrow \text{Pr. of Working at Sector 2}\end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

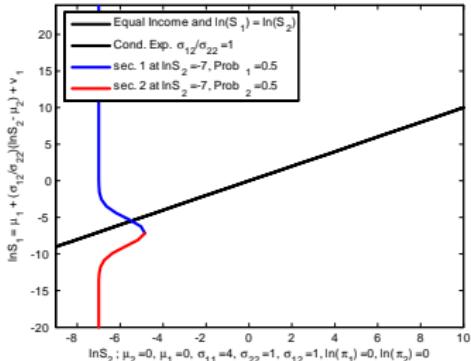


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -8$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -8$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

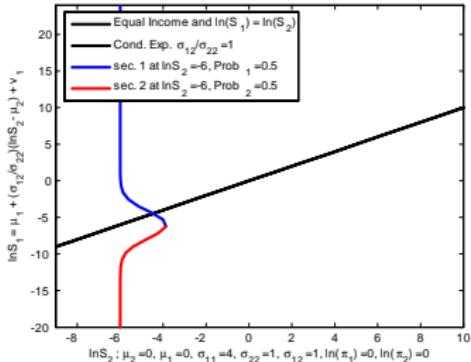


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -7$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -7$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

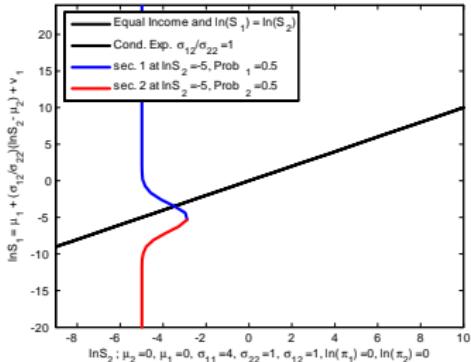


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -6$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -6$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

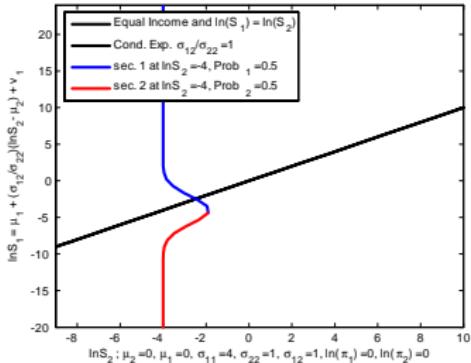


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{11}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

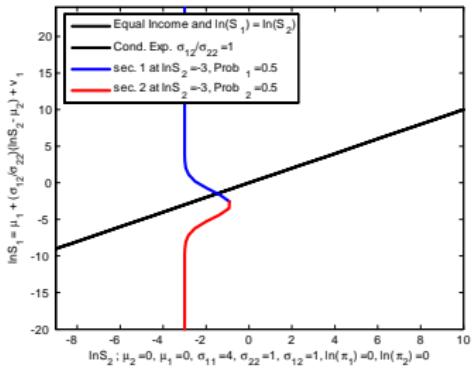


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -4$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -4$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

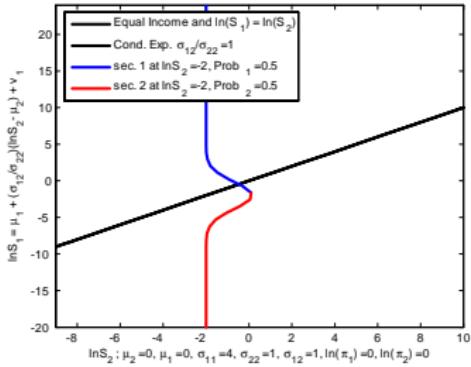


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -3$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -3$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

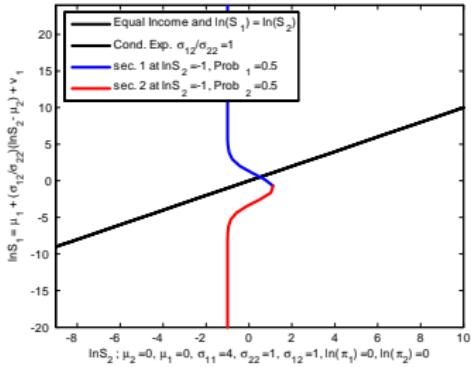


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr( $W_1 > W_2 | \ln S_2 = -2$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr( $W_1 < W_2 | \ln S_2 = 2$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

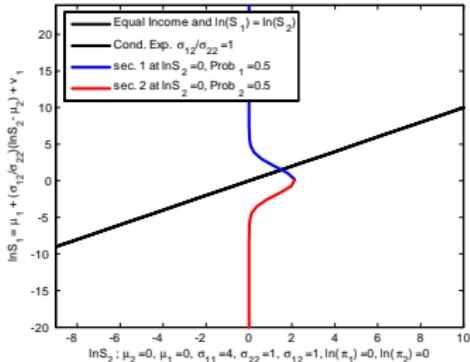


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -1) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

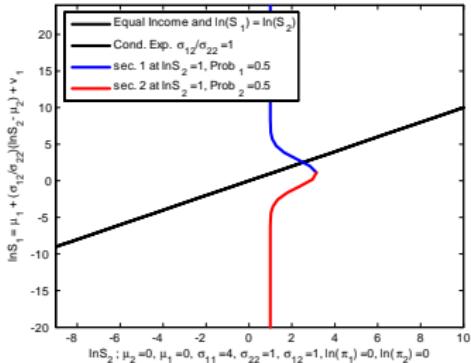


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

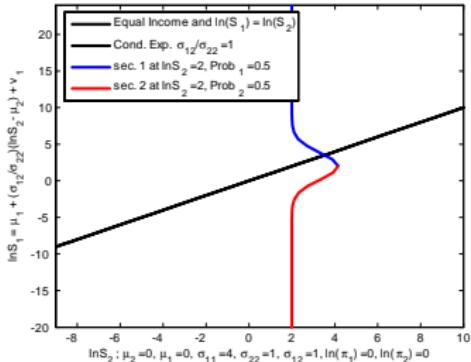


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

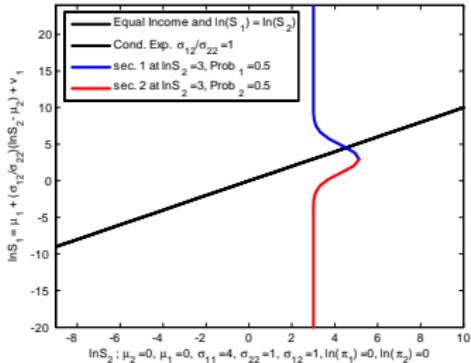


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = 2$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = 2$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$



$$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$$

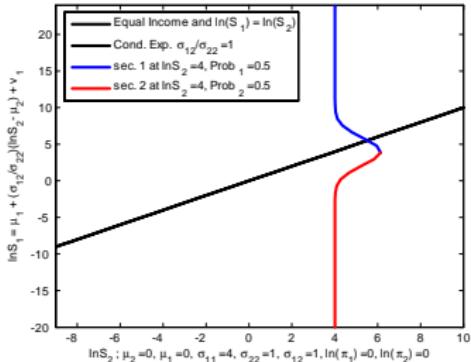
$\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 1}$

$\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

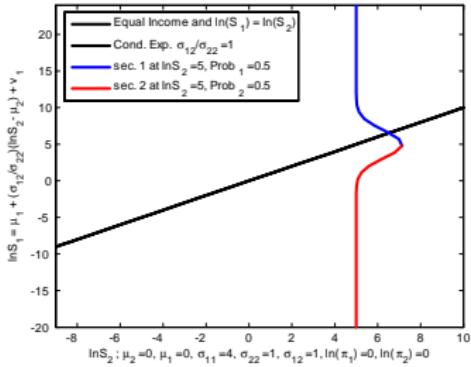


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

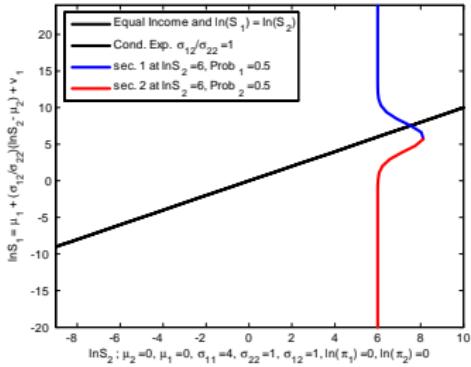


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

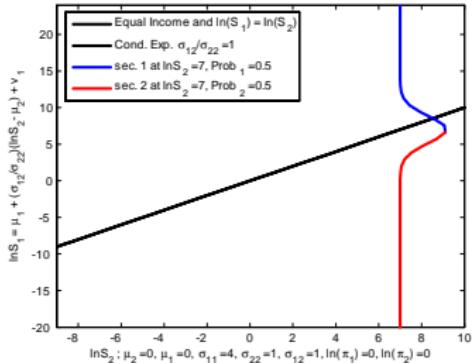


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

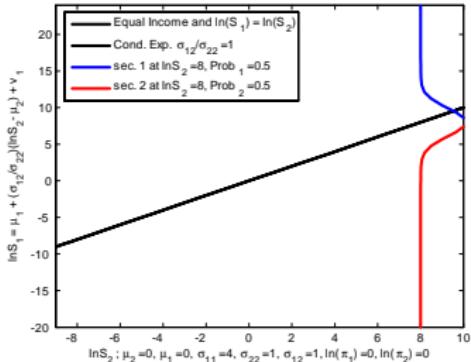


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 7) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

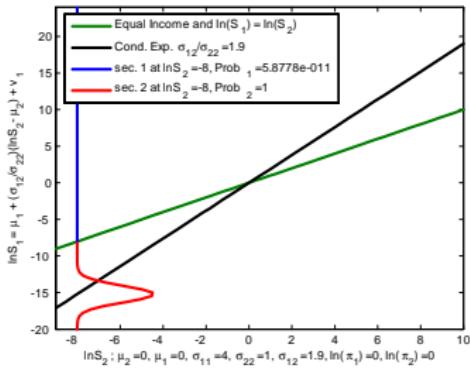
Graph of  $\ln S_1 = f(\ln S_2)$



Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

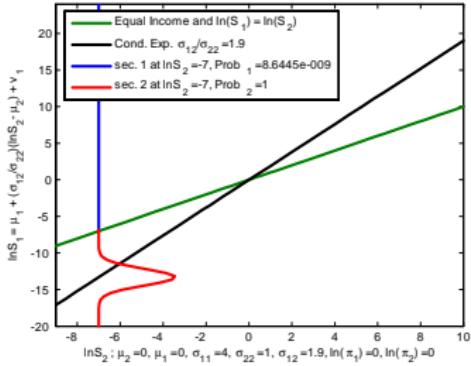


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob $_1 = \Pr(W_1 > W_2 | \ln S_2 = -8) \Rightarrow$  Pr. of Working at Sector 1  
 Prob $_2 = \Pr(W_1 < W_2 | \ln S_2 = -8) \Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

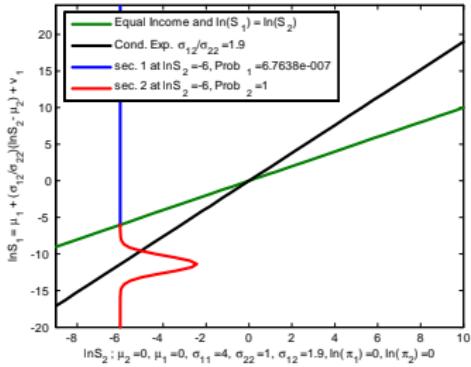


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -7) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -7) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

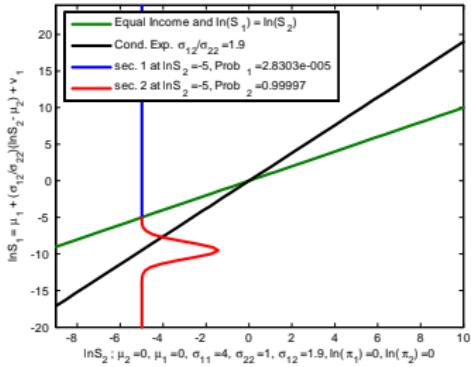


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -6$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -6$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

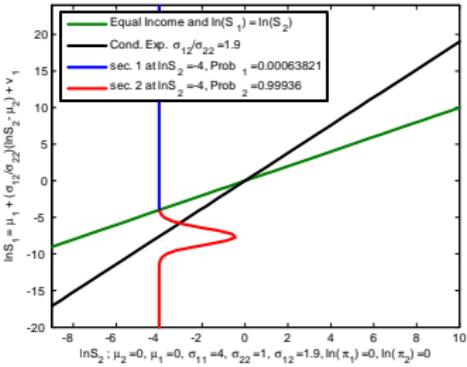


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = -5) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

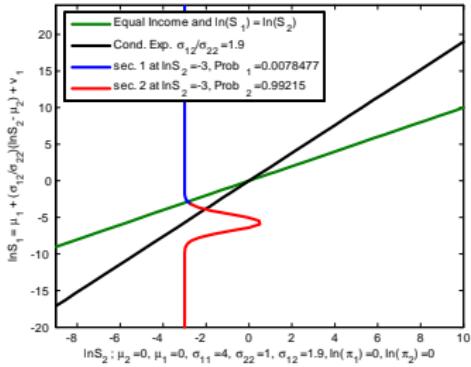


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -4$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = -2$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

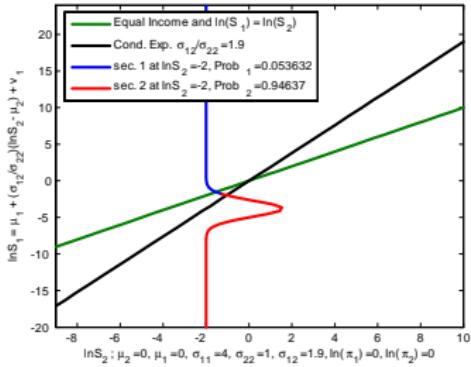


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{12}^2} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = -3) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

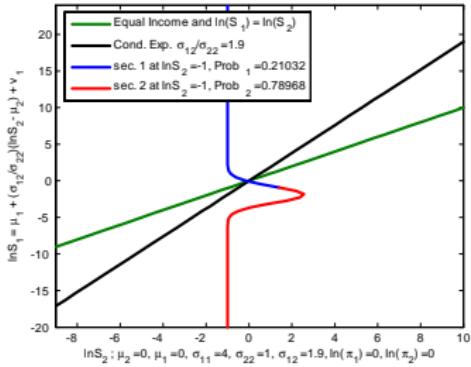


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob $_1 = \Pr(W_1 > W_2 | \ln S_2 = -2) \Rightarrow$  Pr. of Working at Sector 1  
 Prob $_2 = \Pr(W_1 < W_2 | \ln S_2 = -2) \Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

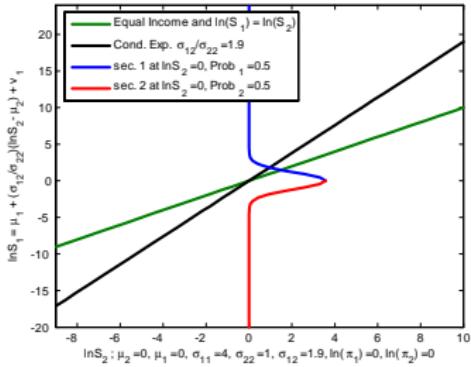


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 Prob<sub>1</sub> = Pr ( $W_1 > W_2 | \ln S_2 = -1$ )  $\Rightarrow$  Pr. of Working at Sector 1  
 Prob<sub>2</sub> = Pr ( $W_1 < W_2 | \ln S_2 = 1$ )  $\Rightarrow$  Pr. of Working at Sector 2

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

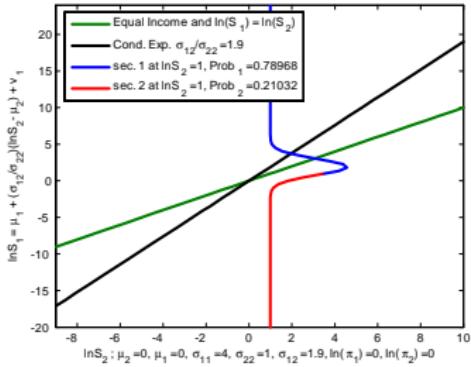


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 0) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

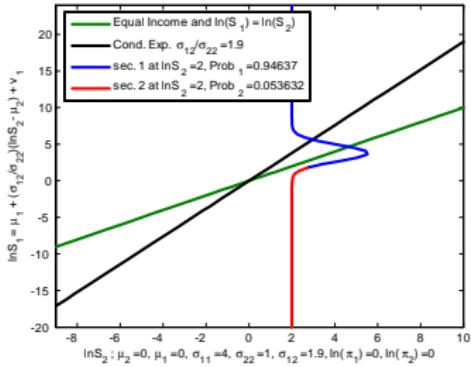


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 1) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

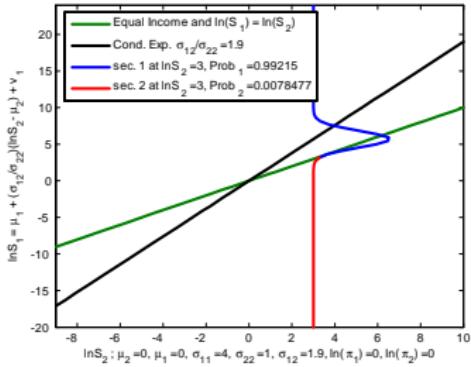


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 2) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

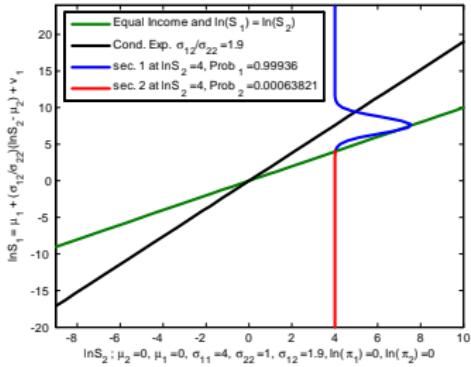


$\ln S_1 = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1$   
 $\text{Prob}_1 = \Pr(W_1 > W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 1}$   
 $\text{Prob}_2 = \Pr(W_1 < W_2 | \ln S_2 = 3) \Rightarrow \text{Pr. of Working at Sector 2}$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

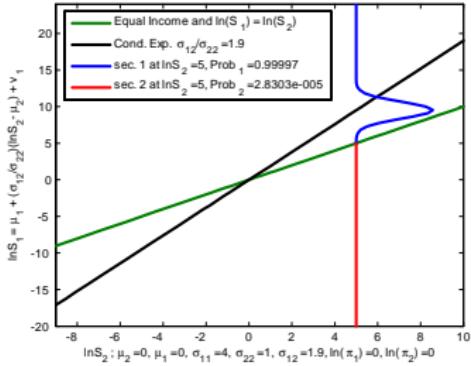


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 4) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$

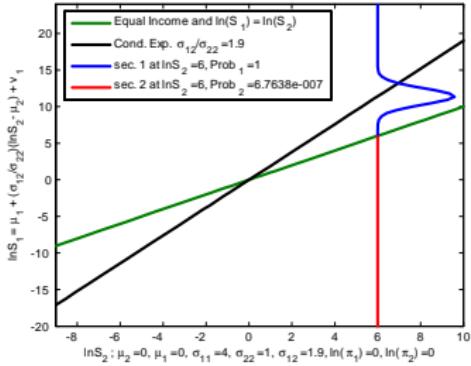


$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 5) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

Graph of  $\ln S_1 = f(\ln S_2)$



$$\begin{aligned} \ln S_1 &= \mu_1 + \frac{\sigma_{12}}{\sigma_{22}} (\ln S_2 - \mu_2) + v_1 \\ \text{Prob}_1 &= \Pr(W_1 > W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 1} \\ \text{Prob}_2 &= \Pr(W_1 < W_2 | \ln S_2 = 6) \Rightarrow \text{Pr. of Working at Sector 2} \end{aligned}$$

Parameters:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 4 & 1.9 \\ 1.9 & 1 \end{bmatrix}, \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \ln(\pi_1) \\ \ln(\pi_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

*Change in skill prices*

$$E[\ln S_1 \mid \ln W_1 > \ln W_2] = \mu_1 + \frac{\sigma_{11} - \sigma_{12}}{\sigma^*} \lambda(-c_1) > \mu_1$$

→ positive selection

$$E[\ln S_2 \mid \ln W_2 > \ln W_1] = \mu_2 + \frac{\sigma_{22} - \sigma_{12}}{\sigma^*} \lambda(-c_2) < \mu_2$$

→ negative selection

, where  $c_i = \ln(\pi_i/\pi_j) + \mu_i - \mu_j$

How does the skill composition react to a change in prices?

$$\frac{E[\ln S_1 \mid \ln W_1 > \ln W_2]}{\partial \ln \pi_1} < 0$$

$$\frac{E[\ln S_2 \mid \ln W_2 > \ln W_1]}{\partial \ln \pi_2} > 0$$

- ▶ What are the underlying economics?

# *Importance of Assignment Mechanism*

Heckman and Honore (1990) show that ...

*For a log normal Roy economy, any random assignment of persons to sectors with the same proportion of persons in each sector as in the Roy economy has higher variance of log earnings provided the proportions lie strictly in the unit interval. This is true whether or not skill prices in the two economies are the same.*

## Choices over Time



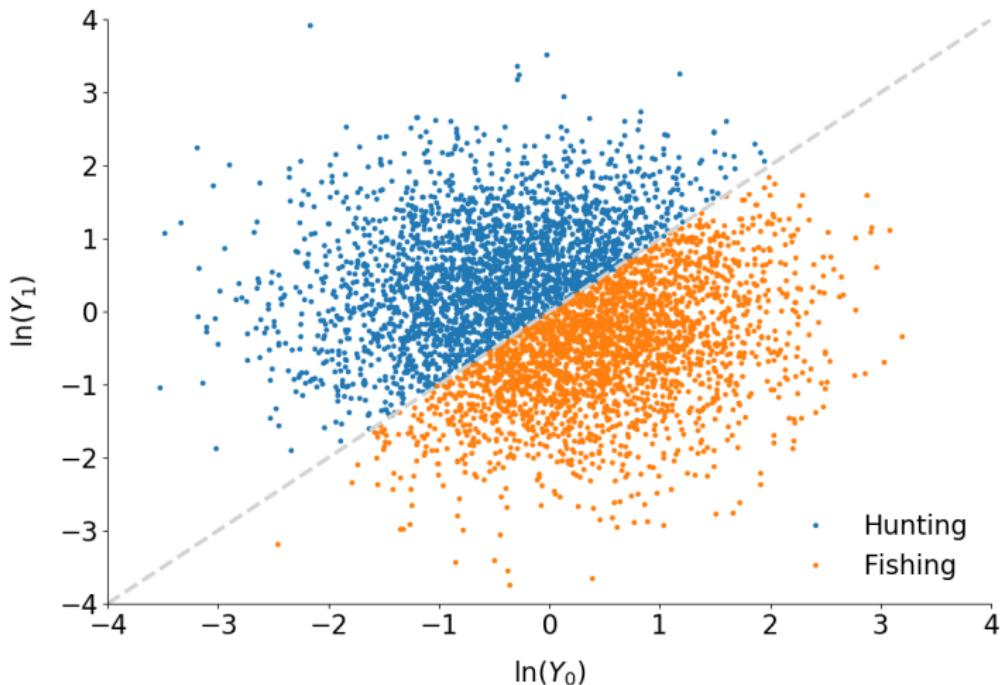
## *Incarnations of the Roy Model*

## Original Roy model

Potential Outcomes	Cost
$W_1 = \pi_1 S_1$	$C = 0$
$W_2 = \pi_2 S_2$	

Observed Outcomes	Choice
$W = DW_1 + (1 - D)W_2$	$S = W_1 - W_2$
	$D = I[S > 0]$

Figure: Occupational sorting in the original Roy model



## Extended Roy model

Potential Outcomes      Cost

$$Y_1 = \mu_1(X) + U_1 \quad C = \mu_D(Z)$$

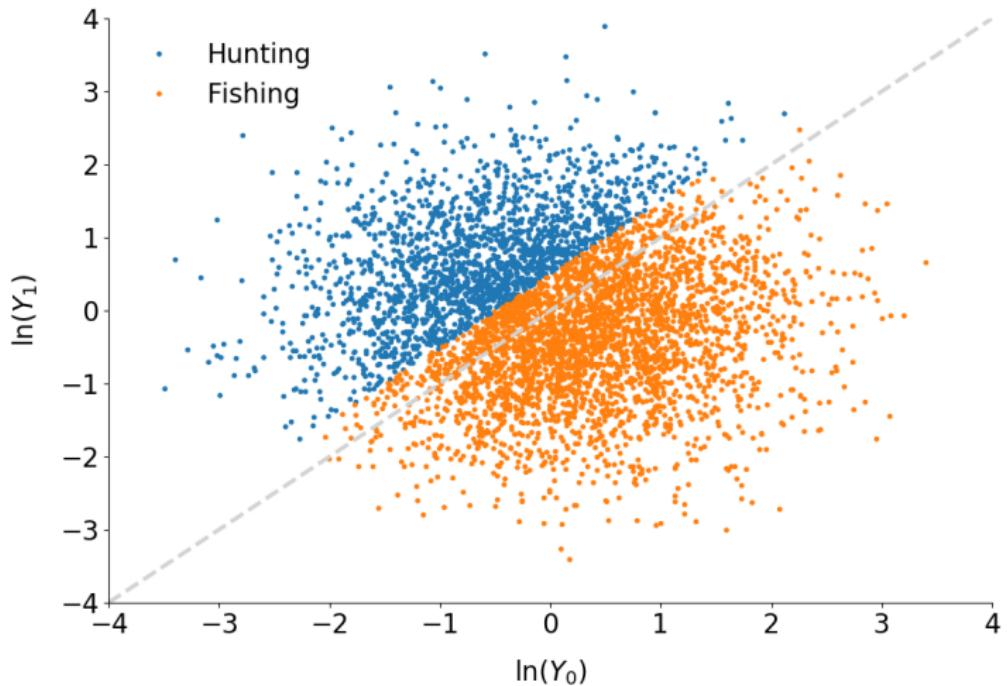
$$Y_0 = \mu_0(X) + U_0$$

Observed Outcomes      Choice

$$Y = DY_1 + (1 - D)Y_0 \quad S = Y_1 - Y_0 - C$$

$$D = I[S > 0]$$

Figure: Occupational sorting in the extended Roy model



## Generalized Roy model

Potential Outcomes      Observed Outcome

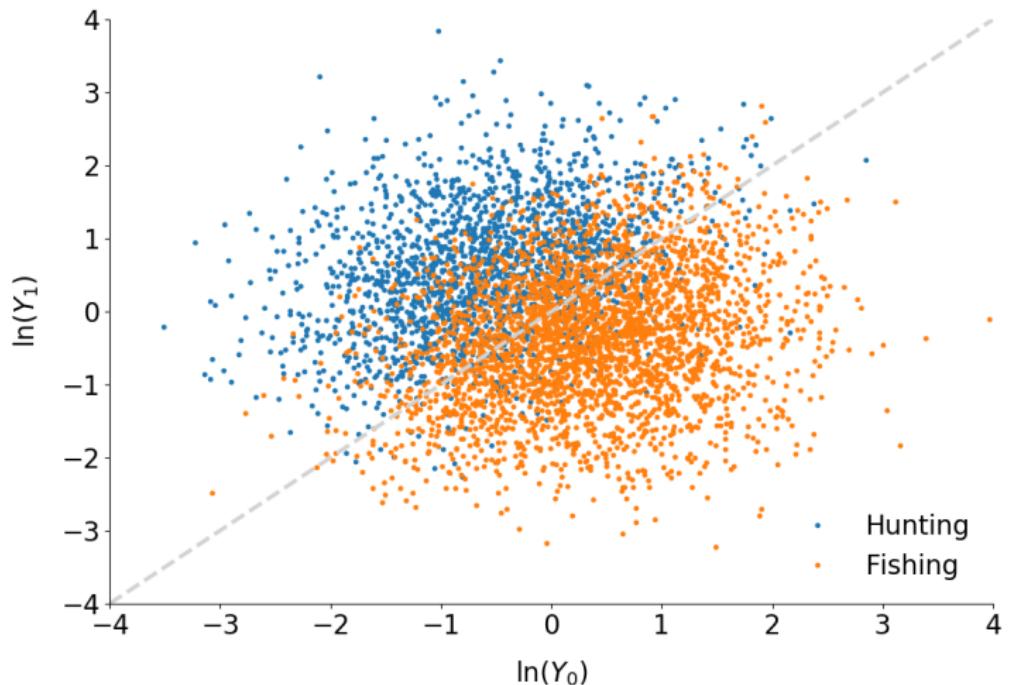
$$Y_1 = \mu_1(X) + U_1 \qquad Y = DY_1 + (1 - D)Y_0$$

$$Y_0 = \mu_0(X) + U_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

**Figure:** Occupational sorting in the generalized Roy model

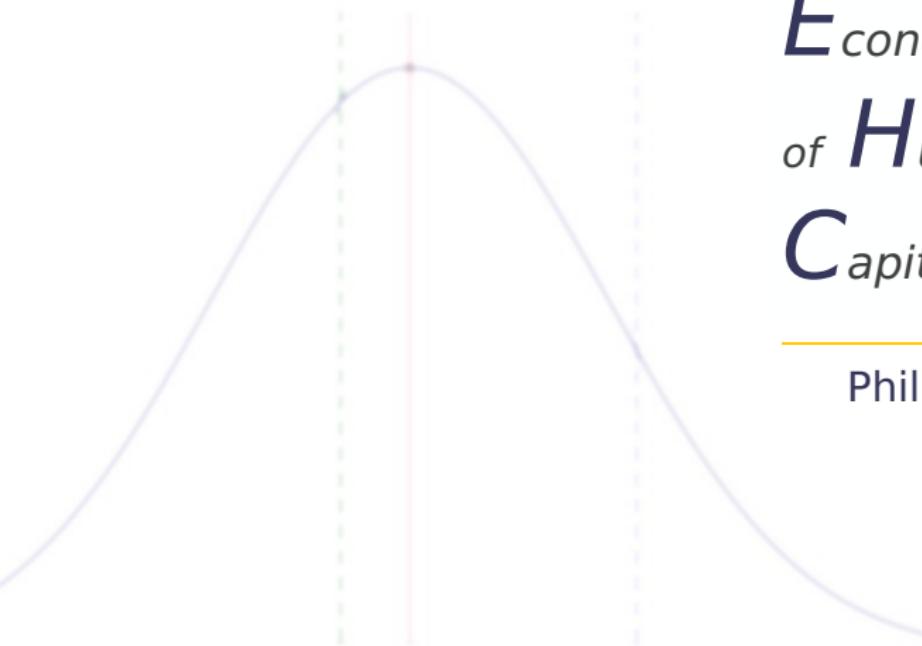


# **Appendix**

# *References*

- Carneiro, P., Heckman, J. J., & Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6), 2754–2781.
- Heckman, J. J. (1990). Selection bias and self-selection. In J. Eatwell, M. Milgate, & P. Newman (Eds.), *Econometrics* (pp. 201–224). London: Palgrave Macmillan.
- Heckman, J. J., & Honore, B. E. (1990). The empirical content of the Roy model. *Econometrica*, 58(5), 1121–1149.

- Heckman, J. J., & Taber, C. (2010). Roy model. In L. E. Blume & S. N. Durlauf (Eds.), *Microeconomics* (pp. 221–228). London: Palgrave Macmillan.
- Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford Economic Papers*, 3(2), 135–146.



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# Parameters of Interest

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Heckman (2008) sets out three tasks for us:

- ▶ Defining the Set of Hypotheticals or Counterfactuals  
⇒ A Scientific Theory
- ▶ Identifying Causal Parameters from Real Data  
⇒ Mathematical Analysis of Data Point or Set Identification
- ▶ Identifying Parameters from Real Data  
⇒ Estimation and Testing Theory

# *Setup*

## The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

## **Useful Notation**

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

$$U_D = F_V(V)$$

**Specification** We follow the parameterization in Heckman and Vytlacil (2005):

$$Y_1 = \gamma + \alpha + U_1 \quad U_1 = \sigma_1 \epsilon \quad \gamma = 0.670 \quad \sigma_1 = 0.012$$

$$Y_0 = \gamma + U_0 \quad U_0 = \sigma_0 \epsilon \quad \alpha = 0.200 \quad \sigma_0 = -0.050$$

$$D = I[Z - V > 0] \quad V = \sigma_V \epsilon \quad \epsilon \sim \mathbb{N}(0, 1) \quad \sigma_V = -1.000$$

$$Z \sim \mathbb{N}(-0.0026, 0.2700) \quad U_D = \Phi\left(\frac{V}{\sigma_V \sigma_\epsilon}\right)$$

# *Individual Heterogeneity*

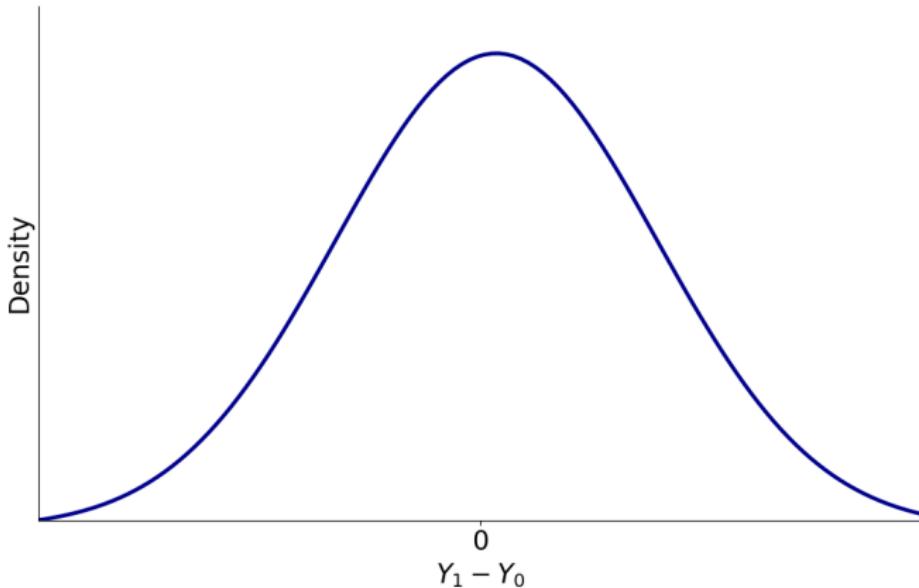
## Individual-specific Benefit of Treatment

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

## Sources of Heterogeneity

- ▶ Difference in Observable Characteristics
- ▶ Difference in Unobservable Characteristics
  - ▶ Uncertainty
  - ▶ Private Information

Figure: Distribution of Benefits

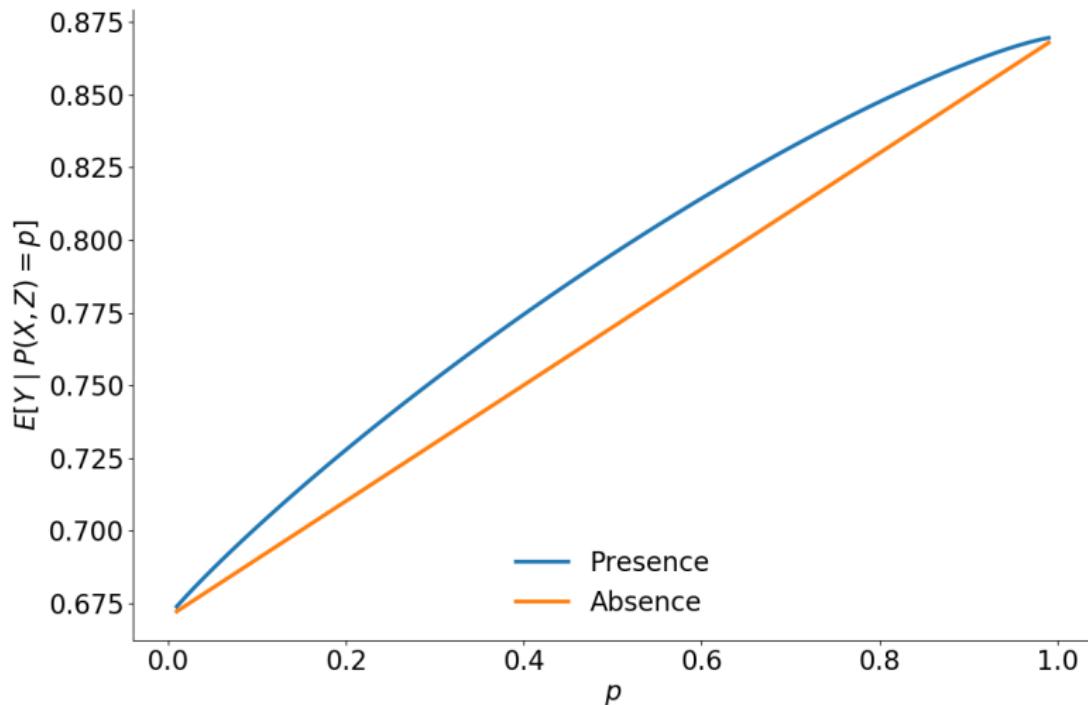


**Essential Heterogeneity Definition:** Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp D \quad | X = x.$$

⇒ consequences for the choice of the estimation strategy

Figure: Conditional Expectation and Essential Heterogeneity



# *Conventional Average Treatment Effects*

## Conventional Average Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

⇒ correspond to *extreme* policy alternatives

## Selection Problem

$$\begin{aligned} E[Y | D = 1] - E[Y | D = 0] &= \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\ &\quad + \underbrace{E[Y_1 - Y_0 | D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &\quad + \underbrace{E[Y_0 | D = 1] - E[Y_0 | D = 0]}_{\text{Selection Bias}} \end{aligned}$$

$$E[Y | D = 1] - E[Y | D = 0] = \underbrace{E[Y_1 - Y_0 | D = 1]}_{B^{TT}} + \underbrace{E[Y_0 | D = 1] - E[Y_0 | D = 0]}_{\text{Selection Bias}}$$

⇒ the bias depends on the parameter of interest

Figure: Distribution of Effects with Essential Heterogeneity

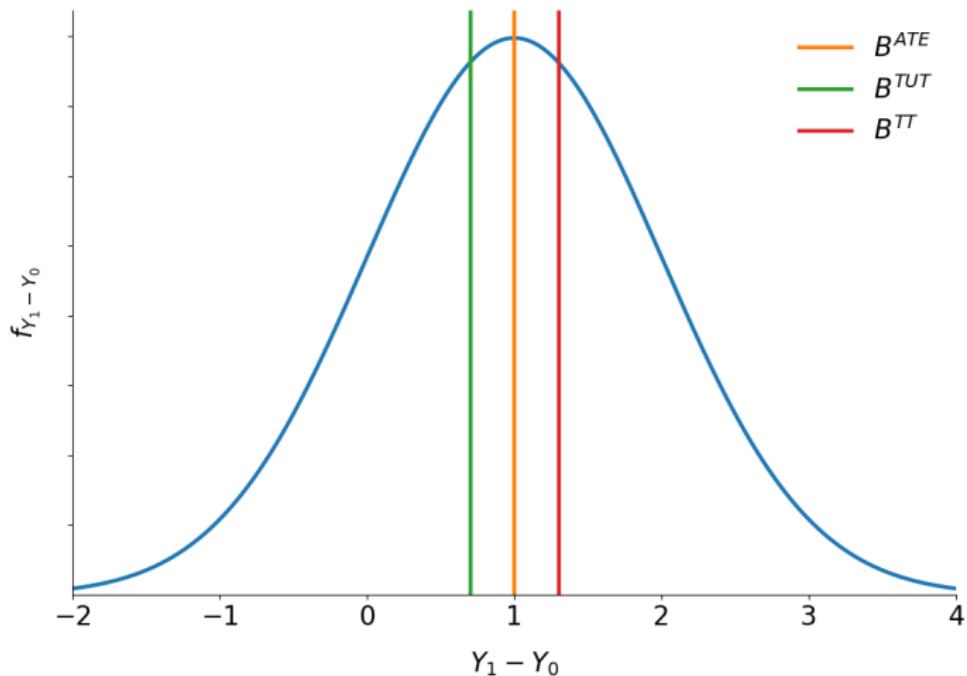


Figure: Distribution of Effects without Essential Heterogeneity

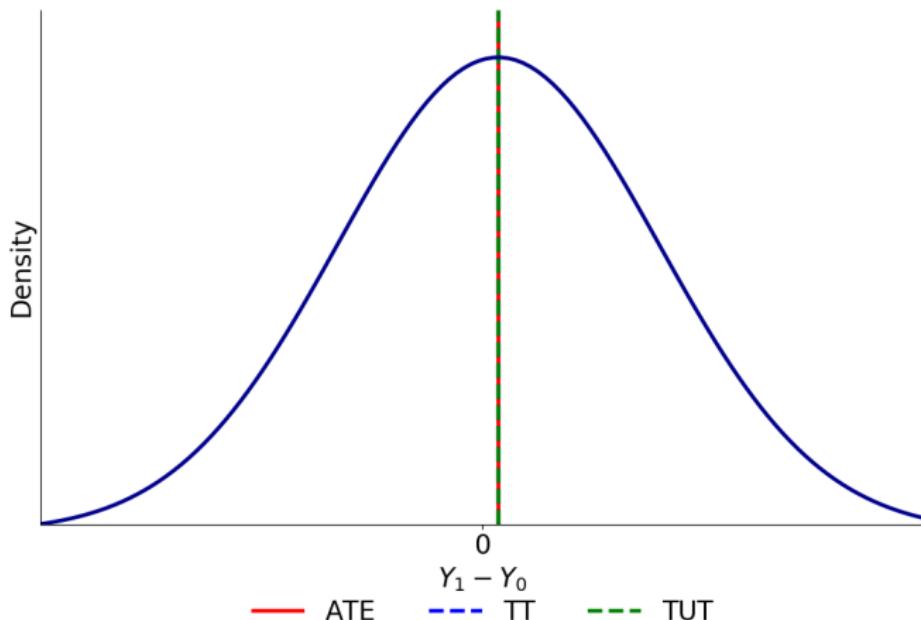
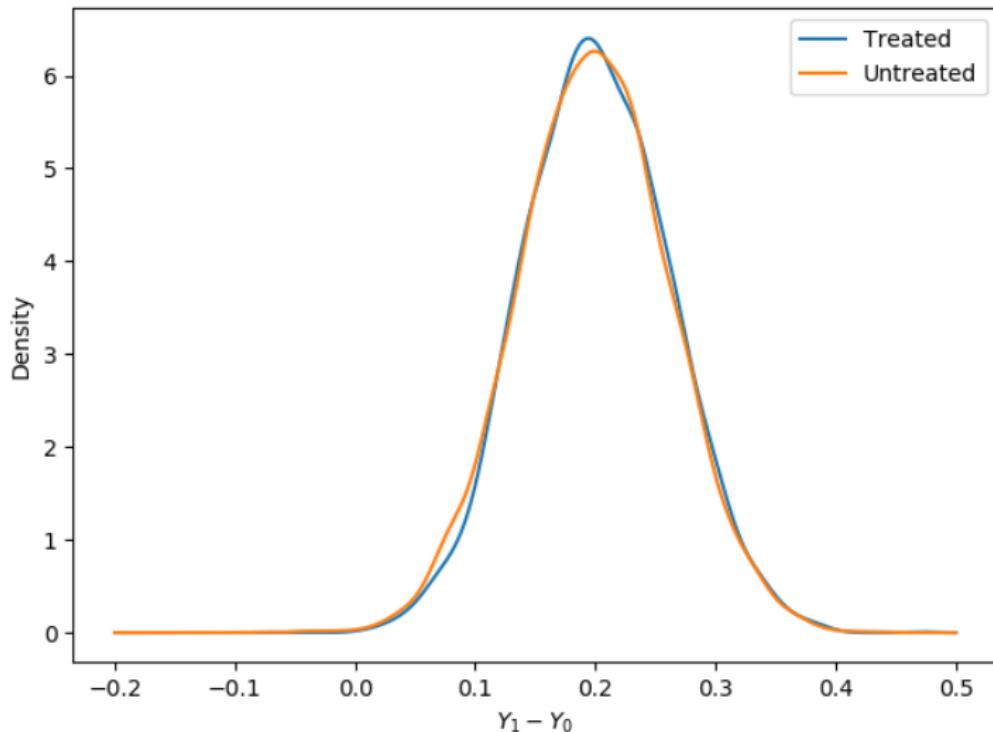


Figure: Distribution of Benefits by Treatment Status



# *Policy-Relevant Average Treatment Effects*

## Observed Outcomes

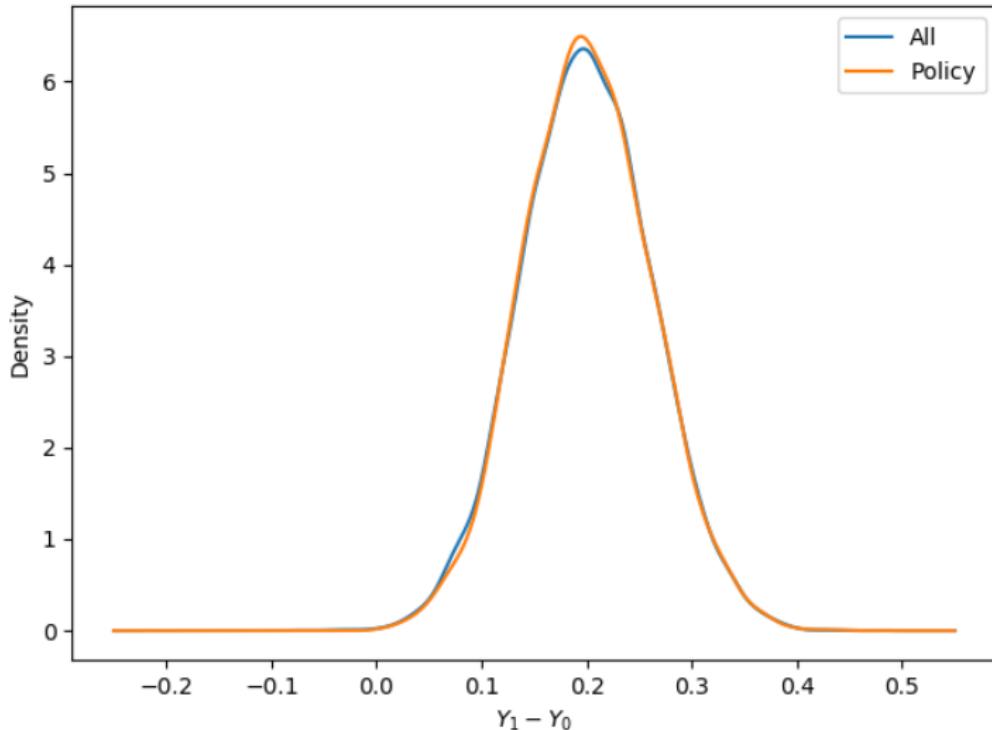
$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$

$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

## Effect of Policy

$$B^{PTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Figure: Distribution of Benefits for Policy



## *Marginal Effect of Treatment*

## Marginal Benefit of Treatment

$$B^{MTE}(x, u_D) = E[Y_1 - Y_0 \mid X = x, U_D = u_D]$$

**Intuition:** Mean gross return to treatment for persons at quantile  $u_D$  of the first-stage unobservable  $V$  or a willingness to pay for individuals at the margin of indifference.

Figure: Margin of Indifference

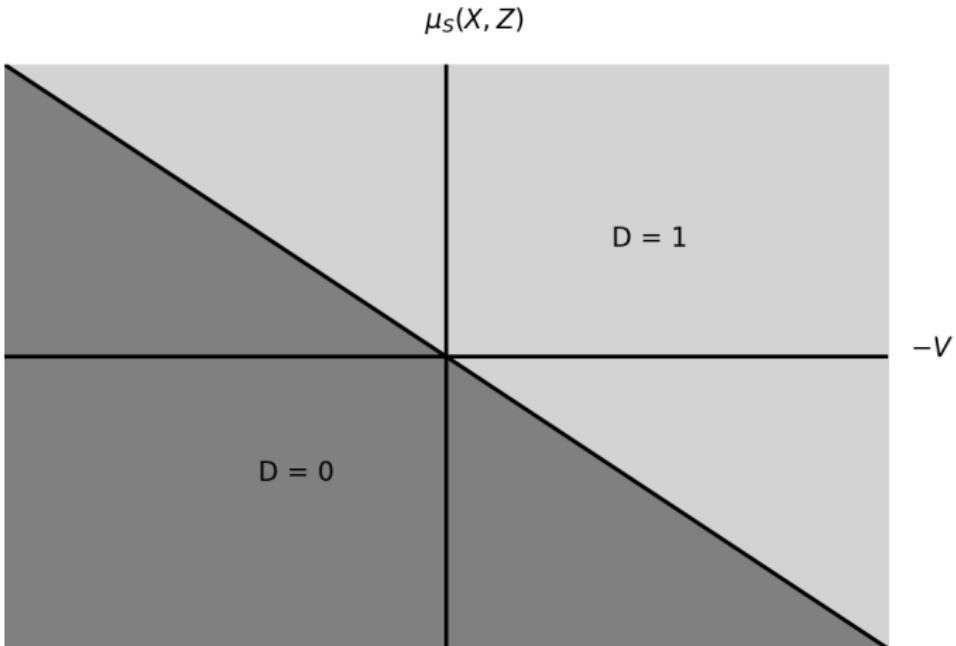
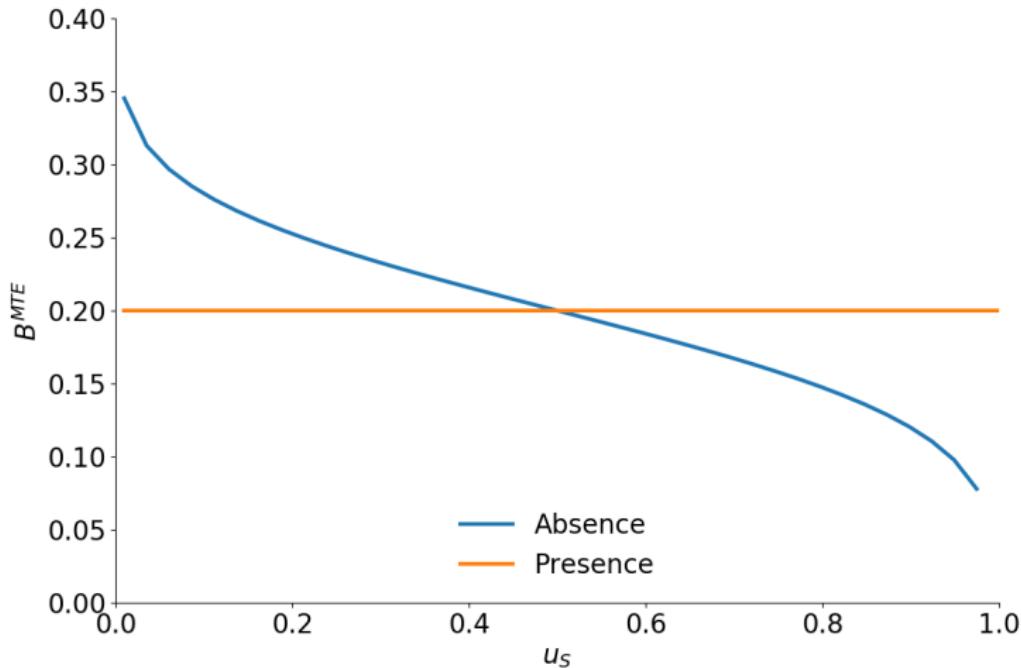


Figure: Marginal Benefit of Treatment



**Effects of Treatment as Weighted Averages** Parameter  $\Delta_j$ , can be written as a weighted average of the  $B^{MTE}(x, u_D)$ .

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D) \omega^j(x, u_D) du_D,$$

where the weights  $\omega^j(x, u_D)$  are specific to parameter  $j$  and integrate to one.

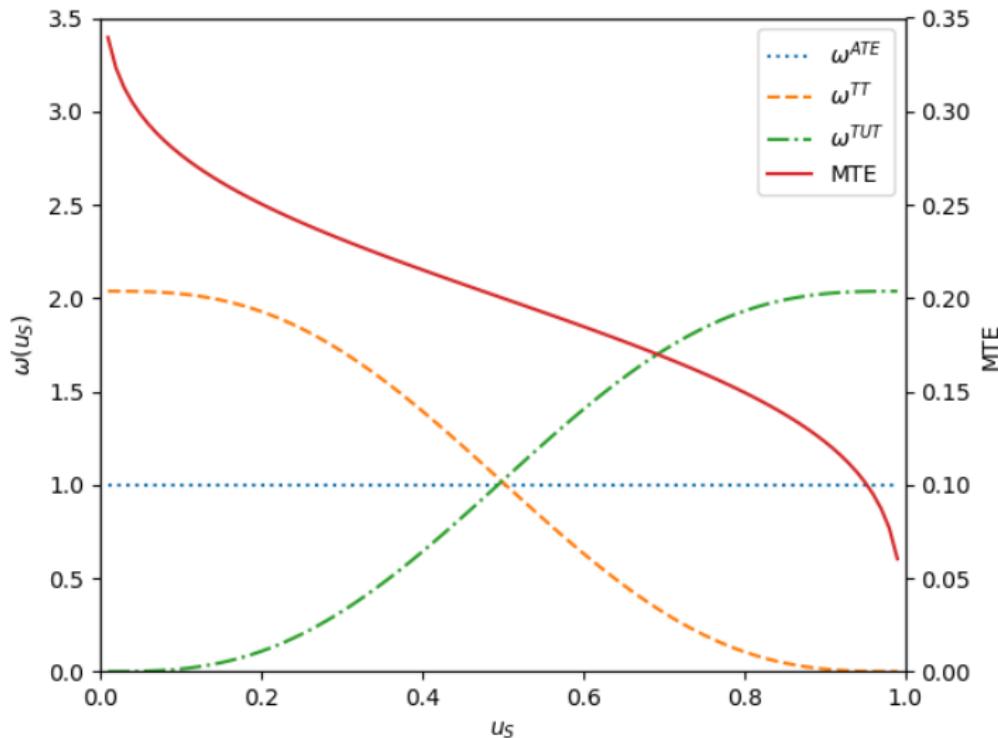
## Weights

$$\omega^{ATE}(x, u_D) = 1$$

$$\omega^{TT}(x, u_D) = \frac{1 - F_{P|X=x}(u_D)}{E[P | X = x]}$$

$$\omega^{TUT}(x, u_D) = \frac{F_{P|X=x}(u_D)}{E[1 - P | X = x]}$$

Figure: Effects of Treatment as Weighted Averages



# *Local Average Treatment Effect*

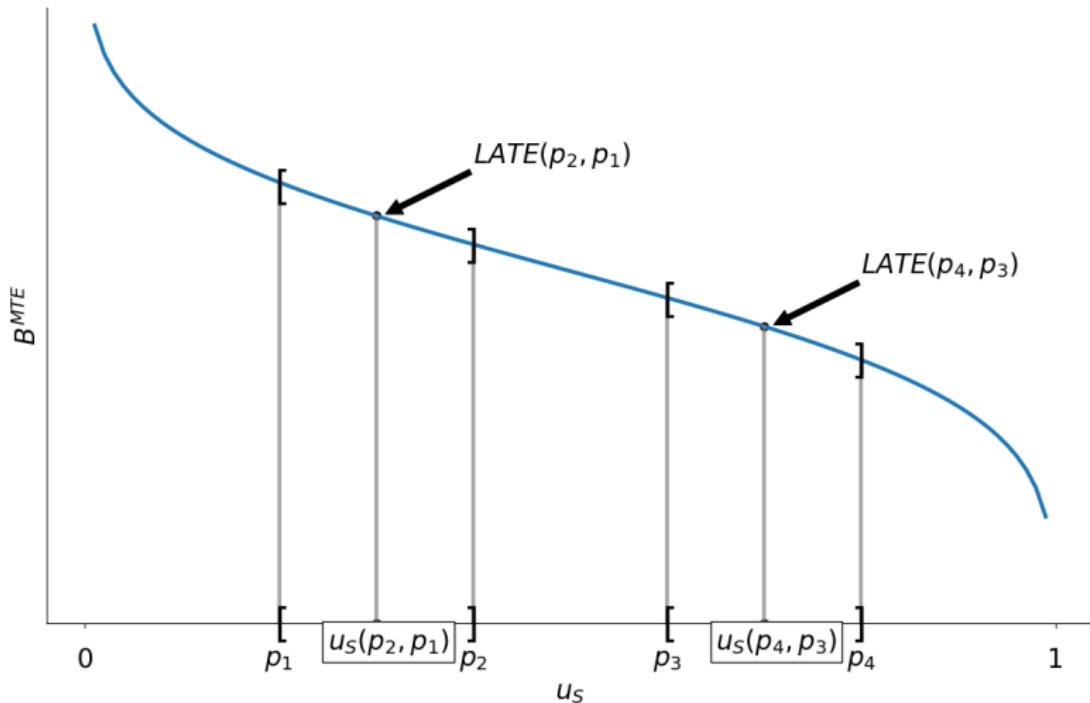
## **Local Average Treatment Effect**

- ▶ **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument. $\Rightarrow$  instrument-dependent parameter
  
- ▶ **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.  
 $\Rightarrow$  deep economic parameter

$$B^{LATE} = \frac{E(Y | Z = z) - E[Y | Z = z']}{P(z) - P(z')}$$

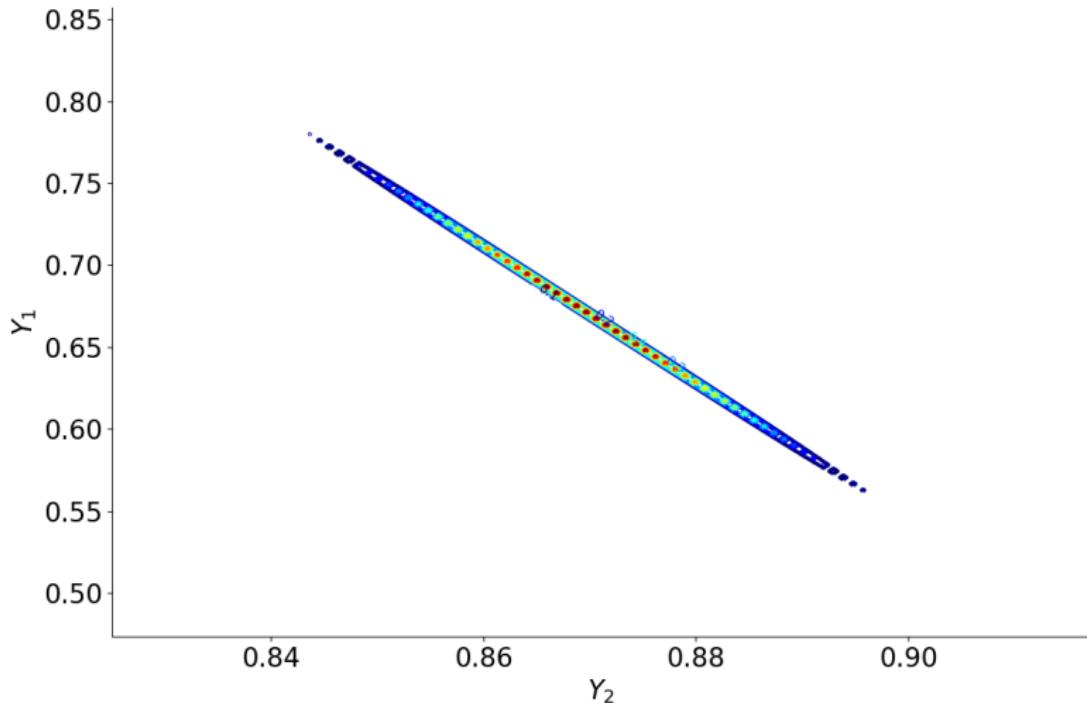
$$B^{LATE}(x, u_D, u_{S'}) = \frac{1}{u_D - u_{D'}} \int_{u_D}^{u_{S'}} B^{MTE}(x, u) du,$$

Figure: Local Average Treatment Effect

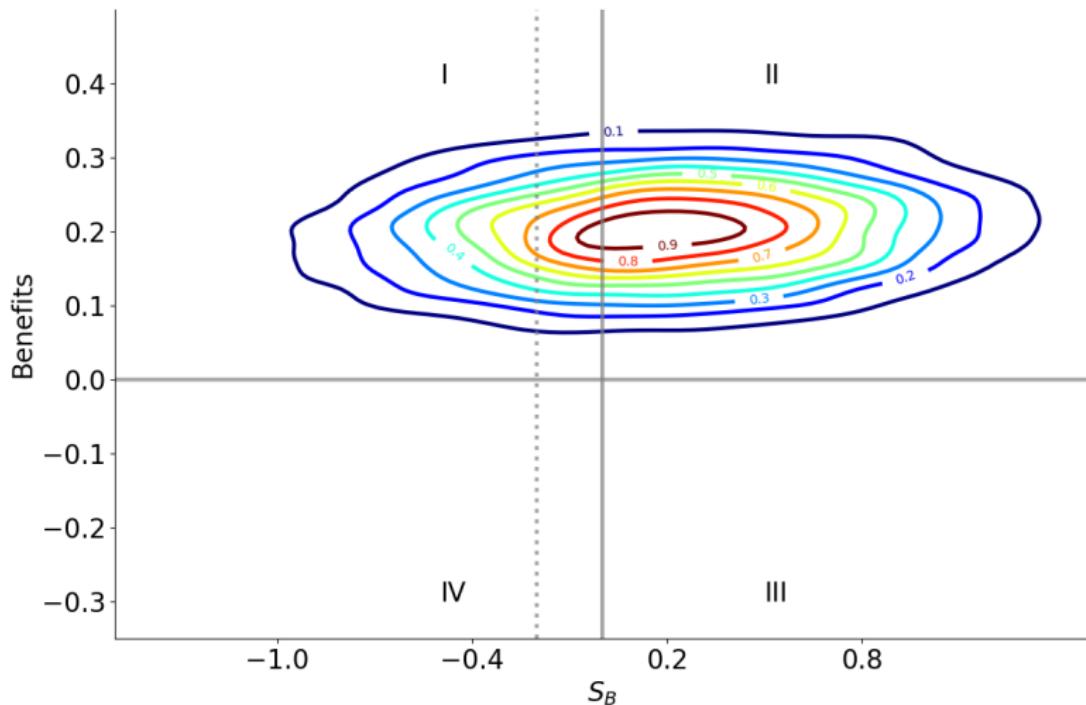


## *Distributions of Effects*

**Figure:** Distribution of Potential Outcomes



**Figure: Distribution of Benefits and Surplus**

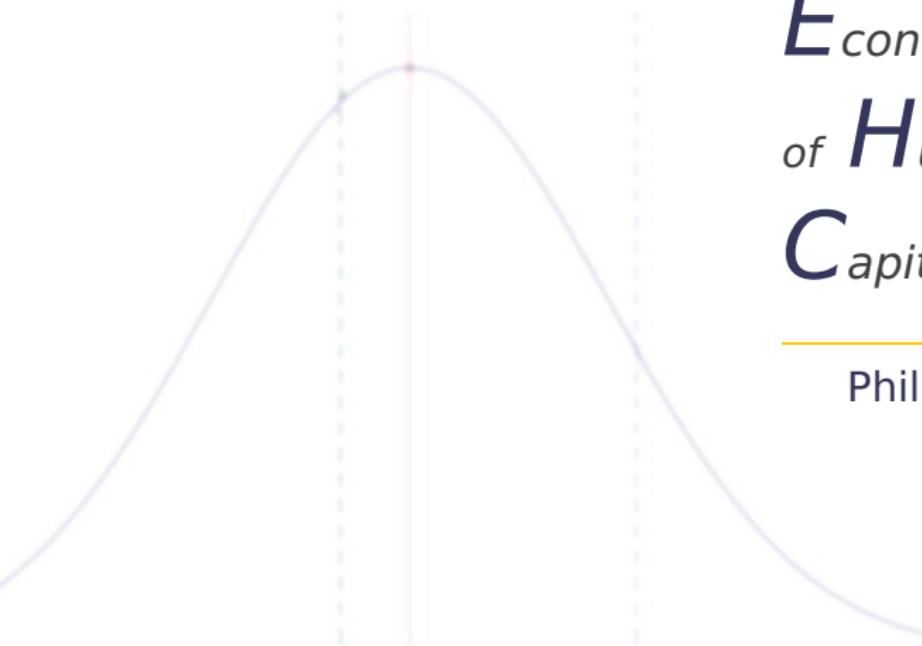


# **Appendix**

# *References*

- Carneiro, P., Heckman, J. J., & Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6), 2754–2781.
- Heckman, J. J. (1990). Selection bias and self-selection. In J. Eatwell, M. Milgate, & P. Newman (Eds.), *Econometrics* (pp. 201–224). London: Palgrave Macmillan.
- Heckman, J. J. (2008). Econometric causality. *International Statistical Review*, 76(1), 1–27.
- Heckman, J. J., & Taber, C. (2010). Roy model. In L. E. Blume & S. N. Durlauf (Eds.), *Microeconomics* (pp. 221–228). London: Palgrave Macmillan.

Heckman, J. J., & Vytlacil, E. J. (2005). Structural equations, treatment effects, and econometric policy evaluation. *Econometrica*, 73(3), 669–739.



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# Estimation Strategies

Philipp Eisenhauer

# **Setup**

## The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

## Key Concept

**Definition:** Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \perp\!\!\!\perp D \quad | X = x.$$

⇒ consequences for the choice of the estimation strategy

## Useful Notation

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

$$U_D = F_V(V)$$

## Key Assumptions

- ▶  $(U_1, U_0, V)$  are independent of  $Z$  conditional on  $X$
- ▶  $\mu_D(X, Z)$  is a nondegenerate random variable conditional on  $X$
- ▶  $0 < \Pr(D = 1 | X) < 1$
- ▶ ...

## Evaluation Problem

$$Y = DY_1 + (1 - D)Y_0 = \begin{cases} Y_1 & \text{if } D = 1 \\ Y_0 & \text{if } D = 0 \end{cases}$$

## Selection Problem

$$\begin{aligned} E[Y | D = 1] - E[Y | D = 0] &= \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\ &\quad + \underbrace{E[Y_1 - Y_0 | D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &\quad + \underbrace{E[Y_0 | D = 1] - E[Y_0 | D = 0]}_{\text{Selection Bias}} \end{aligned}$$

## **Estimation Strategies**

- ▶ Randomization
- ▶ Matching
- ▶ Instrumental Variables
  - ▶ conventional and local
- ▶ Regression Discontinuity
  - ▶ fuzzy and sharp design

# **Randomization**

## Treatment Status

$D$  self-selected

$\xi$  assigned

$A$  actual

## **Key Identifying Assumptions**

$$(Y_1, Y_0) \perp\!\!\!\perp D$$

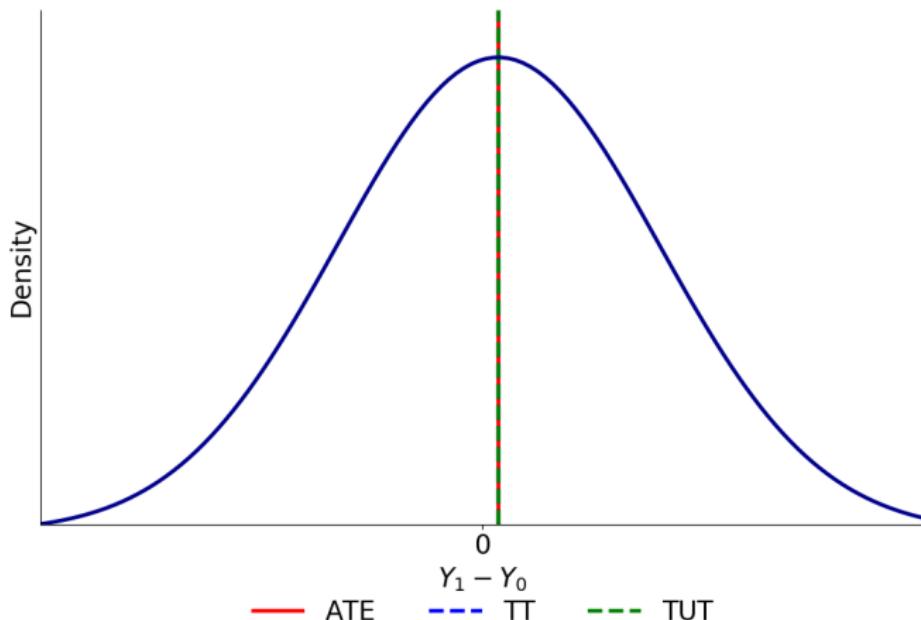
$$(Y_1, Y_0) \perp\!\!\!\perp \xi$$

$$(Y_1, Y_0) \perp\!\!\!\perp A$$

When do we have to worry about compliance?

$$\begin{aligned} E(Y | A = 1) - E(Y | A = 0) \\ &= E(Y_1 | A = 1) - E(Y_0 | A = 0) \\ &\quad \text{(by full compliance)} \\ &= E(Y_1) - E(Y_0) \quad \text{(by randomization)} \\ &= ATE = TT = TUT \end{aligned}$$

Figure: Distribution of Effects



What if we can only deny program participation to individuals who are willing to participate?

$$\begin{aligned}E(Y | D = 1, A = 1) - E(Y | D = 1, A = 0) \\&= E(Y_1 | D = 1, A = 1) - E(Y_0 | D = 1, A = 0) \\&= E(Y_1 | D = 1) - E(Y_0 | D = 1) \\&= TT \neq ATE \neq TUT\end{aligned}$$

## **Issues**

- ▶ Compliance
- ▶ Imperfect Randomization
- ▶ Ethical Concerns
- ▶ Feasibility
- ▶ Expenses
- ▶ External Validity

## Challenges to Scaling Experiments

- ▶ market equilibrium effects
- ▶ spillovers
- ▶ political reactions
- ▶ context dependence
- ▶ randomization or site-selection bias
- ▶ piloting bias

See Banerjee et al. (2017) for a discussion of these challenges and their attempts to address them in their work.

# **Matching**

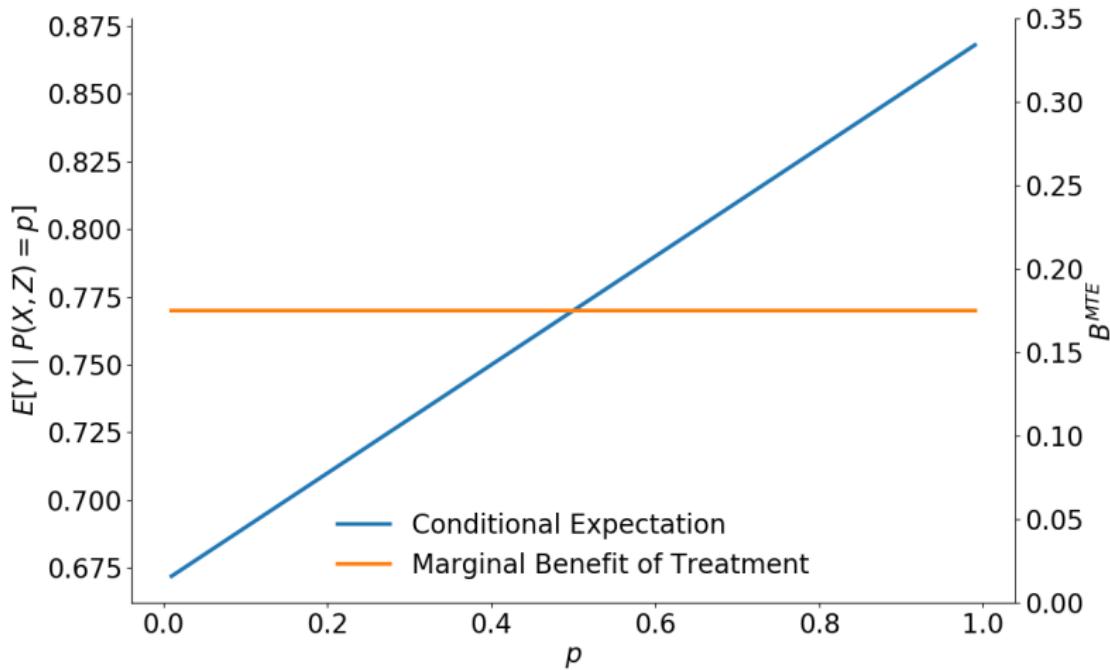
## **Key Identifying Assumption**

$$(Y_1, Y_0) \perp\!\!\!\perp D | X$$

What is in the agent's and econometrician's information set?

- ▶ J. J. Heckman and Navarro-Lozano (2004) highlights the sensitivity of results to different conditioning variables.

Figure: Matching and Essential Heterogeneity



# **Instrumental Variables**

## **Key Identifying Assumption**

$$(Y_1, Y_0) \perp\!\!\!\perp Z | X$$

Even in the best cases, this is sometimes not as obvious as you think. See J. Heckman (1997) for a study of implicit behavioral assumptions used in making program evaluations.

## Conventional Notation

$$Y = \alpha + \beta D + \epsilon,$$

where

$$\alpha = \mu_0(X)$$

$$\beta = (Y_1 - Y_0) = \mu_1(X) - \mu_0(X) + (U_1 - U_0)$$

$$\epsilon = U_0$$

Assume for now that there is no treatment effect heterogeneity, i.e.  $Y_1 - Y_0$  is the same for everybody. If we have access to a variable  $Z$  with the following properties

...

$$\text{cov}(Z, D) \neq 0$$

$$\text{cov}(Z, \epsilon) = 0$$

then the following holds

$$\text{plim } \hat{\beta}_{IV} = \frac{\text{cov}(Z, Y)}{\text{cov}(Z, D)} = \beta$$

What happens if  $\beta$  varies in the population?

- ▶ Do individuals select their treatment status based on gains?  
⇒ essential heterogeneity

Let  $\beta = E[\beta] + \eta$ , where  $U_1 - U_0 = \eta$ , then

$$Y = \alpha + \bar{\beta}D + [\epsilon + \eta D].$$

and

$$\text{plim } \hat{\beta}_{IV} = \bar{\beta} + \frac{\text{cov}(Z, \epsilon + \eta D)}{\text{cov}(D, Z)}$$

So we cannot even learn about the mean effect of treatment unless we rule out essential heterogeneity, i.e. individuals selecting their treatment status based on gains.

## Local Average Treatment Effect

- ▶ Average effect for those induced to change treatment because of a change in the instrument.  
⇒ instrument-dependent parameter

$$\frac{E(Y | Z = z) - E[Y | Z = z']}{P(z) - P(z')} = \\ E(Y_1 - Y_0 | D(z) = 1, D(z') = 0)$$

## *Local Instrumental Variables*

## Local Instrumental Variable

$$\begin{aligned}\frac{\partial E(Y | P(Z) = p)}{\partial p} \Big|_{p=u_D} &= E(Y_1 - Y_0 | U_D = u_D) \\ &= B^{MTE}(u_D)\end{aligned}$$

⇒ we can only identify the  $B^{MTE}(u_D)$  over the support of  $p$  in our data

Figure: Observed Outcome and Essential Heterogeneity

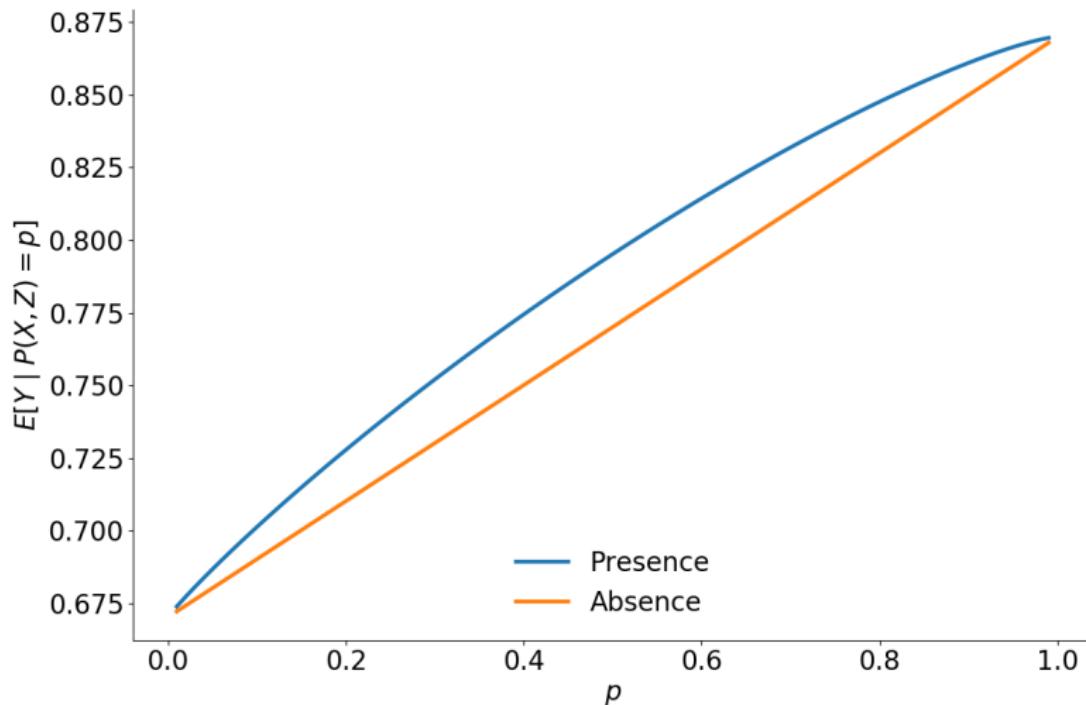
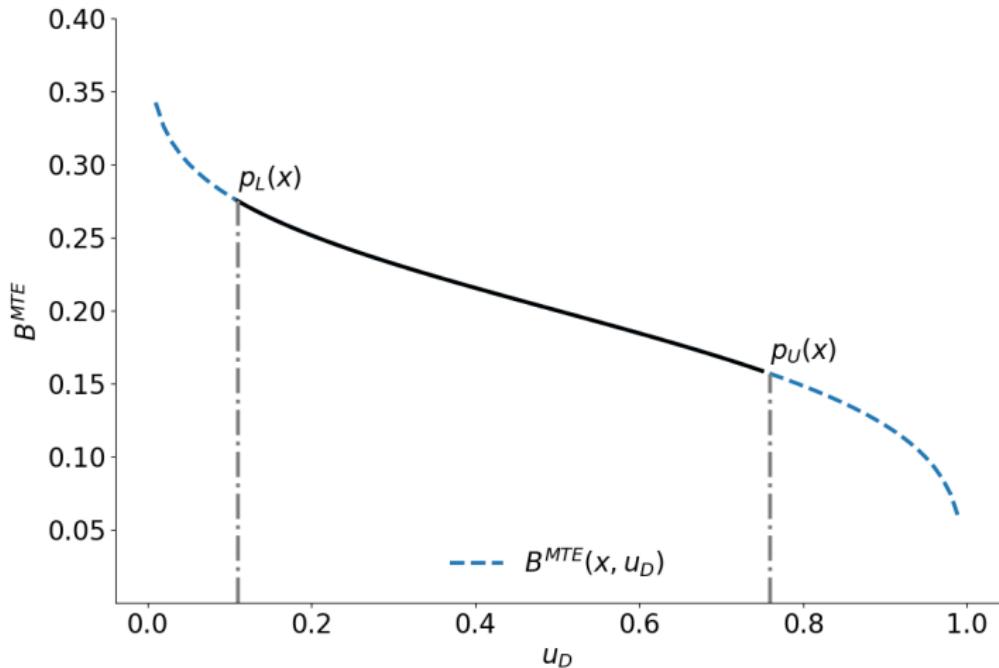


Figure: Identification I



Making  $X = x$  explicit

$$\begin{aligned} E(Y_1 - Y_0 | X = x, U_D = u_D) \\ = (\mu_1(x) - \mu_0(x)) + E(U_1 - U_0 | X = x, U_D = u_D) \end{aligned}$$

but if we are willing to assume  $(U_1 - U_0) \perp\!\!\!\perp X$  then

$$\begin{aligned} E(Y_1 - Y_0 | X = x, U_D = u_D) \\ = (\mu_1(x) - \mu_0(x)) + E(U_1 - U_0 | U_D = u_D) \end{aligned}$$

Figure: Identification II

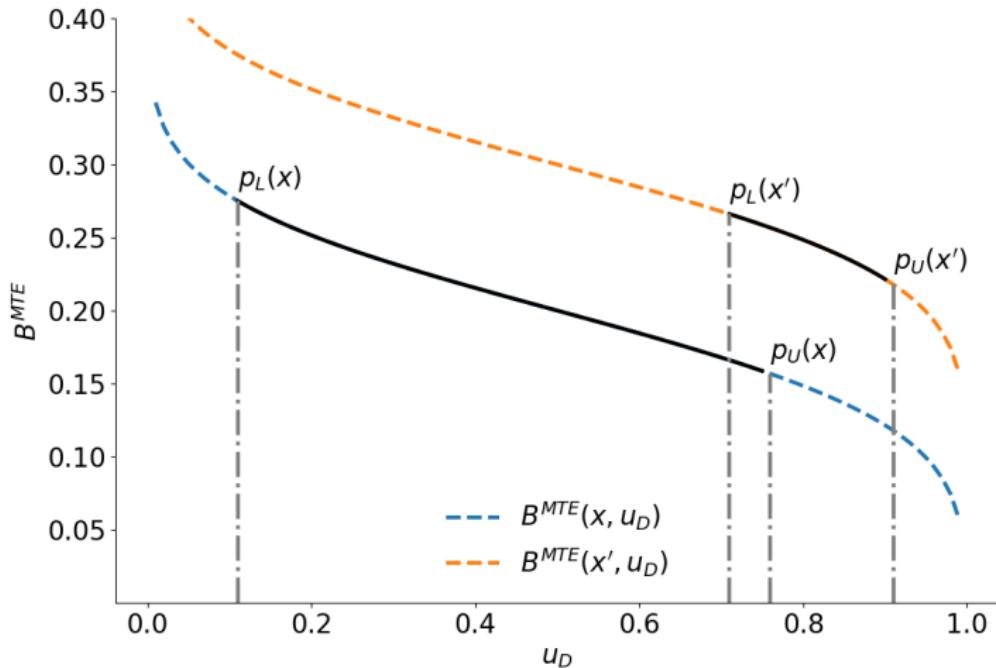
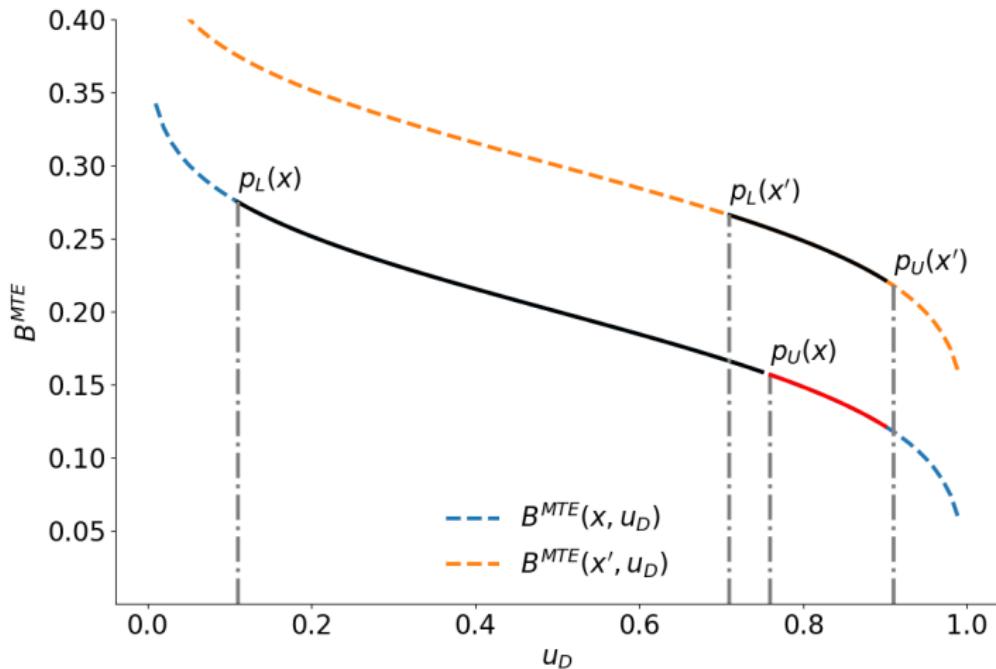


Figure: Identification III



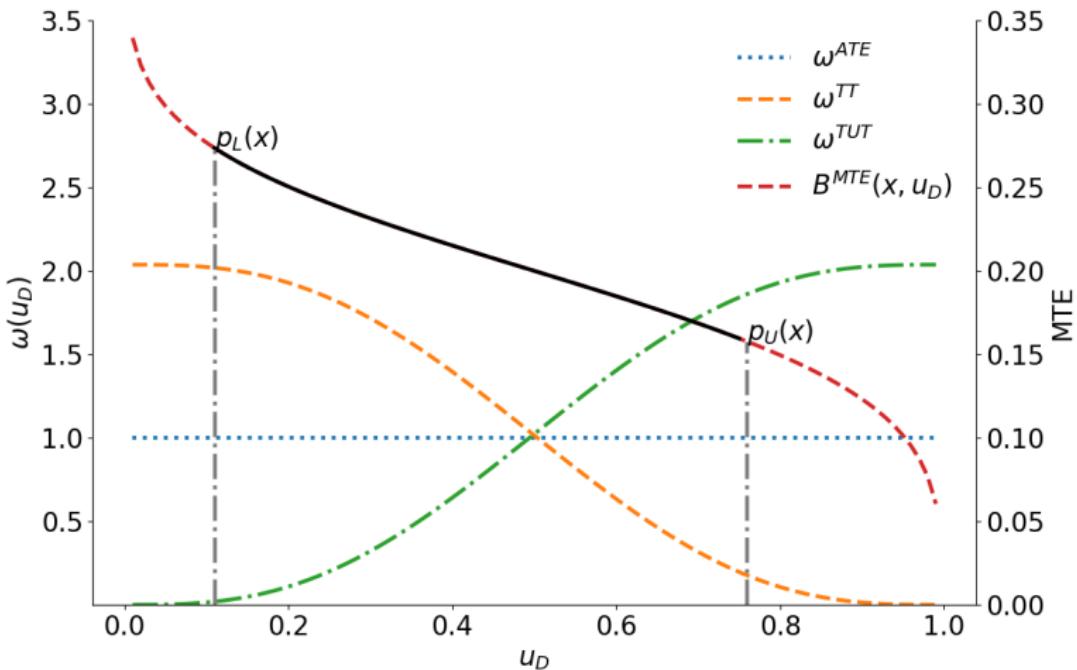
## **Effects of Treatment as Weighted Averages**

Parameter  $\Delta_j$ , can be written as a weighted average of the  $B^{MTE}(x, u_D)$ .

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_D) \omega^j(x, u_D) du_D,$$

where the weights  $\omega^j(x, u_D)$  are specific to parameter  $j$  and integrate to one.

Figure: Identification IV



# **Regression Discontinuity Design**

Suppose  $D = 1$  if  $X \geq x_0$ , and  $D = 0$  otherwise

$$\Rightarrow \begin{cases} E(Y | X = x) = E(Y_0 | X = x) & \text{for } x < x_0 \\ E(Y | X = x) = E(Y_1 | X = x) & \text{for } x \geq x_0 \end{cases}$$

Suppose  $E(Y_1 | X = x)$ ,  $E(Y_0 | X = x)$  are continuous in  $x$ .

$$\Rightarrow \begin{cases} \lim_{\epsilon \searrow 0} E(Y_0 | X = x_0 - \epsilon) = E(Y_0 | X = x_0) \\ \lim_{\epsilon \searrow 0} E(Y_1 | X = x_0 + \epsilon) = E(Y_1 | X = x_0) \end{cases}$$

$$\begin{aligned}& \lim_{\epsilon \searrow 0} E(Y | X = x_0 + \epsilon) - \lim_{\epsilon \searrow 0} E(Y | X = x_0 - \epsilon) \\&= \lim_{\epsilon \searrow 0} E(Y_1 | X = x_0 + \epsilon) - \lim_{\epsilon \searrow 0} E(Y_0 | X = x_0 - \epsilon) \\&= E(Y_1 | X = x_0) - E(Y_0 | X = x_0) \\&= E(Y_1 - Y_0 | X = x_0)\end{aligned}$$

Figure: Probability

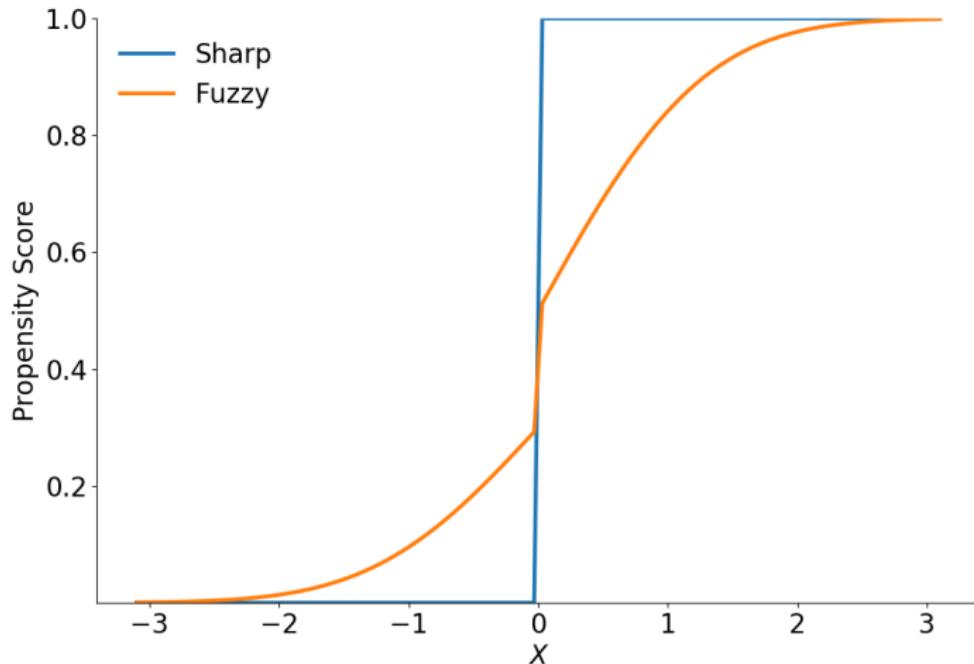
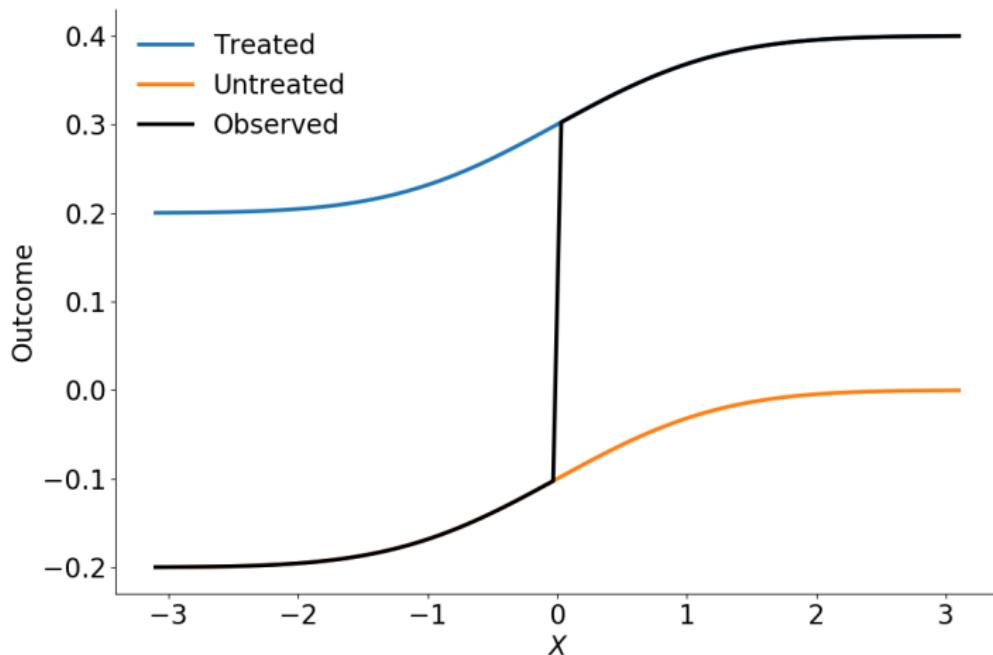
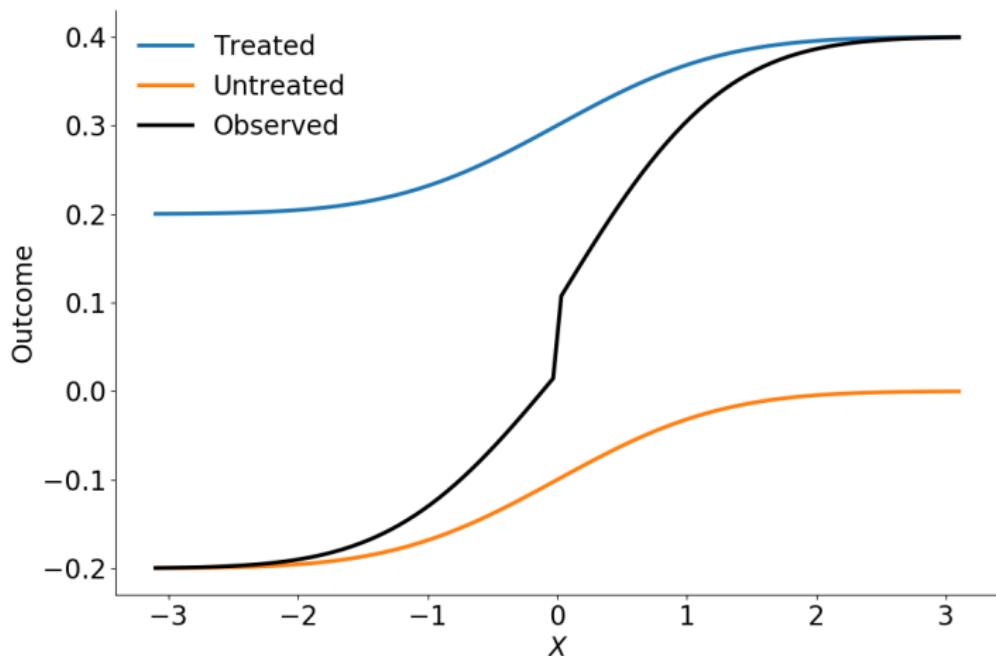


Figure: Observed Outcome in a Sharp Design



**Figure:** Observed Outcome in a Fuzzy Design



# **Conclusion**

*We must not cease from exploration  
and the end of all our exploring will  
be to arrive where we began and to  
know the place for the first time.*

- T. S. Eliot (1943)

# **Appendix**

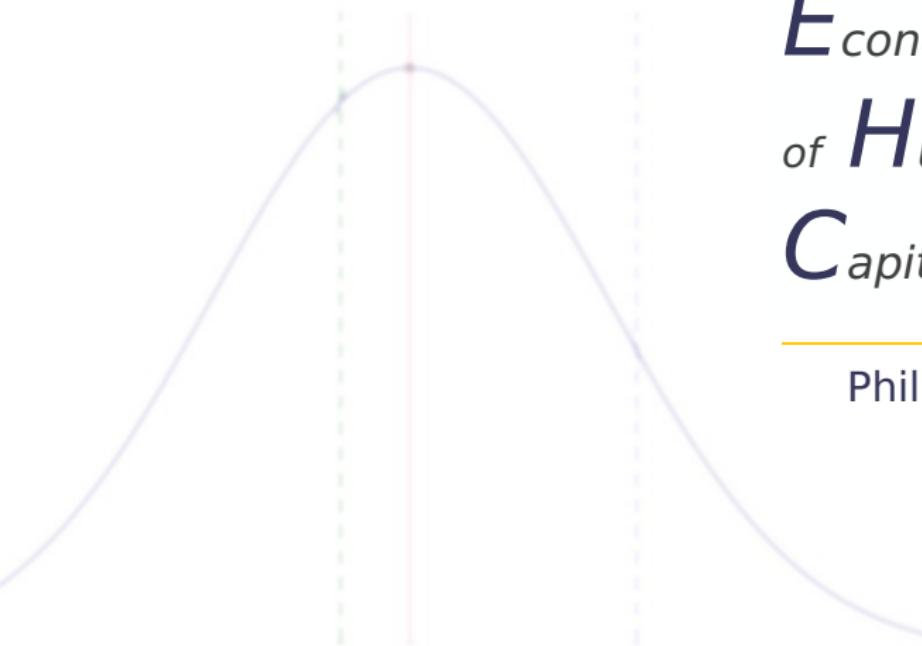
# *References*

- Banerjee, A., Banerji, R., Berry, J., Duflo, E., Kannan, H., Mukerji, S., ... Walton, M. (2017). From proof of concept to scalable policies: Challenges and solutions, with an application. *Journal of Economic Perspectives*, 31(4), 73–102.
- Carneiro, P., Heckman, J. J., & Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6), 2754–2781.
- Eliot, T. (1943). Little gidding. In *Four quartets*. Harcourt.

Heckman, J. (1997). Instrumental variables: A study of implicit behavioral assumptions used in making program evaluations. *The Journal of Human Resources*, 32(3), 441. Retrieved from <http://www.jstor.org/stable/146178?origin=crossref>  
doi: doi: 10.2307/146178

Heckman, J. J. (1990). Selection bias and self-selection. In J. Eatwell, M. Milgate, & P. Newman (Eds.), *Econometrics* (pp. 201–224). London: Palgrave Macmillan.

- Heckman, J. J., & Navarro-Lozano, S. (2004). Using matching, instrumental variables, and control functions to estimate economic choice models. *Review of Economics and Statistics*, 86(1), 30–57.
- Heckman, J. J., & Taber, C. (2010). Roy model. In L. E. Blume & S. N. Durlauf (Eds.), *Microeconomics* (pp. 221–228). London: Palgrave Macmillan.



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Philipp Eisenhauer

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# *grmpy* Tutorial

Sebastian Becker

# **Introduction**

## grmpy

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`grmpy` is an open-source Python package for the simulation and estimation of generalized Roy model. Its main purpose is to serve as a teaching tool to promote the conceptual framework provided by the generalized Roy model to illustrate a variety of issues in the econometrics of policy evaluation.

## *grmpy*

- ▶ ***grmpy*** is ...
  - ▶ ...an open-source Python Package for the simulation and estimation of the generalized Roy model.
  - ▶ ...intended as an useful device to support and improve the understanding of the framework by the opportunity to experience the effect of particular specifications directly.

# **Setup**

## Setup

- ▶ Normal linear-in-parameters version of the generalized Roy model.

Potential Outcomes      Cost

$$Y_1 = \beta_1 X + U_1 \quad C = \gamma Z + U_C$$

$$Y_0 = \beta_0 X + U_0$$

Observed Outcomes      Choice

$$Y = DY_1 + (1 - D)Y_0 \quad S = Y_1 - Y_0 - C$$

$$D = I[S > 0]$$

# **Features**

## Features

- ▶ *grmpy* is currently capable of the following features:
  - ▶ Simulating a dataset based on your own specifications.
  - ▶ Providing some useful information about the simulated dataset for instance:
    - ▶ Distributional outcome characteristics
    - ▶ ATE, TT, TUT
    - ▶ MTE by decile
  - ▶ Estimating the coefficients of interest given a dataset (of a specific form).

## **Install the package**

- ▶ OS, Linux : Use the pip install manager (*pip install grmpy*) or download the package via [GitHub](#) and install it manually.
- ▶ Windows: The same procedure as for Linux, OS but you have to verify that the numpy package is already installed on your machine.

## Initialization file

- ▶ The initialization file provides the user the opportunity to specify all parameters of his/her model, for instance:
  - ▶ Simulation parameters (number of observations, name of the output files)
  - ▶ Estimation parameters (optimization algorithm, start values)
  - ▶ Optimization parameters
  - ▶ Coefficients and covariance parameters, dummy variables...
- ▶ Example
- ▶ for a detailed explanation see: [\*grmpy\*-documentation](#)

## Simulation

- ▶ *grmpy.simulate()*:
  - ▶ Input: path of the initialization file.
  - ▶ The function returns a data frame based on your specifications and different output files.
    - ▶ The data set as a pickle and a txt file.
    - ▶ An [Info file](#) that provides the distributional characteristics of the data as well as information about the different treatment effects.

## **Estimation**

- ▶ *grmpy.estimate()*:
  - ▶ Input: path of the initialization file.
  - ▶ At the moment the estimation process is only capable of two different optimization algorithms:
    - ▶ Broyden Fletcher Goldfarb Shanno (BFGS) algorithm
    - ▶ Powell's conjugate direction method

- ▶ There are two different options for the start values that could be set in the initialization file:
  - ▶ *init*: The estimation process uses the coefficient values specified in the initialization file as the start values for the estimation process.
  - ▶ *auto*: The start values are determined via a simple OLS followed by a Probit regression for the choice indicator.
- ▶ The estimation results are printed to an [output file](#)

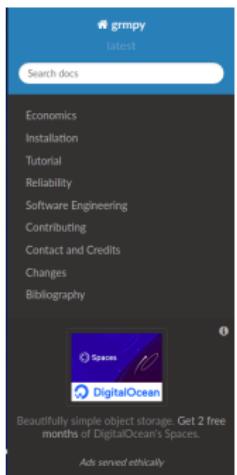
## **Test battery**

- ▶ We also provide a test battery that includes several tests to ensure that the processes perform as intended.
  - ▶ Property-based testing
  - ▶ Reliability testing
  - ▶ Regression testing

# **Application Example**

# **Additional Information**

# Online documentation



The screenshot shows the grmpy documentation website. The top navigation bar includes a logo, a search bar, and links for "Docs" and "Edit on GitHub". The main content area features a title "Welcome to grmpy's documentation!" and a brief introduction about the package. Below the introduction, there are two sections of references. At the bottom, there is a footer with a DigitalOcean advertisement and a note about ethical advertising.

grmpy

latest

Search docs

Economics

Installation

Tutorial

Reliability

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Contributing

Contact and Credits

Changes

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Spaces

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Docs > Welcome to grmpy's documentation!

Edit on GitHub

## Welcome to grmpy's documentation!

[PyPI](#) | [GitHub](#) | [Issues](#)

`grmpy` is an open-source Python package for the simulation and estimation of generalized Roy Model (Heckman & Vytlacil, 2005 [1]). Its main purpose is to serve as a teaching tool to promote the conceptual framework provided by the generalized Roy model which allows to illustrate a variety of issues in the econometrics of policy evaluation.

We build on the following main references:

James J. Heckman and Edward J. Vytlacil. Chapter 70 Econometric Evaluation of Social Programs, Part I: Causal Models, Structural Models and Econometric Policy Evaluation. *Handbook of Econometrics*, 6, 4779 – 4874, 2007.

James J. Heckman and Edward J. Vytlacil. Chapter 71 Econometric Evaluation of Social Programs, Part II: Using the Marginal Treatment Effect to Organize Alternative Econometric Estimators to Evaluate Social Programs, and to Forecast their Effects in New Environments. *Handbook of Econometrics*, 6, 4875 – 5143, 2007.

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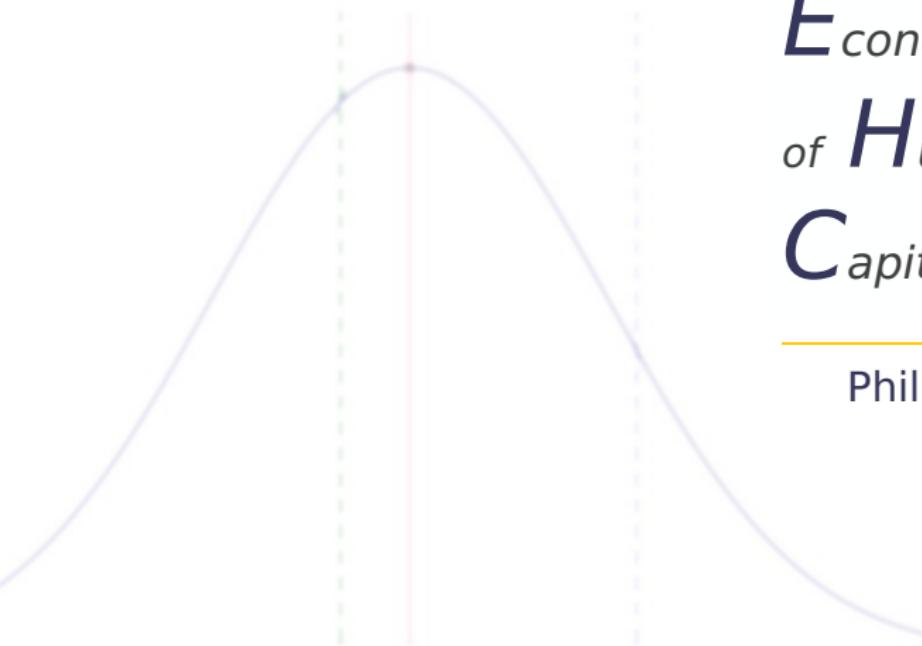
## **Additional Information**

- ▶ [\*grmpy\*-documentation](#)
- ▶ Course material regarding the generalized Roy model
- ▶ GitHub Repository

# **Appendix**

# *References*

- Heckman, J. J., & Vytlacil, E. J. (2007a). Econometric evaluation of social programs, part I: Causal effects, structural models and econometric policy evaluation. In J. J. Heckman & E. E. Leamer (Eds.), *Handbook of econometrics* (Vol. 6B, pp. 4779–4874). Amsterdam, Netherlands: Elsevier Science.
- Heckman, J. J., & Vytlacil, E. J. (2007b). Econometric evaluation of social programs, part II: Using the marginal treatment effect to organize alternative economic estimators to evaluate social programs and to forecast their effects in new environments. In J. J. Heckman & E. E. Leamer (Eds.), *Handbook of econometrics* (Vol. 6B, pp. 4875–5144). Amsterdam, Netherlands: Elsevier Science.



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# Monte Carlo Exploration

Philipp Eisenhauer

# *Introduction*

## **The Econometrics of Policy Evaluation**

- ▶ is important
- ▶ is complicated
- ▶ is multifaceted

## **Fundamental Problems**

- ▶ Evaluation Problem
- ▶ Selection problem
  - ▶ Essential Heterogeneity

## **Objects of Interest**

- ▶ Conventional Average Treatment Effects
- ▶ Policy-Relevant Average Treatment Effects
- ▶ Local Average Treatment Effect
- ▶ Marginal Effect of Treatment
- ▶ Distribution of Effects
- ▶ Effects on Distribution

## **Identification Strategies**

- ▶ Random Assignment
- ▶ Matching
- ▶ Control Functions and Extensions
- ▶ Instrumental Variables

## Generalized Roy Model

*Potential Outcomes*      *Cost*

$$Y_1 = \beta_1 X + U_1 \quad C = \gamma Z + U_C$$

$$Y_0 = \beta_0 X + U_0$$

*Observed Outcomes*      *Choice*

$$Y = DY_1 + (1 - D)Y_0 \quad S = Y_1 - Y_0 - C$$

$$D = I[S > 0]$$

## **Monte Carlo Exploration**

We will touch on all these issues in a Monte Carlo exercise using the **grmpy** package. The notebook is available on the course website.

# **Appendix**

# *References*

- Carneiro, P., Heckman, J. J., & Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6), 2754–2781.
- Heckman, J. J. (1990). Selection bias and self-selection. In J. Eatwell, M. Milgate, & P. Newman (Eds.), *Econometrics* (pp. 201–224). London: Palgrave Macmillan.
- Heckman, J. J., & Taber, C. (2010). Roy model. In L. E. Blume & S. N. Durlauf (Eds.), *Microeconomics* (pp. 221–228). London: Palgrave Macmillan.