Multivariate Distributions

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STAT 4690–Applied Multivariate Analysis

Review of basic concepts

Joint distributions

- Let X and Y be two random variables.
- The *joint distribution function* of X and Y is

$$F(x, y) = P(X \leq x, Y \leq y).$$

- We can recover the *marginal distributions*:

$$F(x) = \lim_{y \to \infty} F(x, y).$$

- More generally, let $Y_1, \ldots, Y_p$ be p random variables. Their *joint distribution function* is

$$F(y_1, \ldots, y_p) = P(Y_1 \leq y_1, \ldots, Y_p \leq y_p).$$

Joint densities

- If F is absolutely continuous almost everywhere, there exists a function f called the *density* such that

$$F(y_1, \ldots, y_p) = \int_{-\infty}^{y_1} \cdots \int_{-\infty}^{y_p} f(u_1, \ldots, u_p) du_1 \ldots du_p.$$

- The *joint moments* are defined as

$$E(Y_1^{n_1} \ldots Y_p^{n_p}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} u_1^{n_1} \ldots u_p^{n_p} f(u_1, \ldots, u_p) du_1 \ldots d$$

- **Exercise**: Show that this is consistent with the univariate definition of $E(Y_1^{n_1})$, i.e. $n_2 = \cdots = n_p = 0$.

Statistical Independence

- The variables $Y_1, \ldots, Y_p$ are said to be *mutually independent* if

$$F(y_1, \ldots, y_p) = F(y_1) \cdots F(y_p).$$

Conditional Distributions