

1.3.4. The formulas

$$f(x) = \frac{1 - \cos(x)}{\sin(x)}, \quad g(x) = \frac{2 \sin^2(x/2)}{\sin(x)}$$

are mathematically identical, but they suggest algorithms that can behave quite differently in floating point.

- (a) Using (1.2.6), find the relative condition numbers of f and g . Show that they are identical either by applying trigonometric identities, or by plotting them over $x \in [-1, 1]$.
- (b) (MATLAB) Set

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format long e
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Compute $f(10^{-6})$ as a sequence of four elementary operations. Using Table 1.1, make a table like that in Example 1.3.3 that shows the result of each elementary step and the numerical value of the condition number of that step.

- (c) (MATLAB) Repeat part (b) for $g(10^{-6})$. (There are six elementary steps.)
- (d) Based on (b) and (c), which of the results is more accurate?