

Midterm 1, 2015

Q5)

$$1. P(X, C | \pi, \sigma_1^2, \sigma_2^2) = \prod_{i=1}^N \left[\pi \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{X_i^2}{2\sigma_1^2}\right) \right]^{S(C_i=1)} \left[(1-\pi) \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{X_i^2}{2\sigma_2^2}\right) \right]^{S(C_i=0)}$$

$$\log P(X, C | \pi, \sigma_1^2, \sigma_2^2) = \sum_{i=1}^N \left(S(C_i=1) \log \pi - \frac{1}{2} S(C_i=1) \log 2\pi\sigma_1^2 - \frac{S(C_i=1) X_i^2}{2\sigma_1^2} + S(C_i=2) \log(1-\pi) - \frac{1}{2} S(C_i=2) \log 2\pi\sigma_2^2 - \frac{S(C_i=2) X_i^2}{2\sigma_2^2} \right)$$

$$2. F(q, \sigma_1^2, \sigma_2^2) = E_q \left[\log P(X, C | \pi, \sigma_1^2, \sigma_2^2) \right]$$

$$= \sum_{i=1}^N \left(q_i \log \pi - \frac{1}{2} q_i \log 2\pi\sigma_1^2 - \frac{q_i X_i^2}{2\sigma_1^2} + (1-q_i) \log(1-\pi) - \frac{1}{2} (1-q_i) \log 2\pi\sigma_2^2 - \frac{(1-q_i) X_i^2}{2\sigma_2^2} \right)$$

$$3. \text{a) For } q_i(C_i=1) = q_i \propto \pi \cdot \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{X_i^2}{2\sigma_1^2}\right), \quad q_i(C_i=0) \propto (1-\pi) \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{X_i^2}{2\sigma_2^2}\right)$$

4. b) To find σ_1^2 , solve $\frac{\partial F}{\partial \sigma_1^2} = 0$ (You should try this)

$$\text{i.e. } \sum_{i=1}^N -\frac{1}{2} \cdot \frac{q_i}{\sigma_1^2} + \frac{q_i X_i^2}{2(\sigma_1^2)^2} = 0$$

$$\text{i.e. } \sigma_1^2 = \frac{\sum_{i=1}^N q_i X_i^2}{\sum_{i=1}^N q_i}$$

Try solving for π yourself.