醫學影像成像原理

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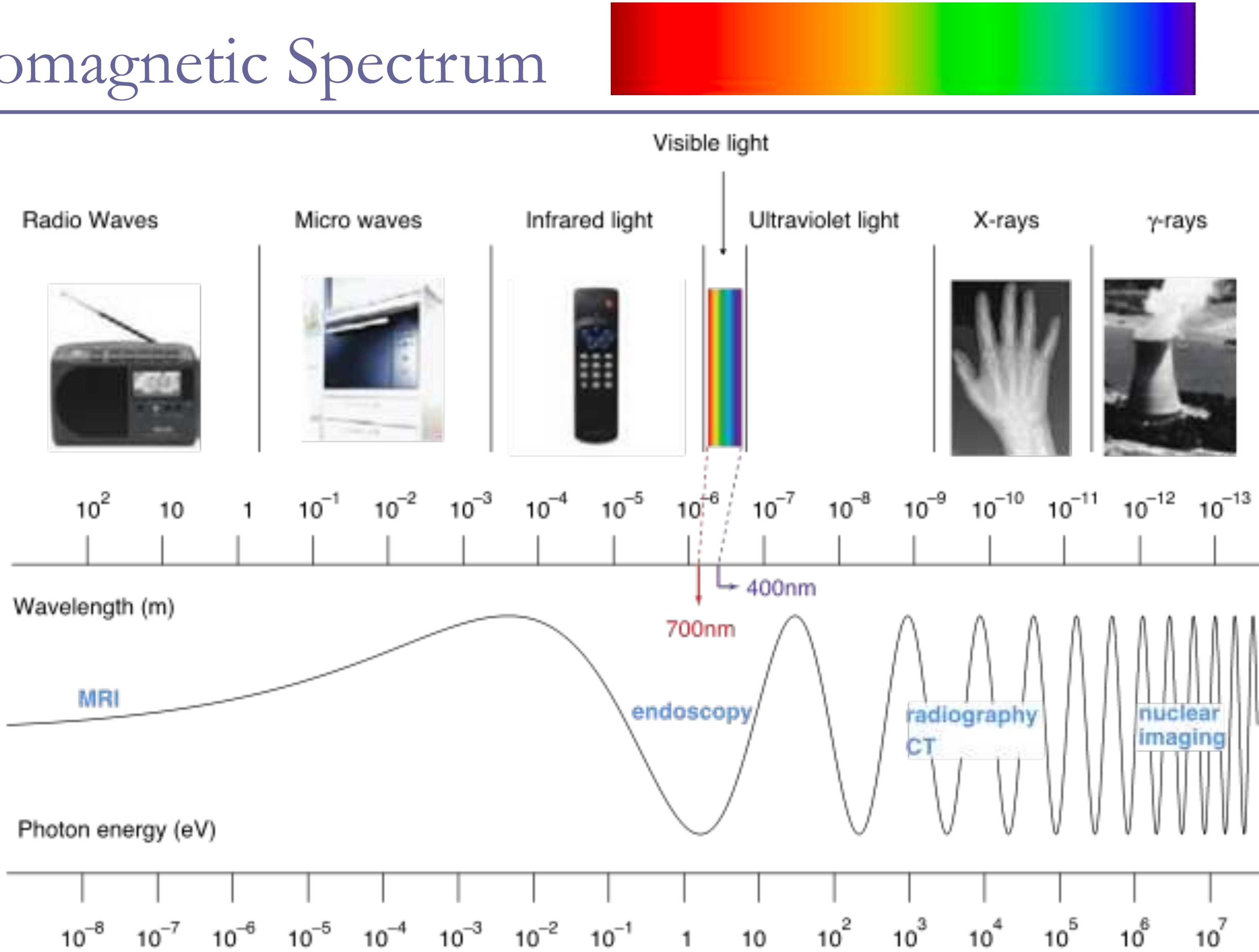
Electromagnetic Spectrum



The EM Spectrum

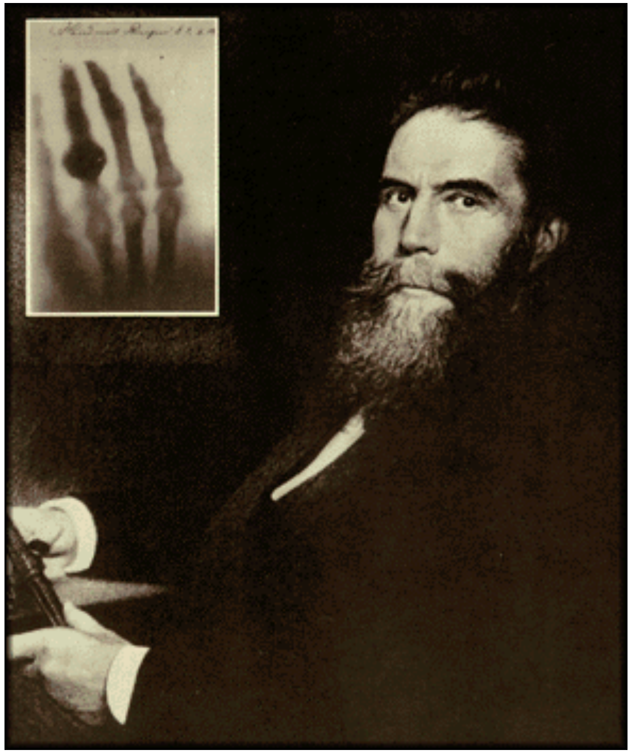
Frequency Range	Wavelengths	Photon Energies	Description
1.0×10^{5}–3.0×10^{10} Hz	3 km–0.01 m	413 peV–124 μeV	Radio waves
3.0×10^{12}–3.0×10^{14} Hz	100–1 μm	12.4 meV–1.24 eV	Infrared radiation
4.3×10^{14}–7.5×10^{14} Hz	700–400 nm	1.77–3.1 eV	Visible light
7.5×10^{14}–3.0×10^{16} Hz	400–10 nm	3.1–124 eV	Ultraviolet light
3.0×10^{16}–3.0×10^{18} Hz	10 nm–100 pm	124 eV–12.4 keV	Soft x-rays
3.0×10^{18}–3.0×10^{19} Hz	100–10 pm	12.4–124 keV	Diagnostic x-rays
3.0×10^{19}–3.0×10^{20} Hz	10–1 pm	124 keV–1.24 MeV	Gamma rays

Medical Imaging Modalities

- X-ray radiography
- Computed tomography
- Nuclear imaging
- Ultrasound Imaging
- Magnetic resonance imaging

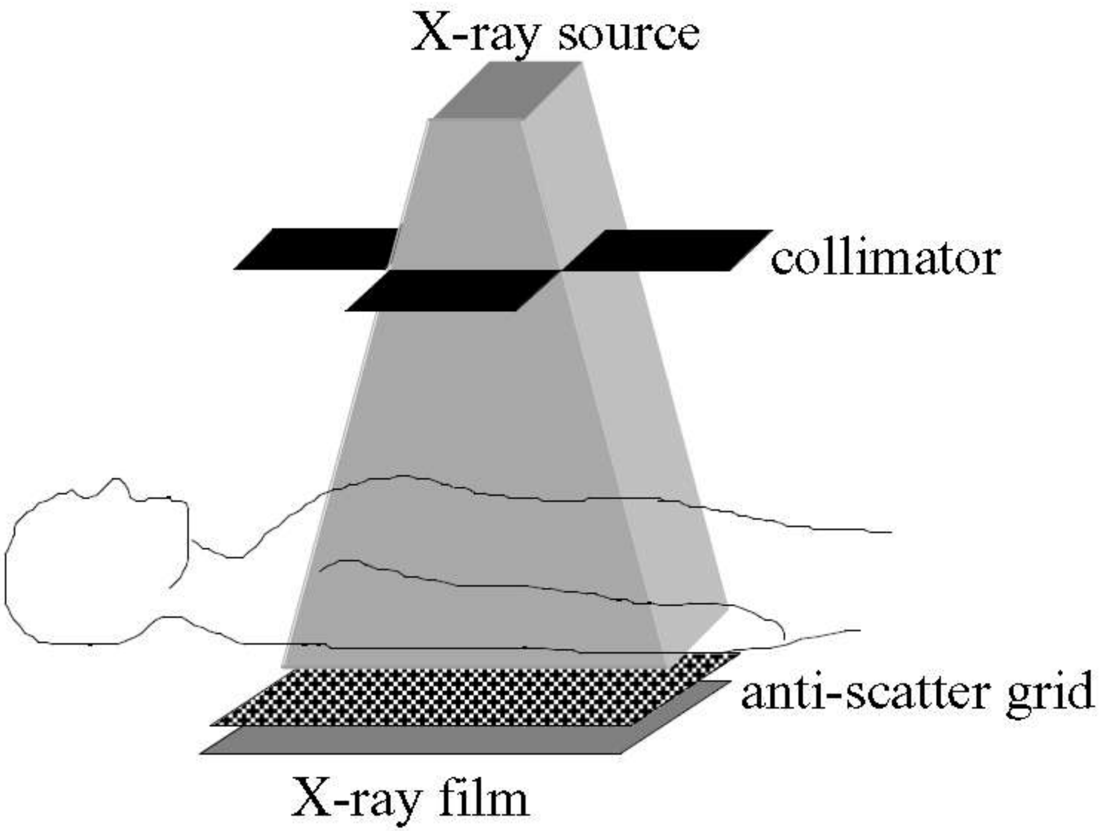
X-RAY SYSTEM

Discovery of X-rays — The Nobel Prize in Physics 1901



Wilhelm Conrad Röntgen
Munich University
Munich, Germany
(1845-1923)

Basic Construct

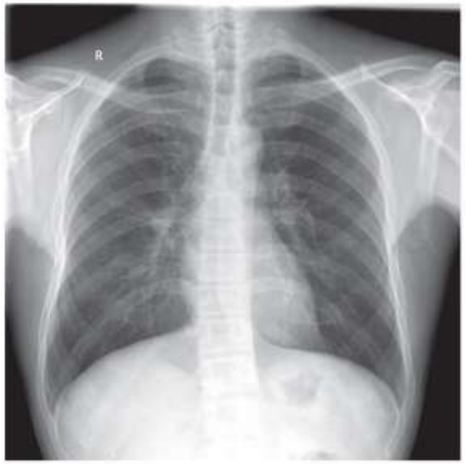


X-ray Transmission Images of Various Parts of the Body


(a)


(b)


(c)

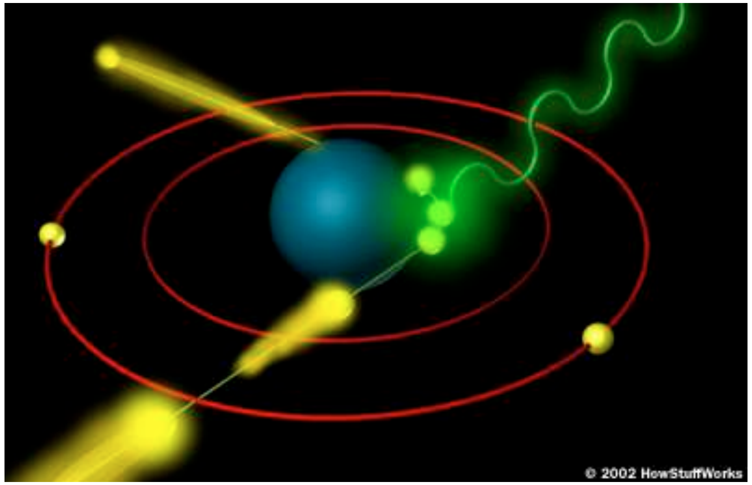

(d)

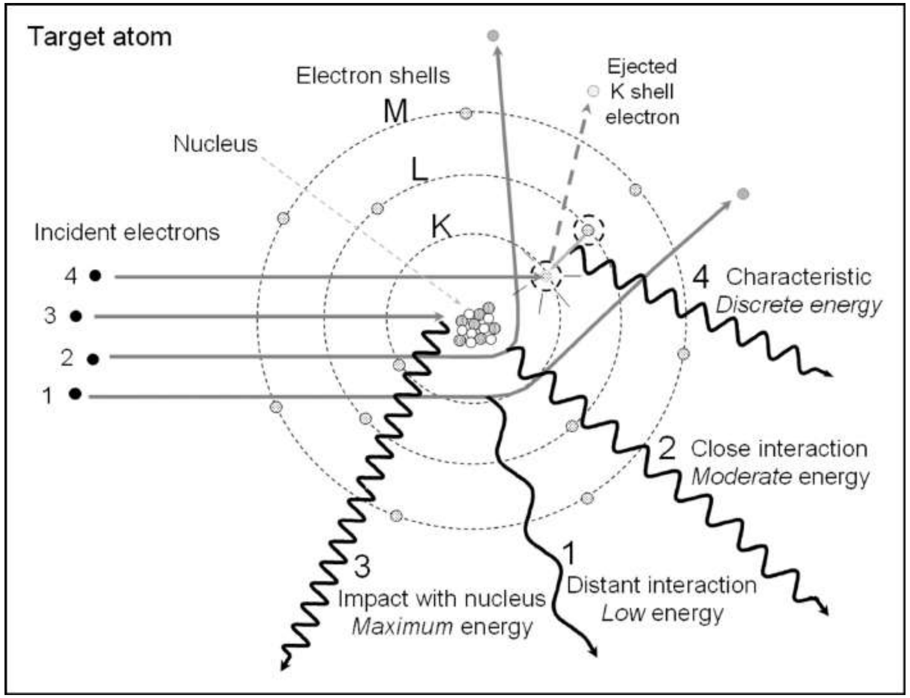

(e)


(f)

X-ray Generation

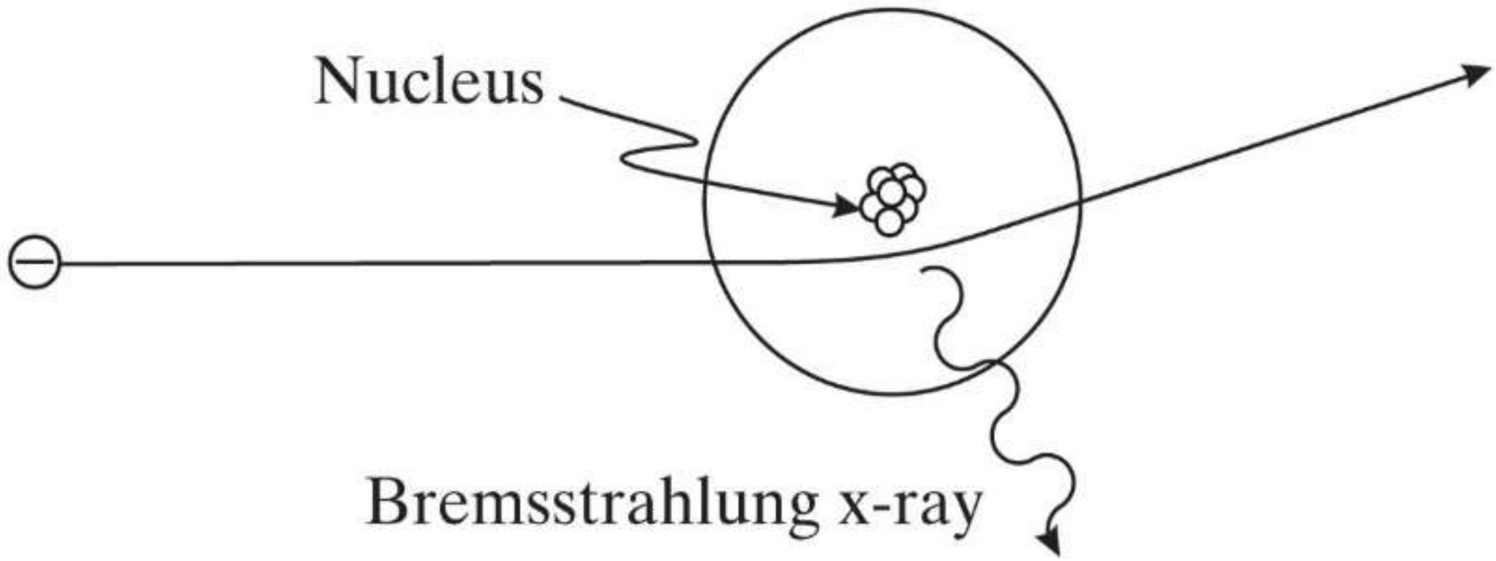
- Two primary methods for X-ray Generation:
  - Rearrangement of atomic-electron configurations "Characteristic" X-rays.
  - The deflection of charged particles in the vicinity of the atomic nucleus "Continuous X-rays or Bremsstrahlung".





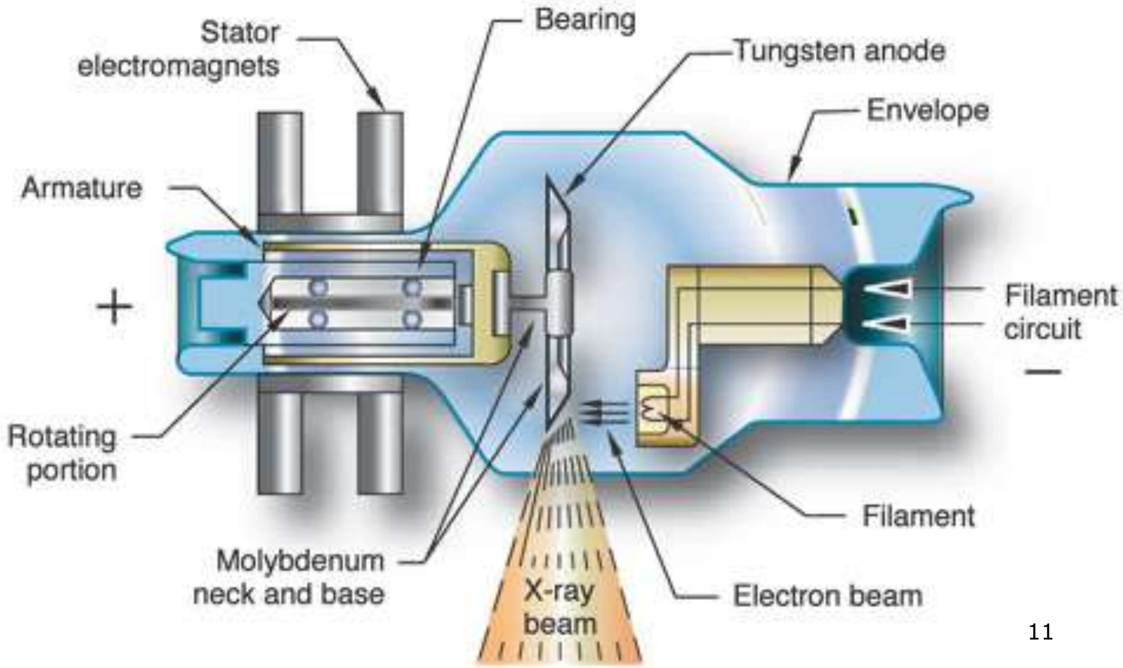
Bremsstrahlung X-ray

- Bremsstrahlung radiation
  - Collide with nucleus or paths are close to the nucleus.
  - More common than characteristic radiation.



X-ray tube components

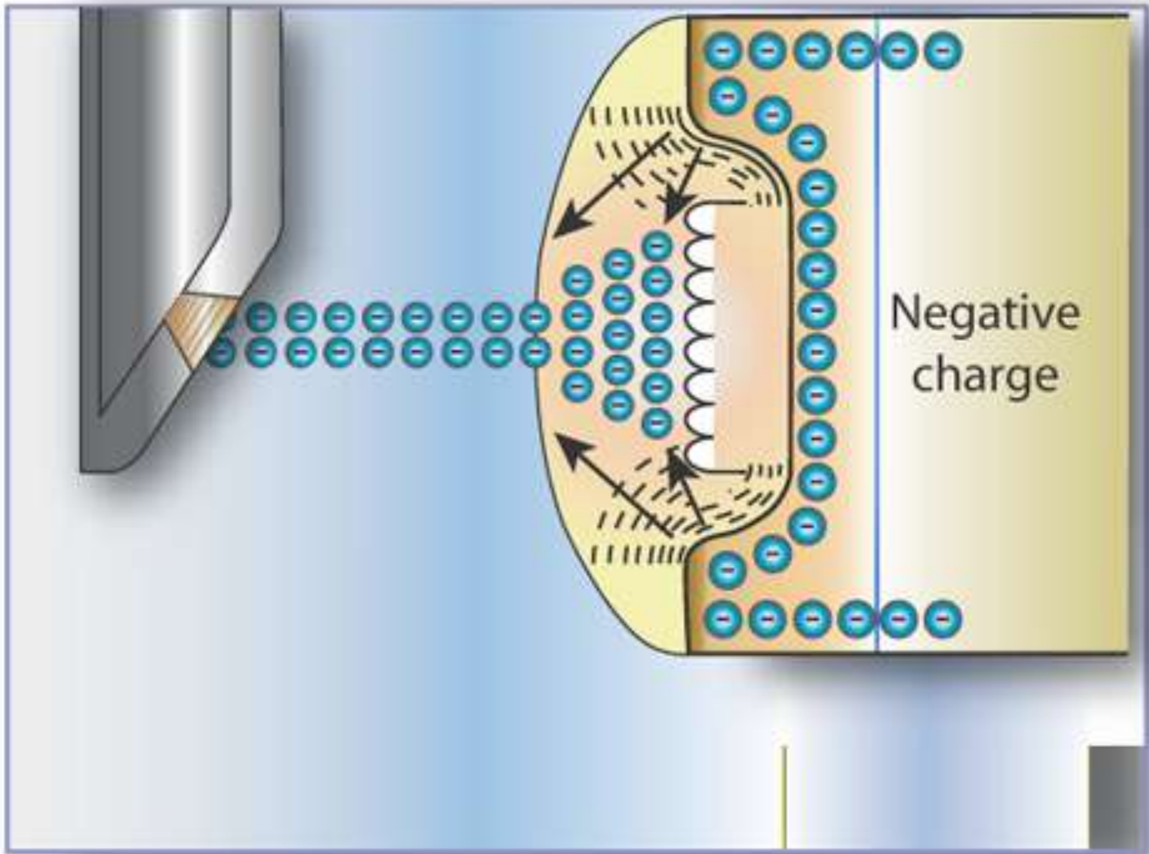
- Filament: controls tube current (mA)
- Cathode and focusing cup
- Anode is switched to high potential
  - 30-150kVp
  - Made of tungsten
  - Bremsstrahlung is 1%
  - Heat is 99%
  - Spins at 3,200-3,600 rpm
- Glass housing: vacuum



Exposure control

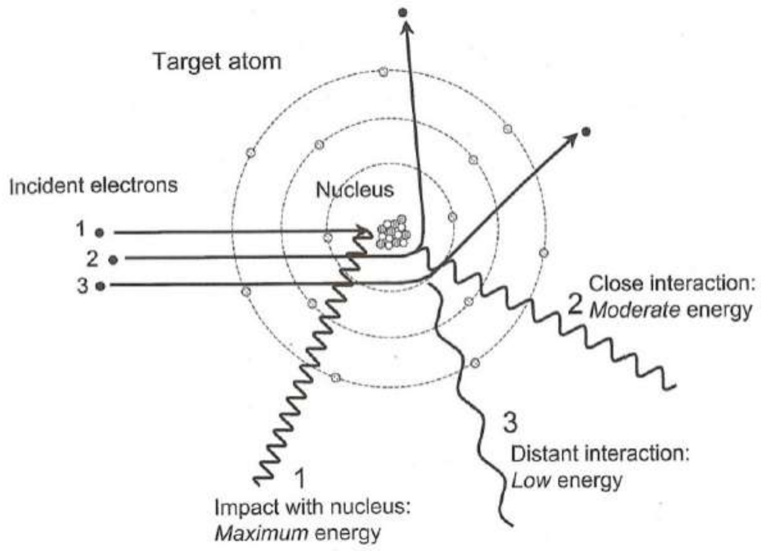
- kVp applied for short duration
  - Fixed timer (silicon controlled rectifier, SCR), or
  - Automatic exposure control (AEC), 5 mm thick ionization chamber triggers SCR
- Tube current mA controlled by
  - Filament current, and kVp
- mA times exposure time yields mAs
  - mAs measures X-ray exposure

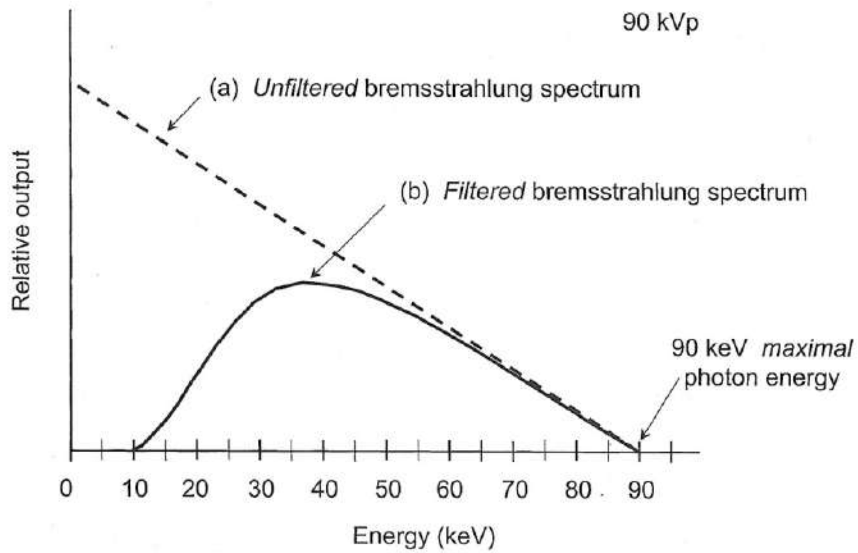
kVp = energy   mAs = amount



Bremsstrahlung (Braking Radiation) Spectrum

- Electrons lose energy through photon emission while being deflected (decelerated) in the electric field of the atomic nucleus.
- For energies < 100 keV, the photons are emitted at 90 deg to the direction of the incident electron.
- For higher energies the direction of the emitted radiation is shifted forward.





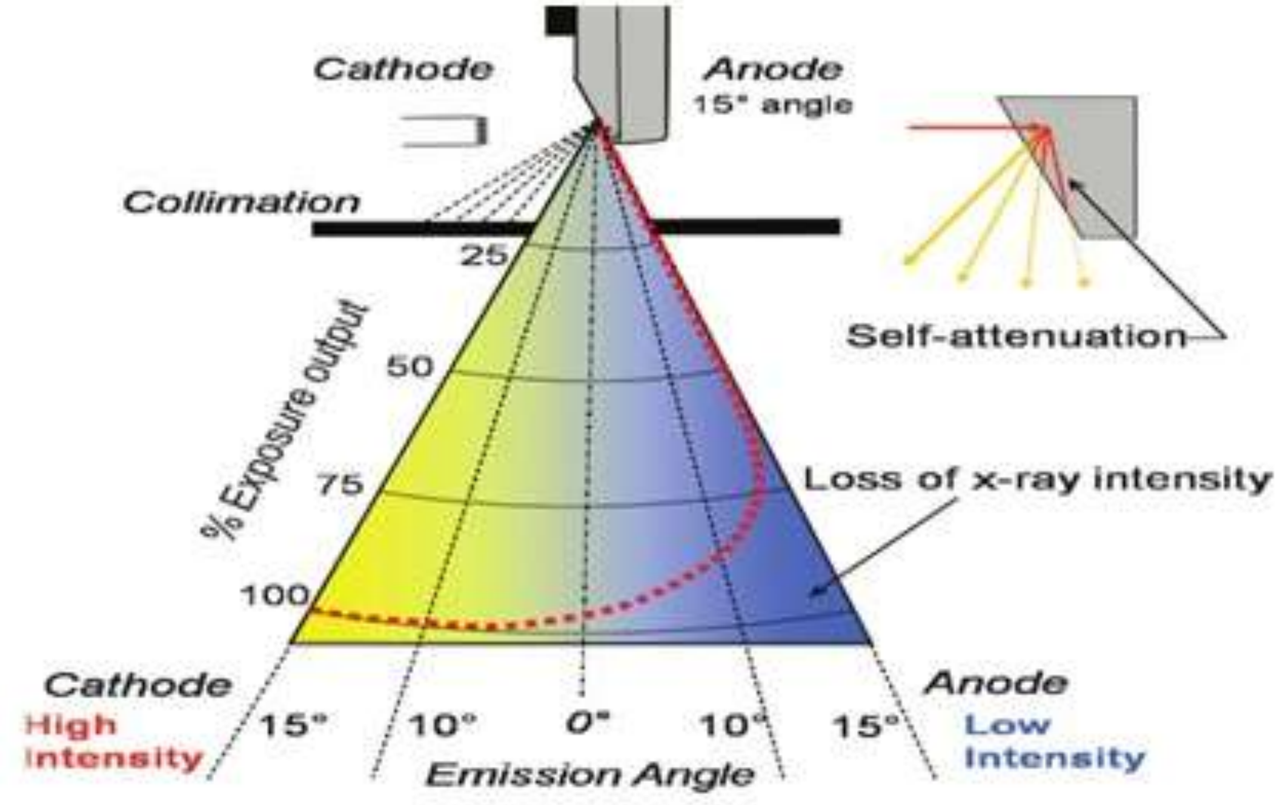
Radiative vs. Collisional Energy Loss

- Predominant energy loss of electrons at energies below 1 MeV are collisions leading to <u>excitation</u> and <u>ionization</u> resulting in heat.

- Assuming a tungsten (W, Z=74) target:
  - X-ray diagnostic
    - At 100 keV the ratio is about 0.9%, therefore, ~99% of the energy result in heat.
  - Radiation therapy
    - At 6 MeV the ratio is about 0.54, therefore, most of the energy results in X-rays.

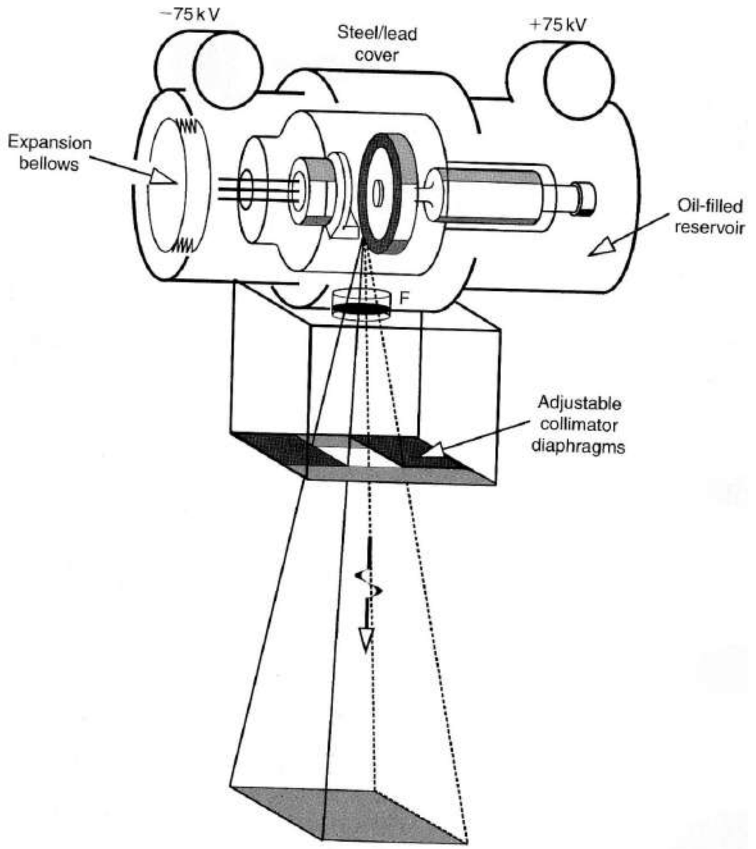
Heel Effect

- Heel effect refers to a reduction in beam intensity toward the anode side due to the attenuation in the anode.



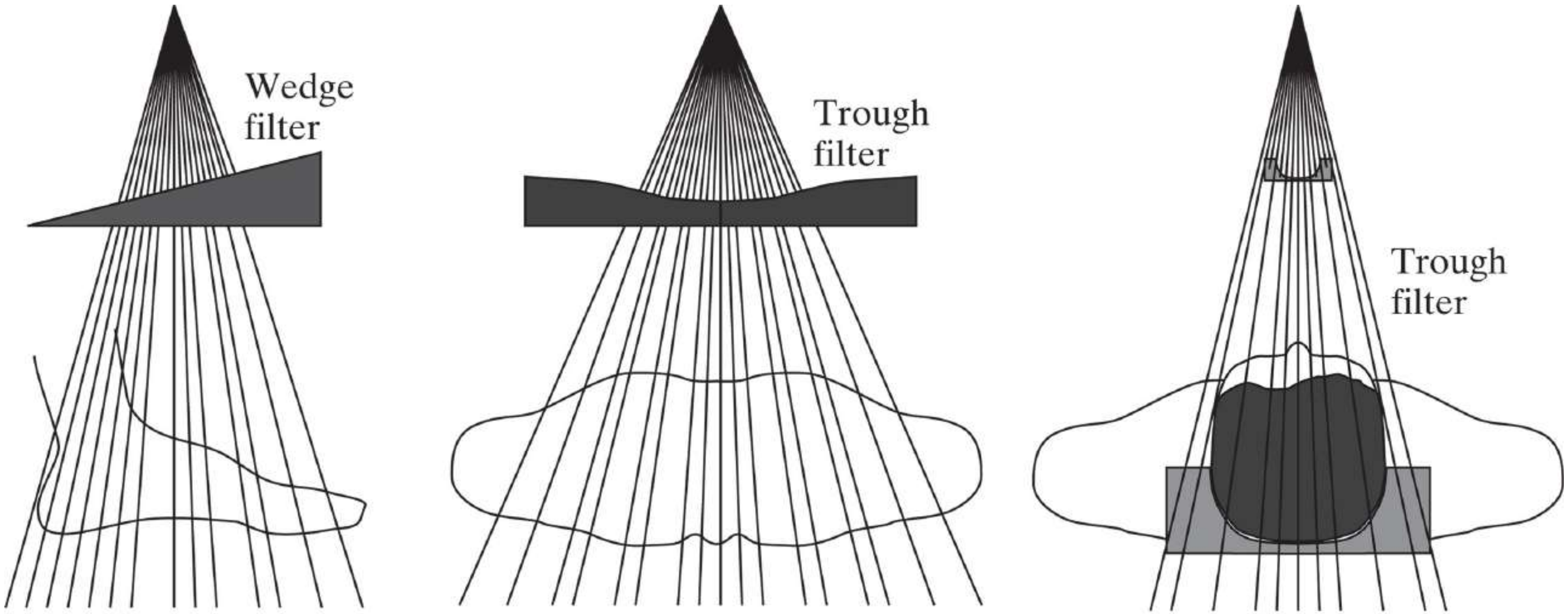
Filtration and Collimation

- Inherent filtration due to tube window (1-2 mm thickness) made of glass (SiO_2) or aluminum, effectively removing all radiation below 15 keV.
- Added filtration due to added metal sheets to intentionally adjust the effective energy (main goal is further reduction in low-energy flux which will be absorbed in patient and only lead to unnecessary dose).
- Collimators adjust the size and shape of the X-ray field emerging from the tube port and the additional tungsten due to internal filament and anode evaporation.
- Compensation or equalization filters are used to deliver a more uniform exposure to the detector compensating inhomogeneous attenuation in the patient.



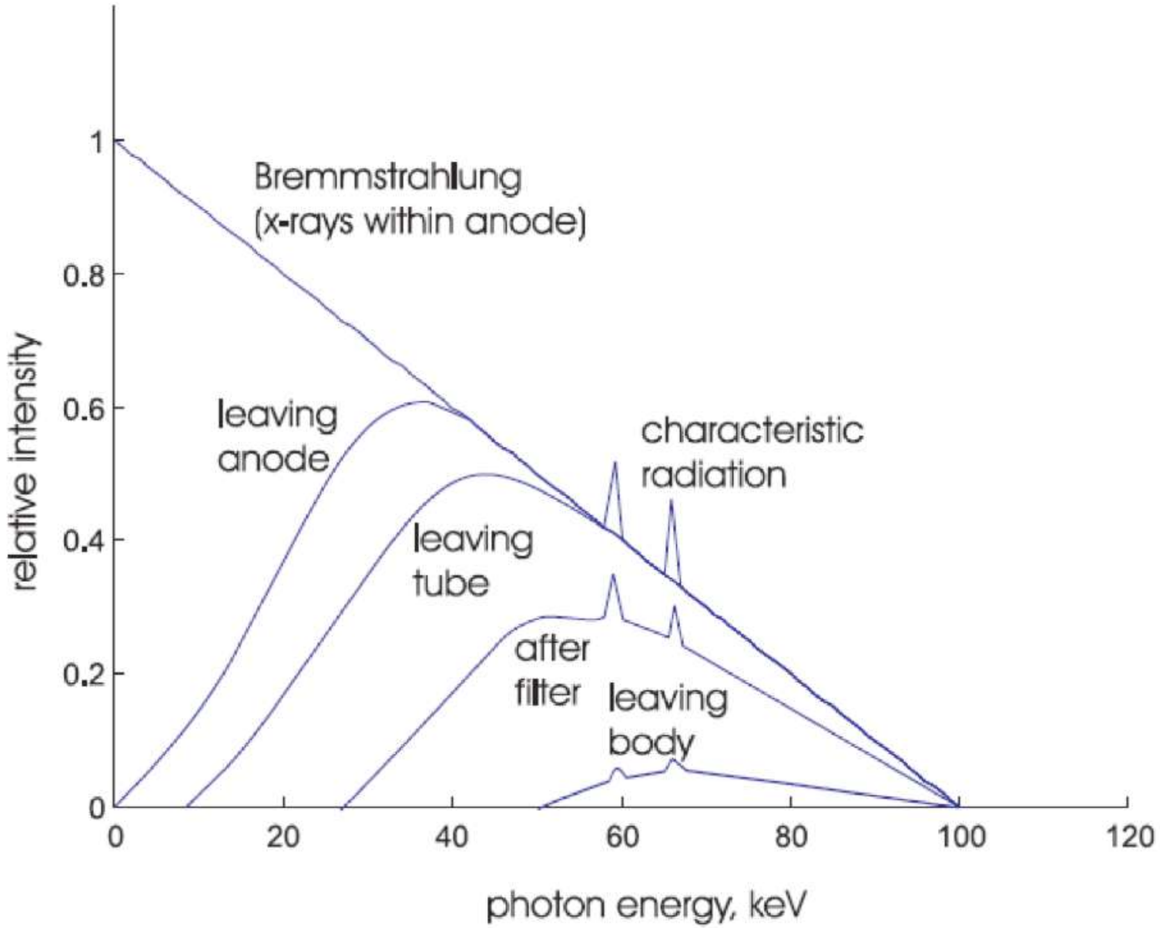
Compensation filters

- Goal: to even out film exposure



(a) (b) (c)

X-ray spectrum



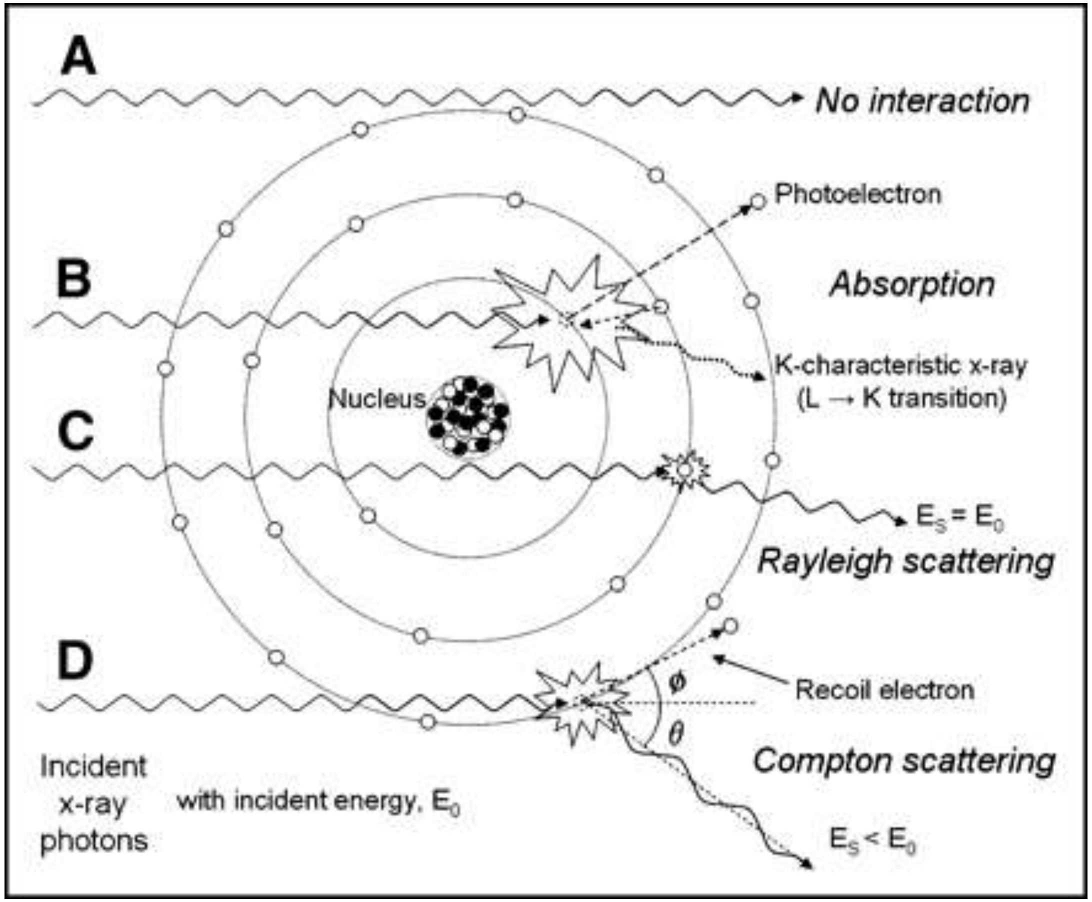
INTERACTIONS OF X-RAYS

Interaction of X-rays with Tissues

- **Primary radiation**: a certain fraction of X-rays pass straight through the body and undergo **no interactions** with tissue.
- **Secondary radiation**: X-rays can be **scattered**, an interaction that alters their trajectory between source and detector.
- **Absorbed radiation**: X-rays can be absorbed completely in tissue and not reach the detector at all.

Interactions in Diagnostic Radiology

- In the X-ray energy range (25–150 keV) used for diagnostic radiology, three dominant mechanisms describe the interaction of X-rays with tissue:
  - Coherent scatter and Compton scatter are both involved in the production of secondary radiation.
  - Photoelectric interactions result in X-ray absorption.

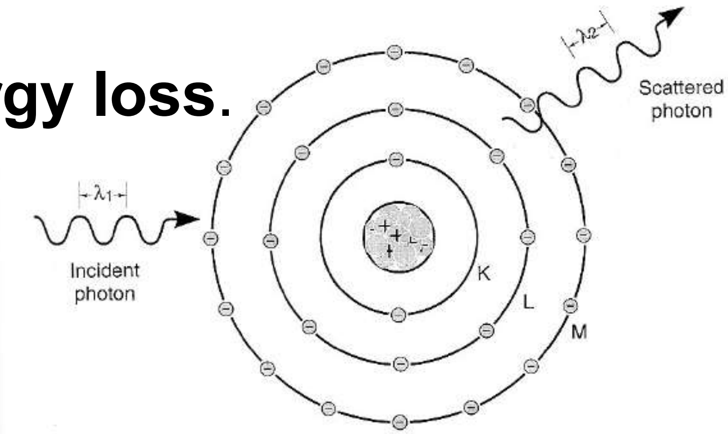


Rayleigh Scattering

- **Elastic (coherent, classical) scattering**
  - **Scatter with direction change but no energy loss.**

$$h\upsilon_{incident} = h\upsilon_{scattered}\ ;$$

$$\lambda_{incident} = \lambda_{scattered}$$



$\lambda_1 = \lambda_2$

- **Characteristics**
  - An interaction of the x-ray with the entire atom via a refractory mechanism (wavelength ~ atomic radius).
  - Occurs for longer wavelengths (low x-ray energies in the mammography range - 15 - 30keV).
  - Reduces imaging quality in x-ray imaging.

E(keV)	P(%)
~30	<12
>70	<5

P: probability

Compton Scattering

- Compton scattering refers to the interaction between an incident X-ray and a loosely bound electron in an outer shell of an atom in tissue.
- A fraction of the X-ray energy is transferred to the electron, the electron is ejected, and the X-ray is deflected from its original path.
- The probability of an X-ray undergoing Compton scattering is
  - Essentially independent of the effective atomic number of the tissue.
  - Linearly proportional to the tissue electron density.
  - Weakly dependent on the energy of the incident X-ray.

Photoelectric Effect (I)

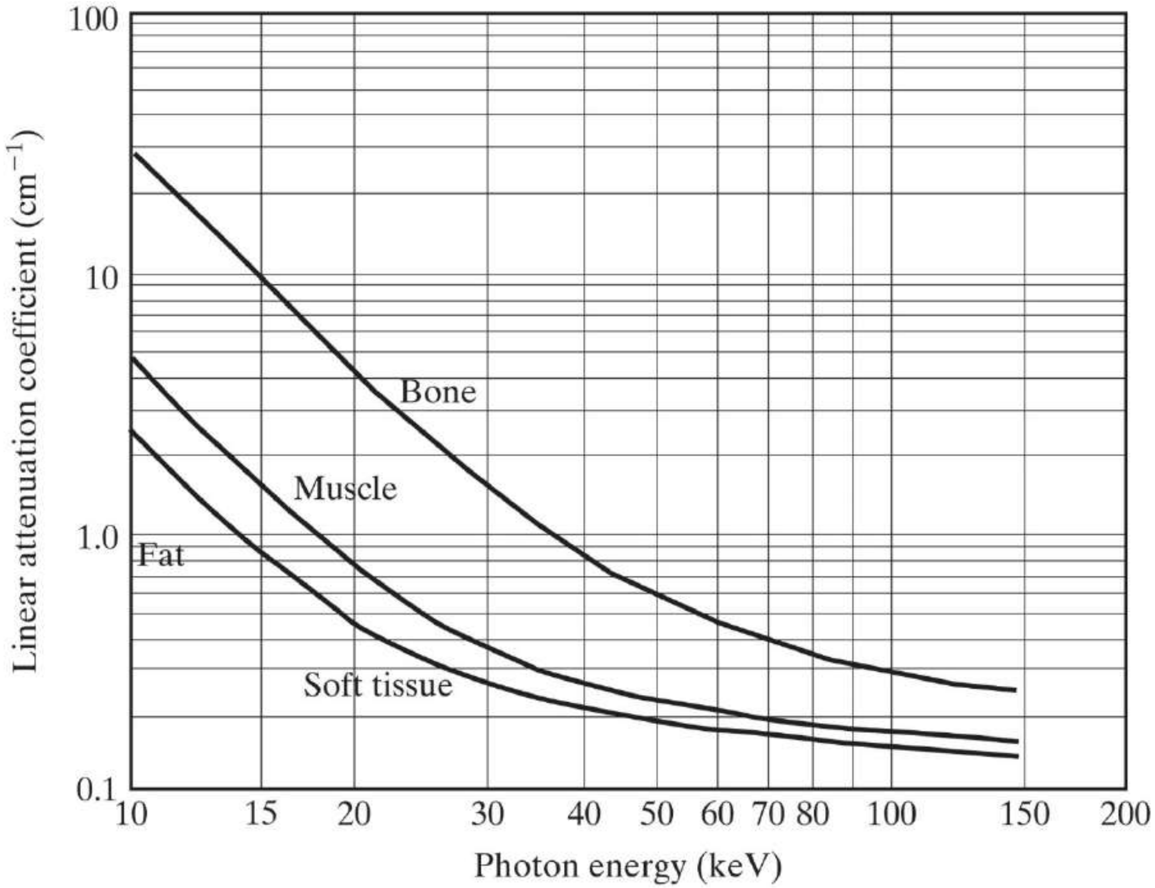
- Photoelectric interactions in the body involve the energy of an incident X-ray being absorbed by an atom in tissue, with a tightly bound electron being emitted from the *K* or *L* shell as a "photoelectron."

- The kinetic energy of the photoelectron is equal to the difference between the energy of the incident X-ray and the binding energy of the electron.

- A second electron from a higher energy level then fills the "hole" created by the ejection of the photoelectron, a process accompanied by the emission of a "characteristic" X-ray with an energy equal to the difference in the binding energies of the outer electron and the photoelectron.

X-Ray Mass Attenuation Coefficients for Some Materials

Material	20 keV (cm^2/g)	100 keV (cm^2/g)
Air	0.7779	0.1541
Water	0.8096	0.1707
Bone	0.4001	0.1855
Muscle	0.8205	0.1693
Lung	0.8316	0.1695
Brain	0.8281	0.1701



Source: Hubbell and Seltzer, NIST online tables.

SCATTER CONTROL AND DETECTORS

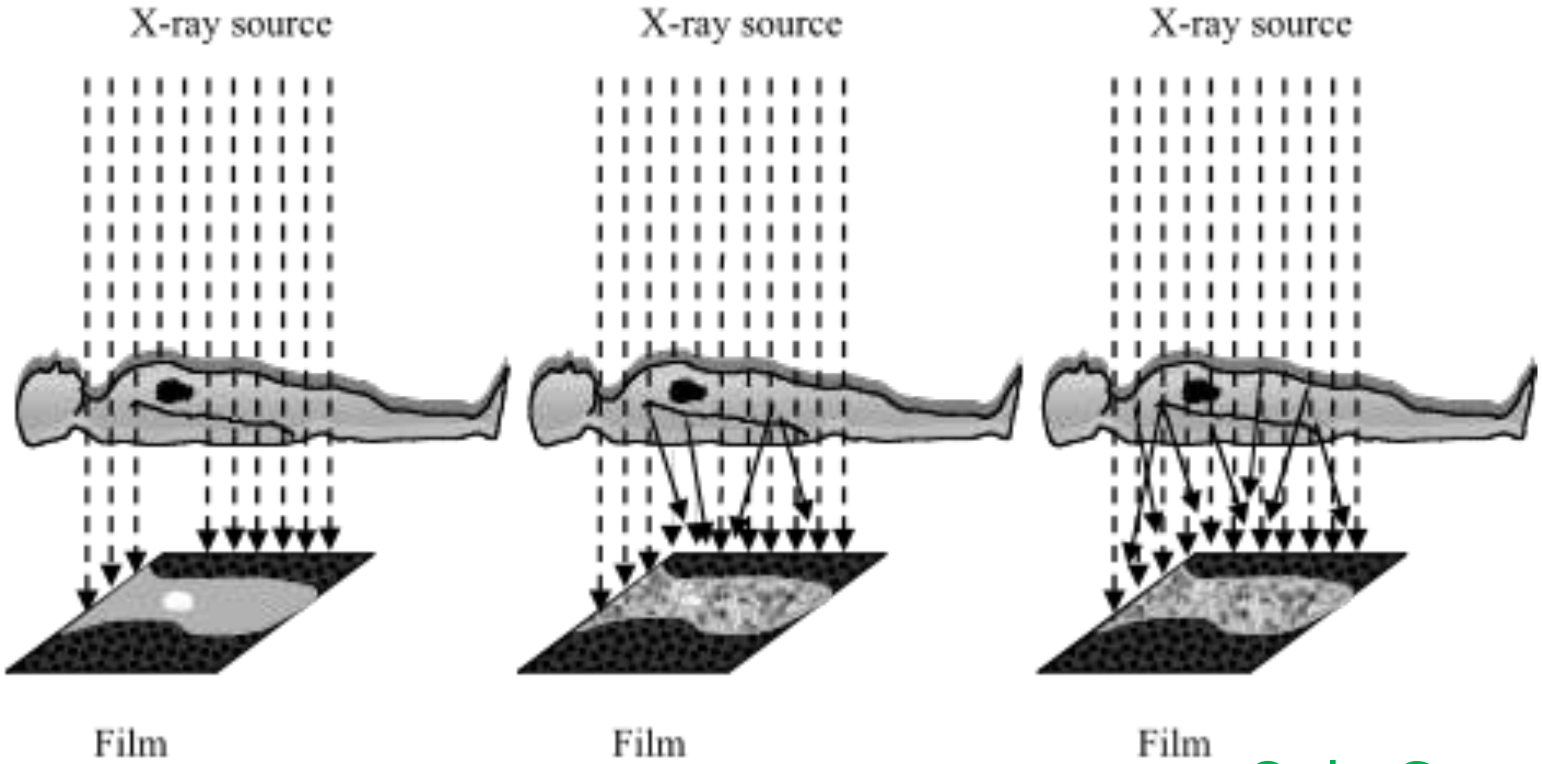
Scatter control

- Ideal X-ray path: a line

- Compton scattering causes blurring

- How to reduce scatter
  - Airgap
  - Scanning slit
  - Grid

Antiscatter Grids(I)

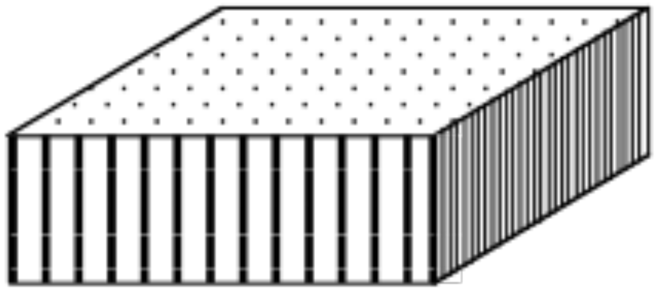


Ideal situation

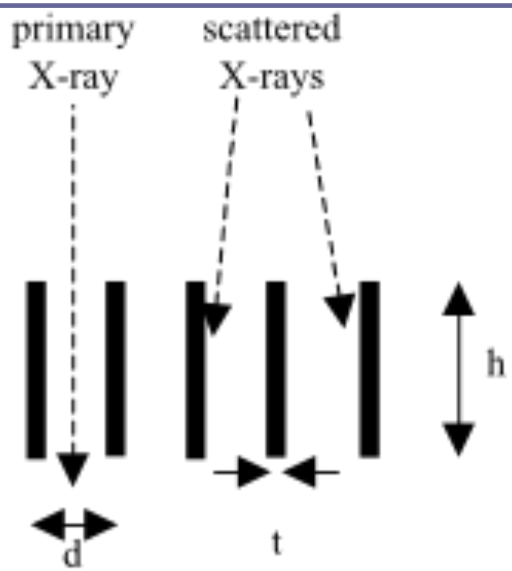
Contribution of Compton scattered X-rays

Only Compton-scattered X-rays

Antiscatter Grids(II)



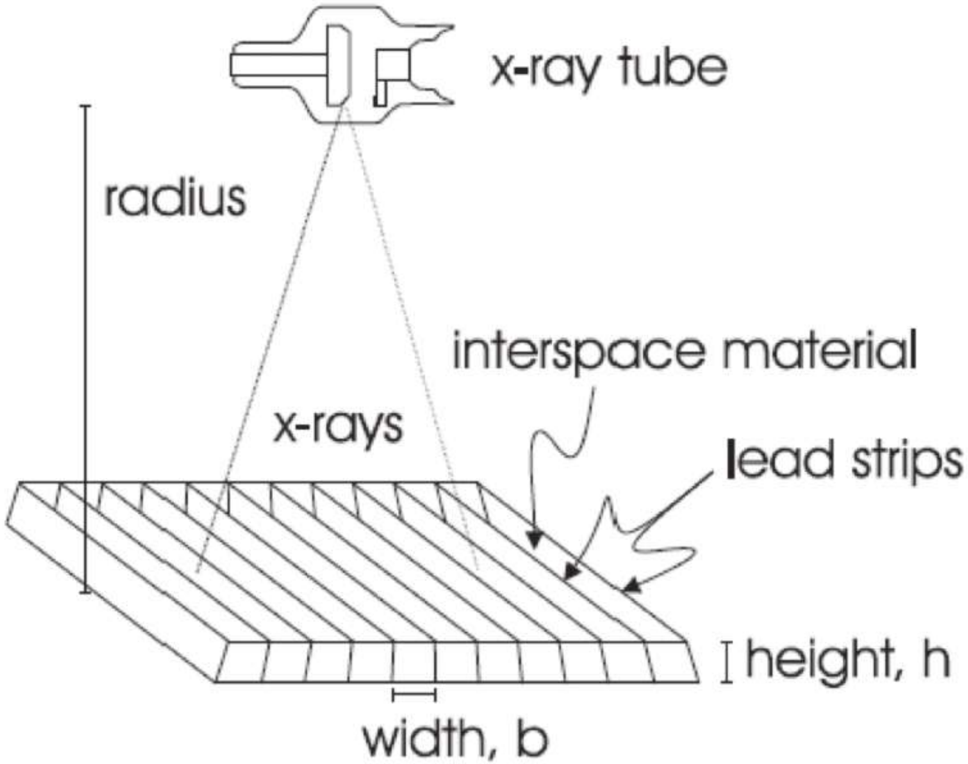
The black areas represent thin lead septa, separated by the aluminum support.



- A one-dimensional representation of the antiscatter grid.
- Primary X-rays pass between the lead septa, whereas those X-rays that have undergone a significant deviation in trajectory from Compton scattering are absorbed by the septa.

Grids

- Effectiveness in scatter reduction
- Grid ratio= h/b
- Radiography
  - 6:1 to 16:1
- Mammography
  - 2:1 to 5:1



Stationary grids

- You can change the mAs or the kVp to compensate the grids, but it is not recommended to do both.
- Grids clean up scatter, they do not create scatter.
- Know the grid ratios!!

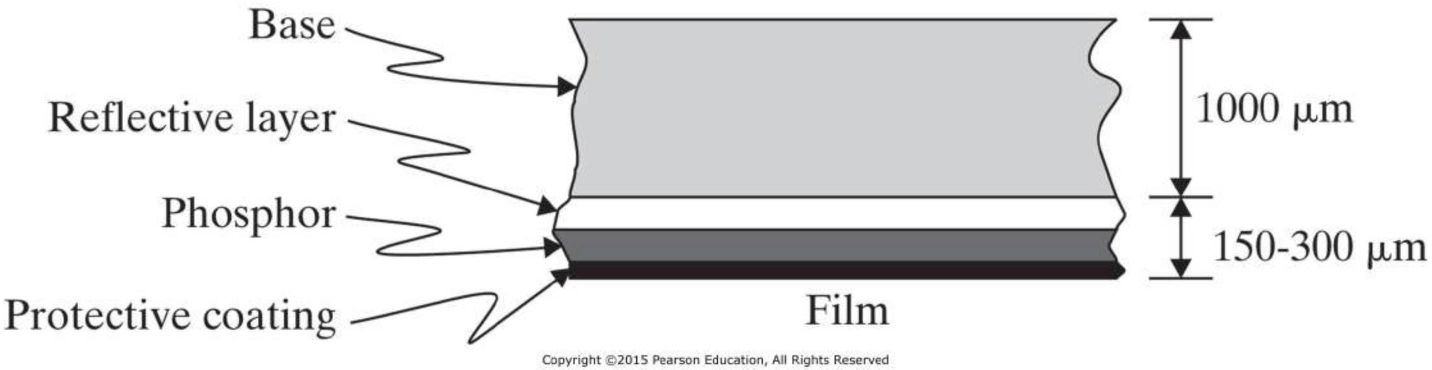
Grid Ratio	mAs increase	kVp increase
No grid	1x	1x
5:1	2x	+8~10
8:1	4x	+13~15
12:1	5x	+20~25
16:1	6x	+30~40

Problem with grids

- Radiation is absorbed by grid
  - Grid conversion factor
    - The amount of additional exposure required for a particular grid.
  - $$GCF = \frac{\text{mAs w/ grid}}{\text{mAs w/o grid}}$$
  - Typical range 3< GCF < 8
- Grid visible on X-ray film
  - Move grid during exposure
  - Linear or circular motion

Film-Screen Detector

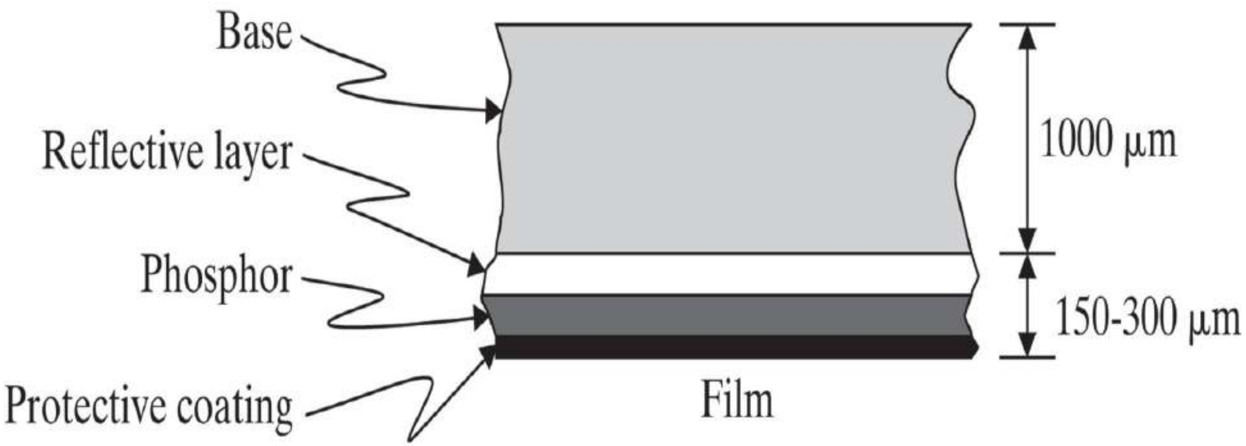
- Film stops only 1~2% of X-rays
- Film stops light really well

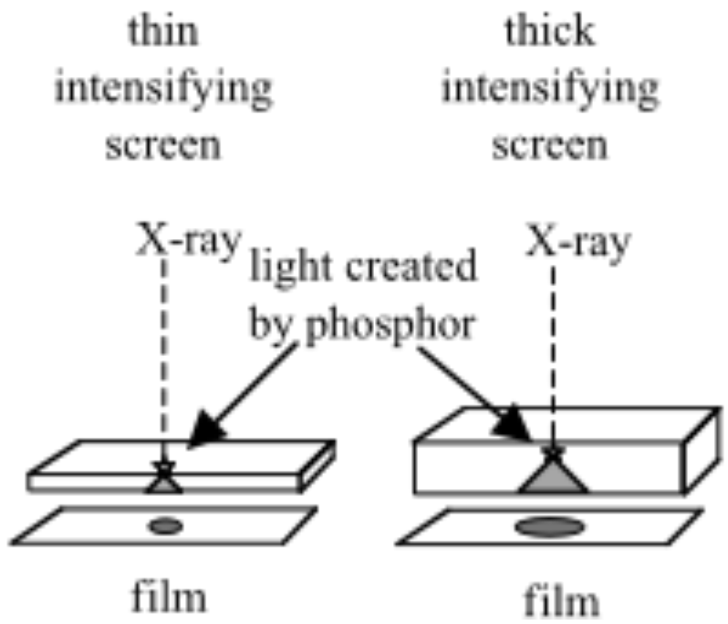


- Phosphor (calcium tungstate $CaWO_4$) transforms x-ray photons into light photons.
- Flash of light lasts 1×10^{-10} second.
- About 1,000 light photons per 50 keV X-ray photon.

Intensifying Screen

- The intrinsic sensitivity of photographic film to X-rays is very low,
  - i.e., its use would require high patient doses of radiation to produce high-quality images.



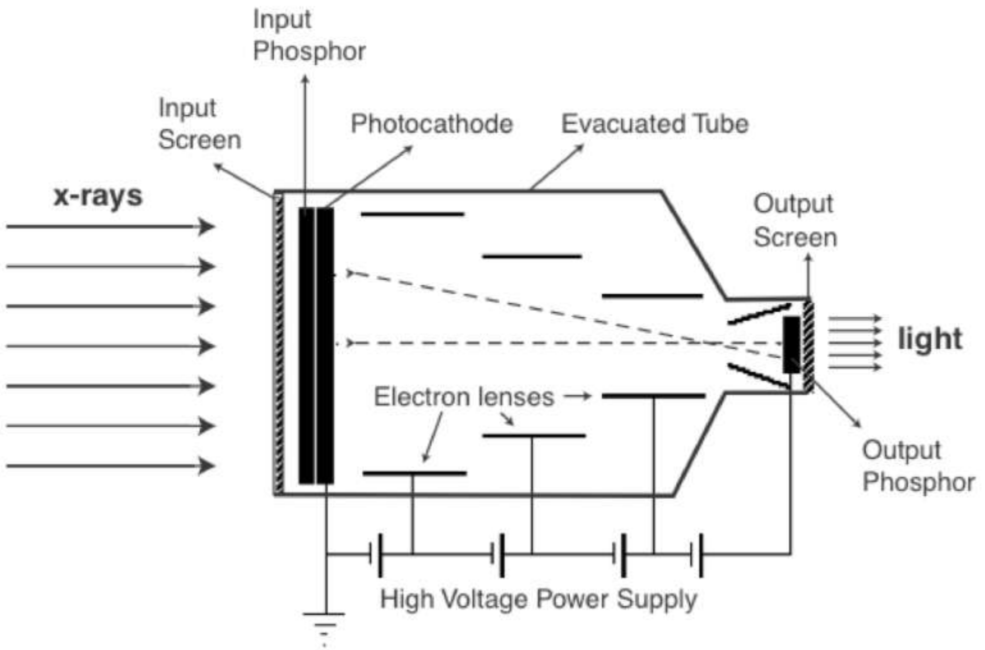


Effect of the thickness of the intensifying screen on the spatial resolution of the image.

X-ray Image Intensifier — used in fluoroscopy

- Image intensifier, coupled to camera
- At phosphor screen: X-rays converted into light; at photocathode: then into electrons.
- Electrons accelerated and focused in tube with electromagnetic fields.
- Converted back into light at output screen, coupled to camera.

- Advantages:
  - Dynamic real-time imaging
- Disadvantages:
  - Worse spatial resolution (camera)
  - More noise (conversions)
  - Geometric pin cushion
  - Distortion, especially at edges



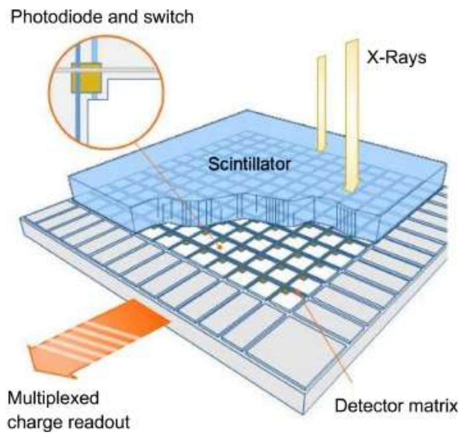
Digital radiology

- Replace the intensifying screen/X-ray film by
  - Flat panel detectors (FPD) using thin-film transistor (TFT) arrays.
  - A scintillator
    - Consisting of many thin, rod-shaped cesium iodide (CsI) crystals.
- When an X-ray is absorbed in a CsI rod, the CsI scintillate and produces light.
- The light is converted into an electrical signal by a photodiode in the TFT array.
- The electrical signal is amplified and converted to a digital value using an A/D converter.
- A typical commercial DR system has flat panel dimensions of 41x41 cm, with an TFT array of 2048x2048 elements.

Ref: Webb, Introduction to biomedical imaging, Sec. 1.5.5.

Current main-stream X-ray detector technologies

- Integrating X-ray detectors predominantly based on
  - Scintillators (e.g. CsI or Gd_2O_2S)
  - Arrays of photodiodes made from crystalline silicon (Si) or amorphous silicon (a-Si)
  - Or semiconductors (e.g. Se) with active a-Si readout matrices.



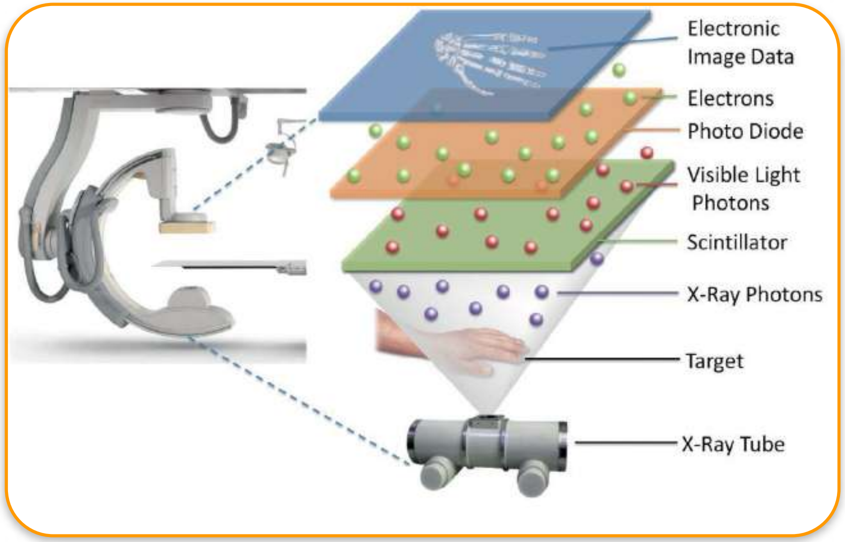


IMAGE CONTRAST & QUALITY

Measures of X-ray beam

- Photon counts
  - Photon fluence $\Phi = \dfrac{N}{A}$
  - Photon fluence rate $\phi = \dfrac{N}{A\Delta t}$
- Energy flow
  - Energy fluence $\Psi = \dfrac{Nh\nu}{A}$
  - Energy fluence rate $\psi = \dfrac{Nh\nu}{A\Delta t}$
  - Intensity $(= \psi)$ $I(E) = \dfrac{NE}{A\Delta t}$

Spectrum of X-ray

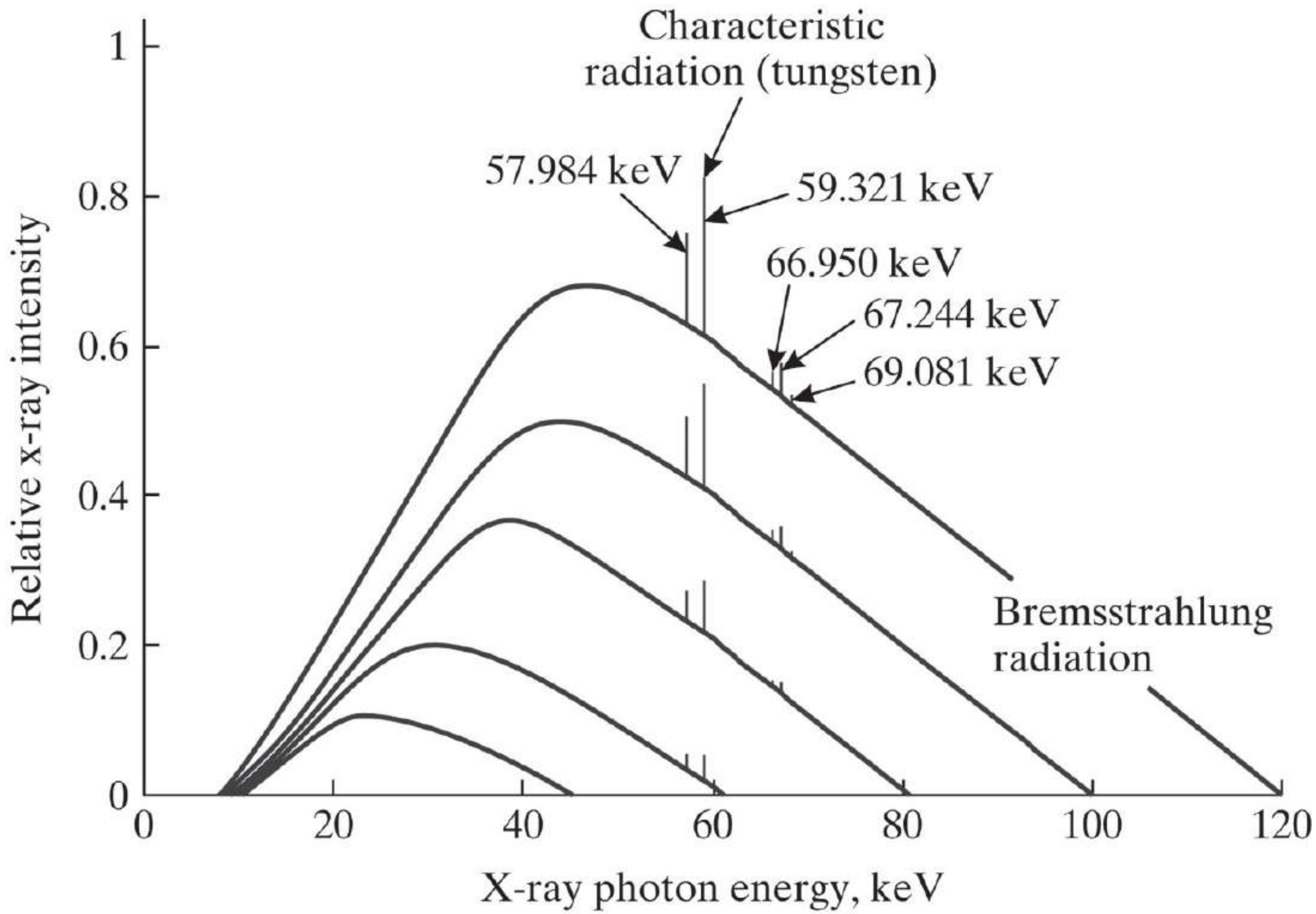
- The x-ray beam produced by an x-ray tube (mainly Bremsstrahlung) is polyenergetic (consisting photons with different energies or frequencies)
- X-ray spectrum $S(E)$:
  - The number of photons with energy E per unit area per unit time
    - Photon fluence rate from spectrum

$$\phi = \int_0^\infty S(E')dE'$$

    - Intensity from spectrum

$$I = \int_0^\infty E'S(E')dE'$$

Spectrum of X-ray

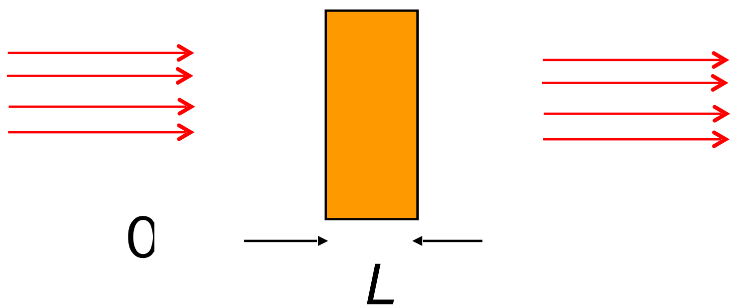


Beer's law

- The reduction in the beam intensity should be a property of the object along the line.

$$\frac{-dI}{I} = \mu dz$$

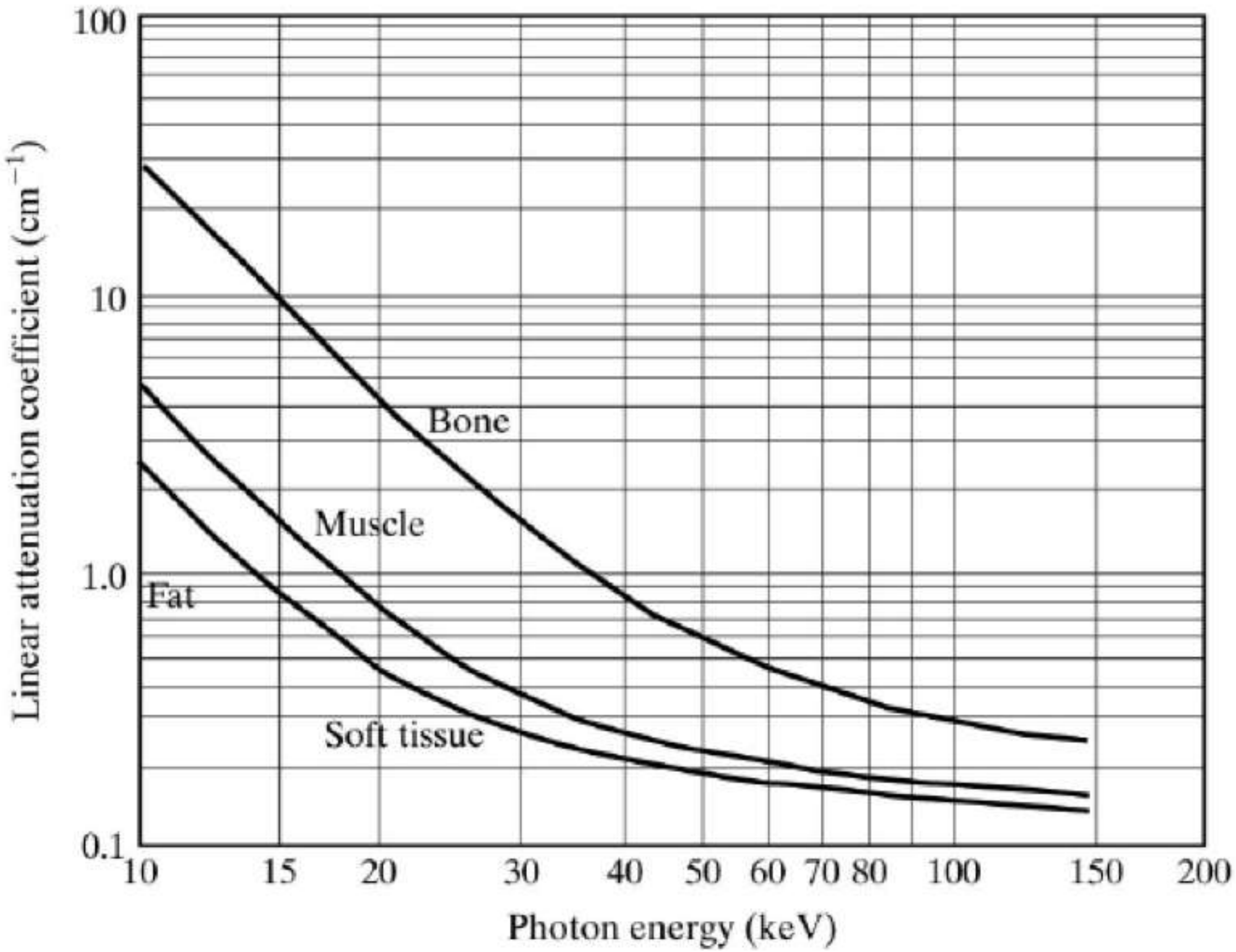
- Where μ is the linear attenuation coefficient and in general is a function of x, y, and z $\rightarrow \mu(x, y, z)$.



$$I = I_0 \mathrm{e}^{-\mu L}$$

$$N = N_0 \mathrm{e}^{-\mu L}$$

Linear attenuation coefficient of biological samples



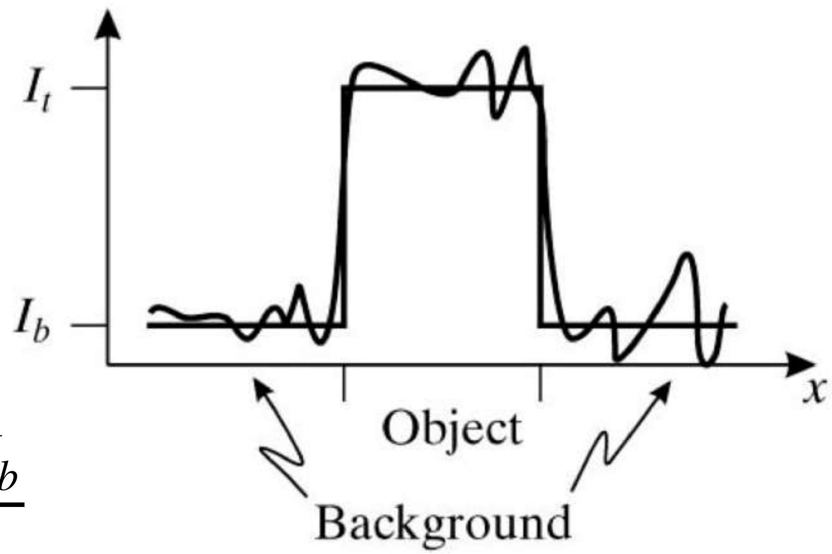
Medical imaging signals and systems, by J. L. Prince and J. Links.

Quality Metrics

- SNR (Signal to noise ratio)
- DQE (Detective quantum efficiency)
- MTF (Modulation transfer function)
- NPS (Noise power spectrum)
- NEQ (Noise equivalent quanta)

Effect of Noise

- Source of noise:
  - Detector does not faithfully reproduce the incident intensity.
  - X-rays arrive in discrete packets of energy. This discrete nature can lead to fluctuations in the image.
  - Local contrast $C = \frac{I_t - I_b}{I_b}$.
  - Signal is $I_t - I_b$.
  - Noise is due to Poisson behavior.
  - Variance of noise in background: σ^2
  - Signal to noise ratio $SNR = \frac{I_t - I_b}{\sigma_b} = \frac{CI_b}{\sigma_b}$



Example

- What is the local contrast of the blood vessel?
- What is the local contrast of the blood vessel when contrast agent is injected?

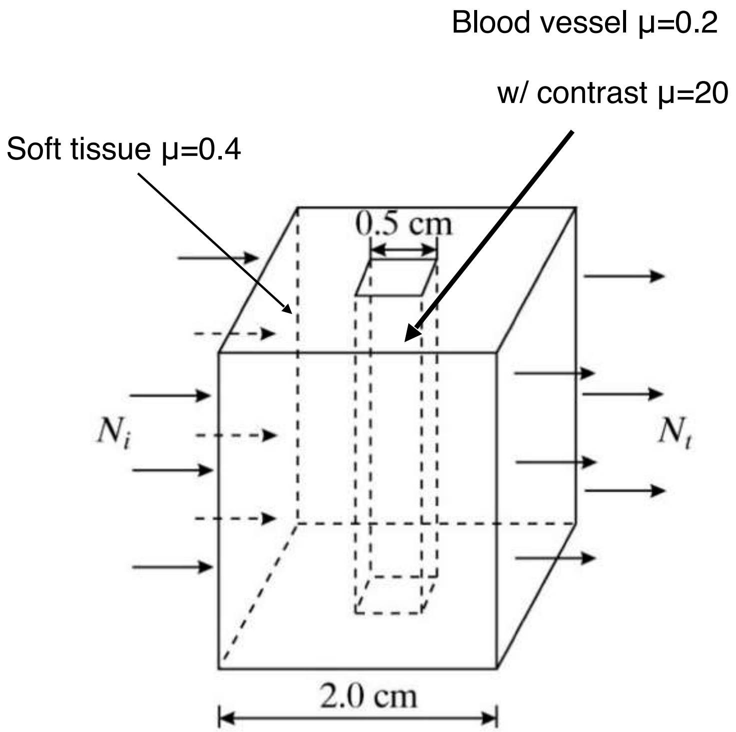
w/o contrast :

$$I_b = I_{\min} = I_0 e^{-(0.4*2.0)};$$

$$I_o = I_{\max} = I_0 e^{-(0.4*1.5+0.2*0.5)}$$

$$\text{Local contrast}: C_1 = \frac{I_o - I_b}{I_b};$$

$$\text{Global contrast}: C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$



Poisson Noise Analysis

- Assuming the number of photons in each burst follows the Poisson distribution

$$P(N = k) = \frac{a^k}{k!} e^{-a}$$

Variance = mean = a

- Let N_b denote the average number of photons per burst per area.
- Let $h\nu$ denote the effective energy for the X-ray source.
- The average background intensity is $I_b = \frac{N_b h\nu}{A\Delta t}$.
- The variance of photon intensity is $\sigma_b^2 = N_b \left(\frac{h\nu}{A\Delta t}\right)^2$.
- The $SNR = C\sqrt{N_b}$.
- SNR can be improved by
  - Increasing incident photon count
  - Improving contrast

Detective Quantum Efficiency

- Related to image quality in radiography and refers to the efficiency of information transfer from the input to the output of the system.
  - Consider
    - Potential SNR before detection.
    - Actual SNR upon detection.
  - Detective quantum efficiency

$$DQE = \left(\frac{SNR_{out}}{SNR_{in}} \right)^2$$

  - Degradation of SNR during detection.

Example

- Suppose an X-ray tube is set up to fire bursts of photons each with N=10000 photons and the detector's output (# of detected photons per burst) x has a mean =8000, variance=40000. What is its DQE?
- Solution:
  - The actual # of photons fired at the X-ray tube follows the Poisson process (mean=variance=10000)

$$SNR_{in} = \text{mean}/\sqrt{\text{variance}} = \sqrt{10000} = 100$$

  - The # of detected photons has mean=8000, variance=40000

$$SNR_{out} = \text{mean}/\sqrt{\text{variance}} = 8000/\sqrt{40000} = 40$$

  - This means that only about 16%of photons are detected correctly.

$$DQE = \left(\frac{SNR_{out}}{SNR_{in}}\right)^2 = 0.16$$

Effect of Compton Scattering

- Compton scattering causes the incident photons to be deflected from their straight line path
  - Add a constant intensity I_s in both target and background intensity ("fog")
  - Decrease in image contrast
  - Decrease in SNR

W/o scattering:

target intensity: I_t

background intensity: I_b

contrast $C = \dfrac{I_t - I_b}{I_b}$

$$\text{SNR} = C\frac{I_b}{\sigma_b} = C\sqrt{N_b}$$

W/ scattering:

target intensity: $I_t + I_s$

background intensity: $I_b + I_s$

contrast $C' = \dfrac{I_t - I_b}{I_b + I_s} = \dfrac{I_b}{I_b + I_s} C = \dfrac{C}{1 + I_s/I_b}$

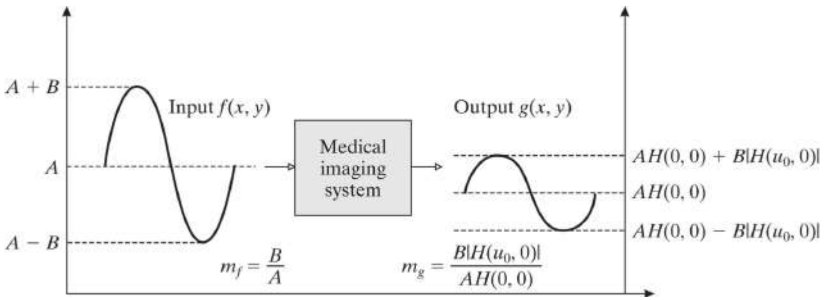
$$\text{SNR}' = C\frac{I_b}{\sigma_b'} = C\frac{N_b}{\sqrt{N_b + N_s}} = C\frac{\sqrt{N_b}}{\sqrt{1 + N_s/N_b}} = \text{SNR}\frac{1}{\sqrt{1 + I_s/I_b}}$$

Modulation

- Modulation of a periodic signal $f(x, y)$

$$m_f = \frac{f_{max} - f_{min}}{f_{max} + f_{min}}$$

- For a sinusoidal signal $f(x, y) = A + B\sin(2\pi u_0 x)$, $m_f = B/A$



$$g(x, y) = AH(0,0) + B\,|H(u_0,0)|\sin(2\pi u_0 x)$$

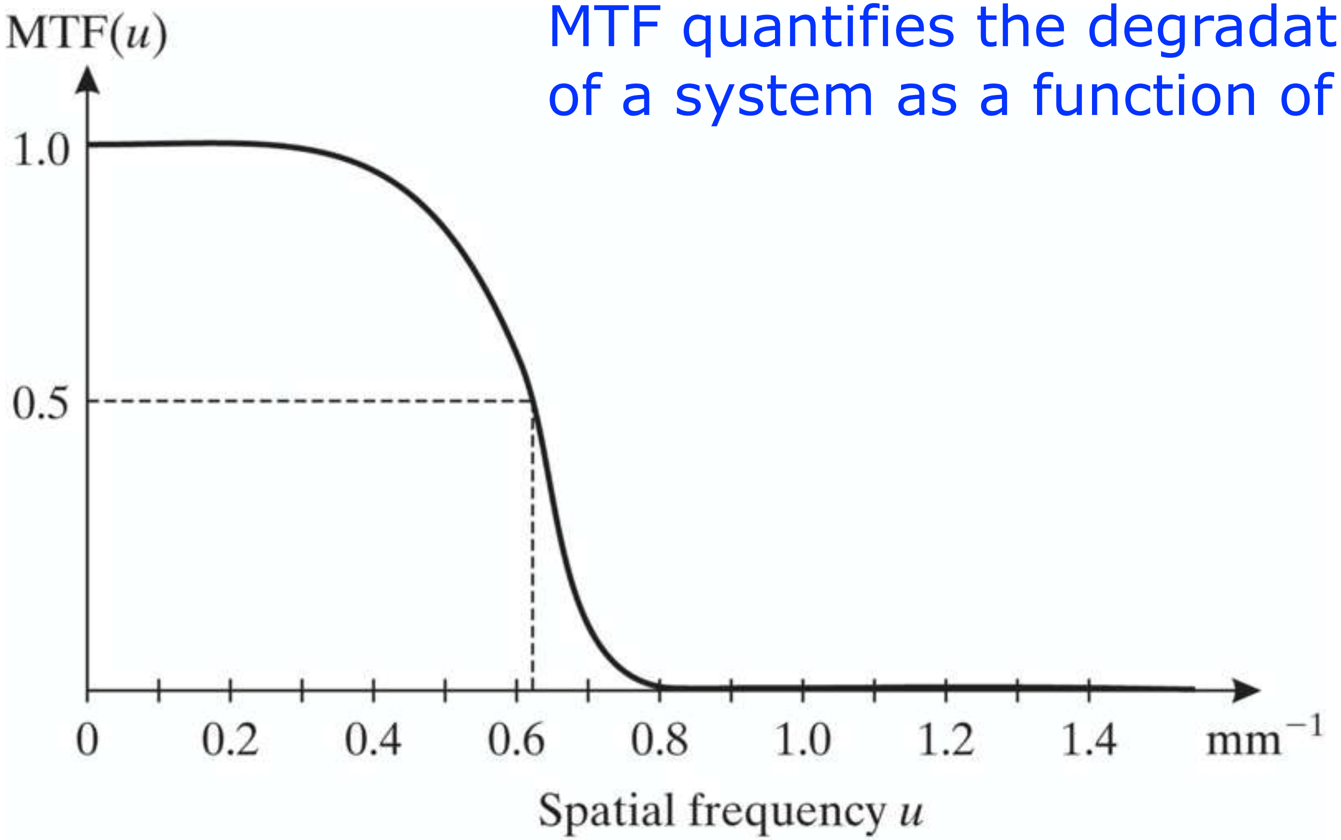
$$g_{\max} = AH(0,0) + B\,|H(u_0,0)|$$

- Modulation transfer function (MTF)
  - The ratio of the output modulation to the input modulation as a function of spatial frequency.

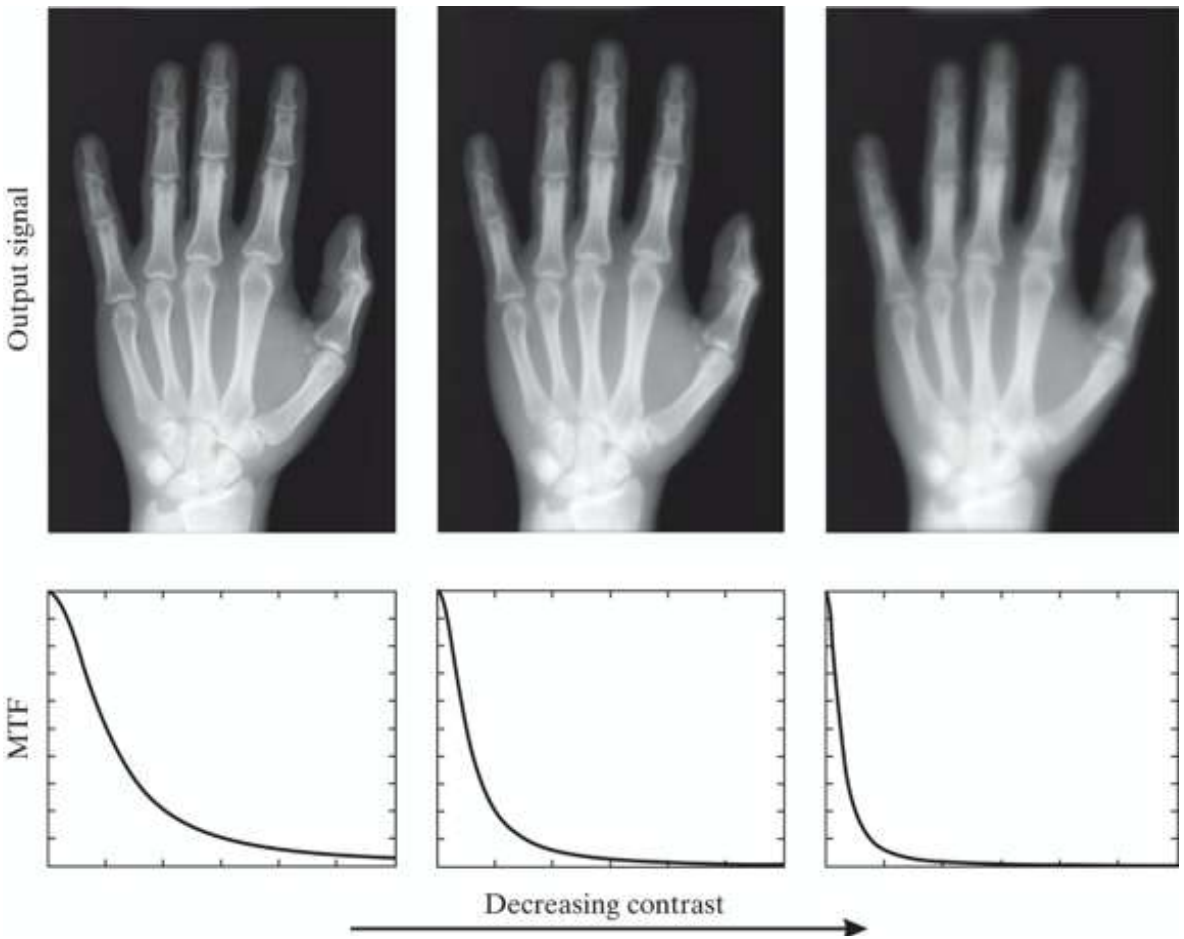
$$MTF(u) = \frac{m_g}{m_f} = \frac{|H(u,0)|}{H(0,0)}$$

A Typical MTF of a Medical Imaging System

MTF quantifies the degradation of the contrast of a system as a function of spatial frequency.



Impact of the MTF of a medical imaging system



Noise Power Spectrum

- Also known as the power spectral density, of a signal
- The Fourier transform of the noise autocorrelation.
- It gives the intensity of noise as a function of spatial frequency.

$$W(u,v) = < \lim_{X,Y\to\infty} \frac{1}{2X}\frac{1}{2Y} \left| \int_{-X}^{+X}\int_{-Y}^{+Y} \Delta g(x,y) e^{-2\pi i(ux+vy)} dx dy \right|^2 >$$

Noise Equivalent Quanta

- $NEQ(\omega) \equiv T \times DQE(\omega)$
  - T: number of quanta incident on the detector per unit area
  - ω: spatial frequency
- An equivalent quanta per unit area required by an ideal imaging system to give the same SNR achieved by an actual system.
- Gives the minimum number of x-ray quanta required to produce a specified SNR.

$$DQE = \frac{NEQ}{T}$$

The information content available in an x-ray image relative to an ideal detector.

Summary

- Projection radiography system consists of an x-ray tube, devices for beam filtration and restriction, compensation filters, grids, and a filmscreen detector (or digital detector, filmless).

- The detector reading (or image gray level) is proportional to the number of unabsorbed x-ray photons arriving at the detector, which depends on the **overall attenuation in the path** from the source to the detector.

- The above relation must be modified to take into account of inverse square law, obliquity, anode heel effect, extended source and detector impulse response.

- Both detector noise and Compton scattering reduce contrast and SNR of the formed image.

COMPUTED TOMOGRAPHY (CT)

CT

- The Nobel Prize in Physiology or Medicine 1979
  for the development of computer assisted tomography (CT)

Alan M. Cormack
Tufts University
Medford, MA, USA
(1924-1998)

Sir Godfrey N. Hounsfield
Central Research
Laboratories, EMI
London, UK
(1919-2004)

CT number

- Consistency across CT scanners is desired.
- CT number is defined as

$$h = 1000 \times \frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}}$$

  - h: Hounsfield units (HU)
- Usually rounded or truncated to nearest integer.
- Range: -1,000 ~ 3,000

HUs observed in several materials and tissue classes in human body

Material/Tissue	HU
Air	-1000
Lung	-600 to -400
Fat	-100 to -60
Water	0
Muscle	10 to 40
Blood	30 to 45
Soft tissue	40 to 80
Bone	400 to 3000

Tomographic vs. Projection Images

Parallel projection

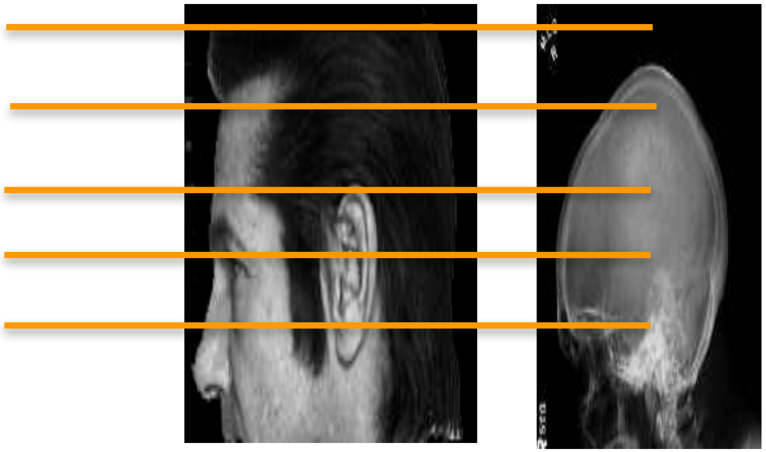
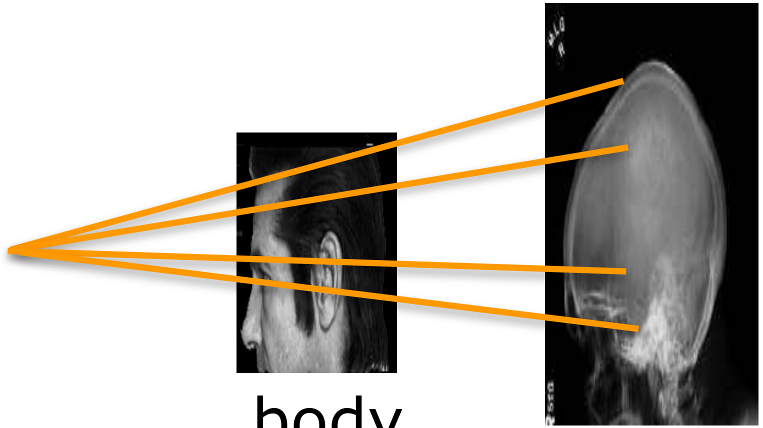


Image
(shadow)

Cone beam



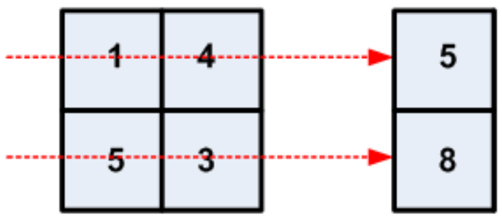
body

Image
(shadow)

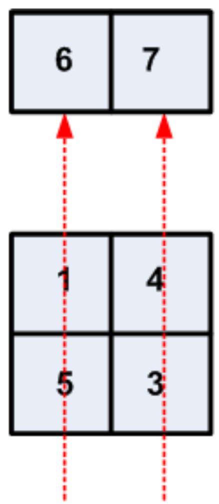
Tomographic image

Illustration of forward projecting 4 pixels image into 2 projections

- Projection
  - An integral operation along the path of the ray.
  - It sums the pixel values along its path, generating a vector of projection values.



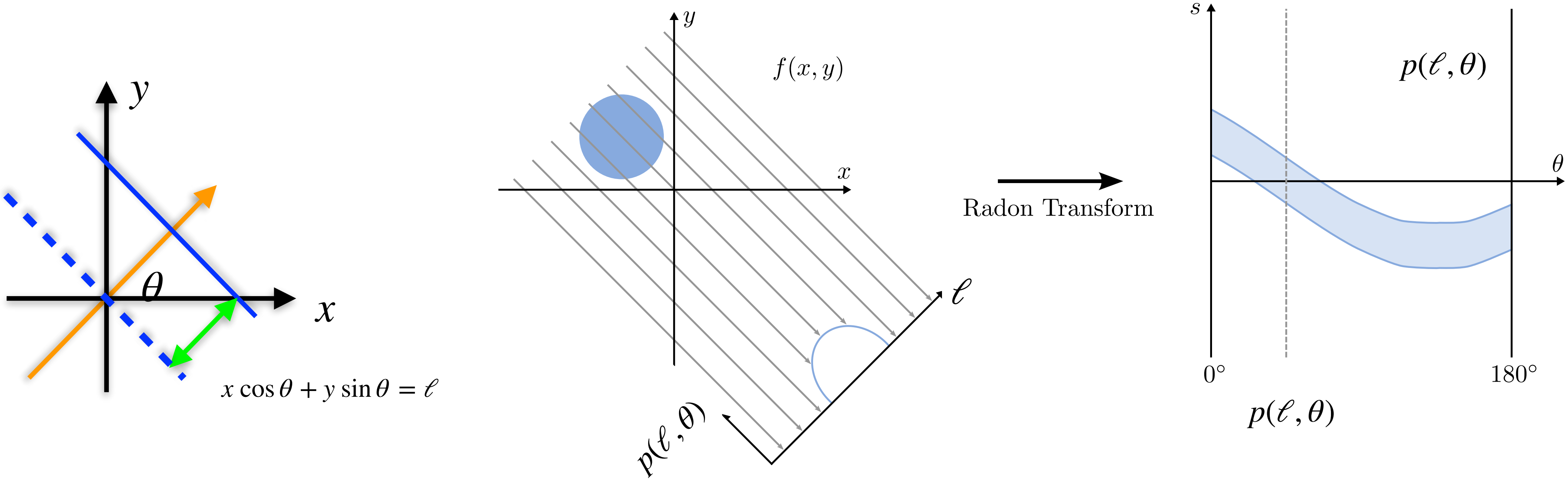
First projection at angle 0 degrees



Second projection at angle 90 degrees

Radon transform

- $$p(\ell,\theta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta+y\sin\theta-\ell)dxdy$$



Fourier Slice Theorem

- 1D Fourier transform of a parallel projection equals a slice of the 2D Fourier transform of the original object.
- Given projection data, it should then be possible to estimate the object by simply performing the 2D inverse Fourier transform.

Start by defining the 2D Fourier transform of the object function as

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

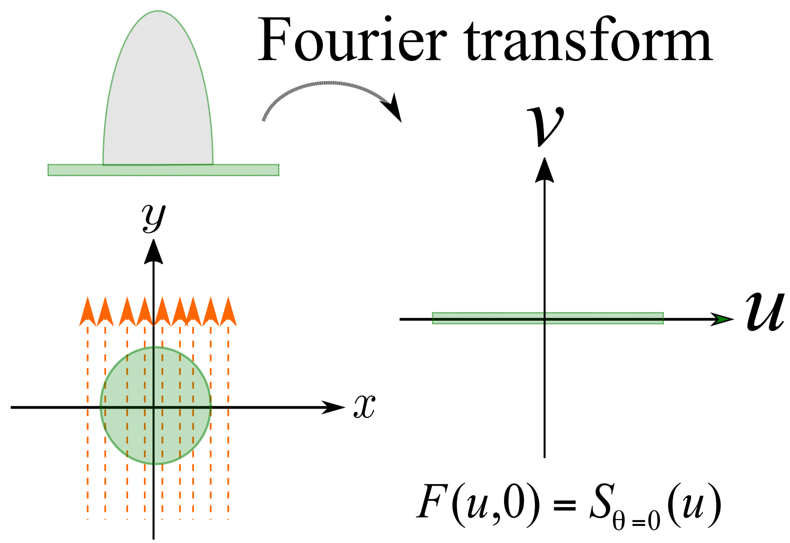
Projection at θ=0,

$$P_{\theta=0}(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

Leads to $v = 0$

$$F(u,0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi ux}dxdy$$

$$F(u,0) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(x,y)dy\right]e^{-j2\pi ux}dx = \int_{-\infty}^{\infty} P_{\theta=0}(x)e^{-j2\pi ux}dx$$

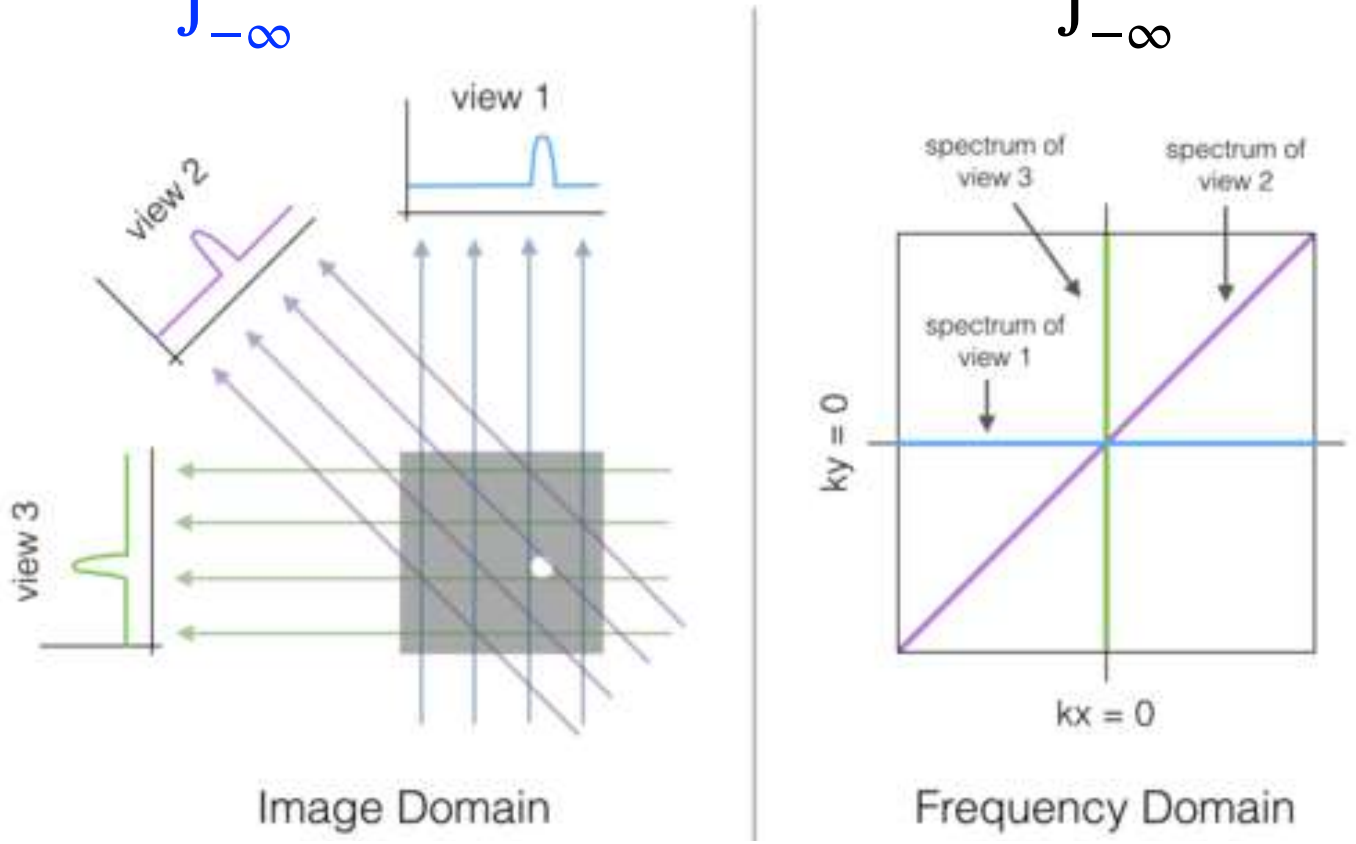


$$F(u,0) = S_{\theta=0}(u)$$

Fourier Slice Theorem

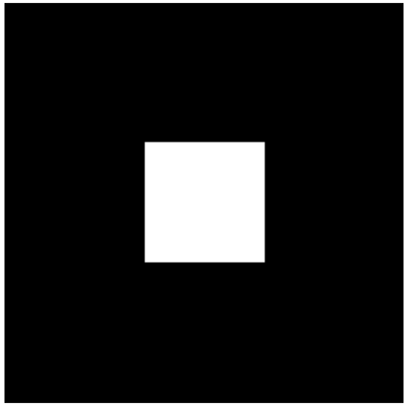
$$P_{\theta=0}(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

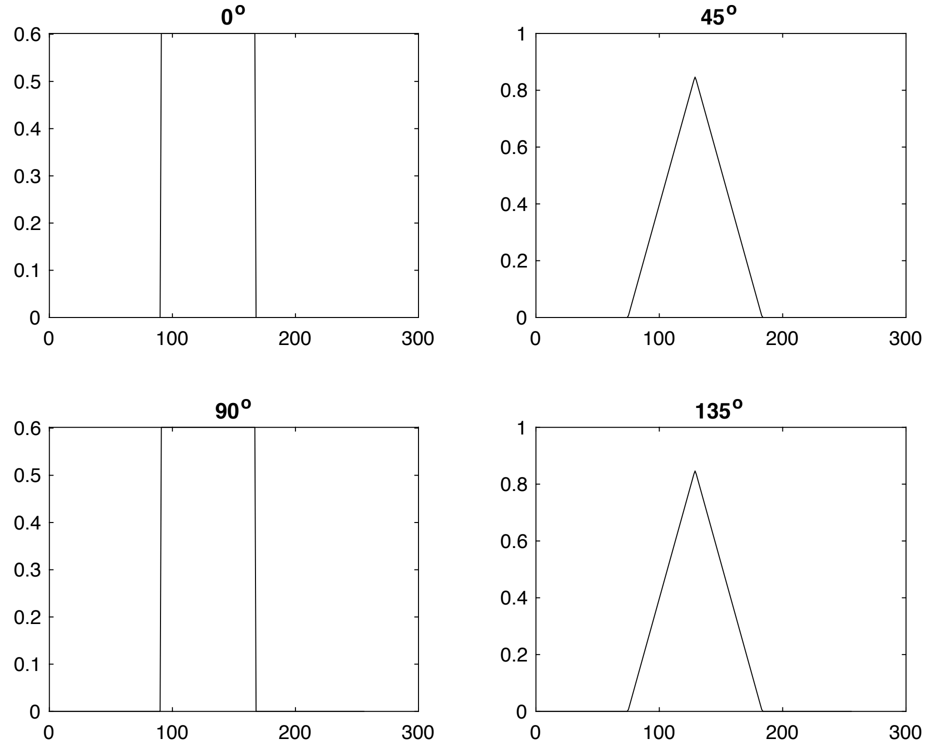
$$S_{\theta}(w) = \int_{-\infty}^{\infty} P_{\theta}(t)e^{-j2\pi\rho t}dt$$



Example

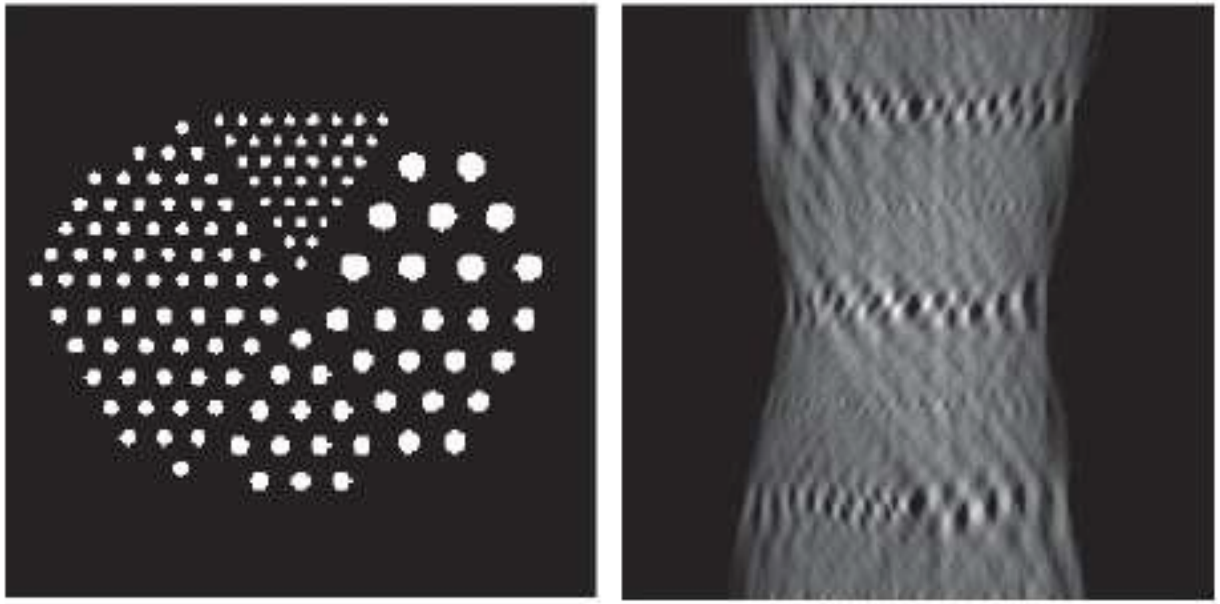
- Consider an image slice which contains a single square in the center. What are its projections along 0, 45, 90, 135 degrees?



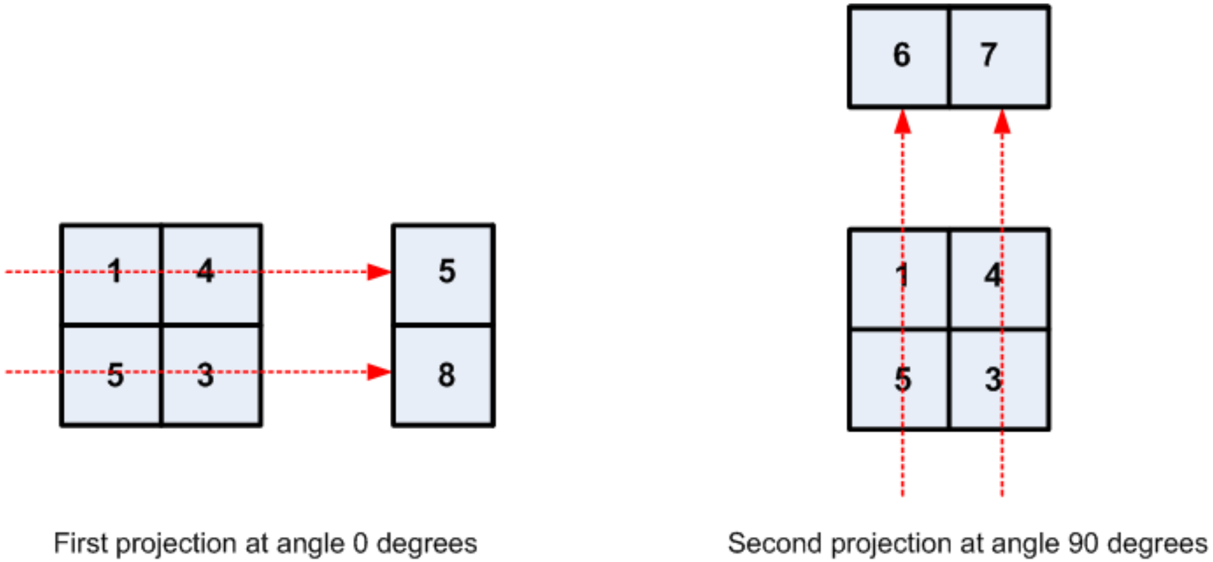


Sinogram

- CT data acquired for a collection of ℓ and θ
- CT scanner acquires a sinogram



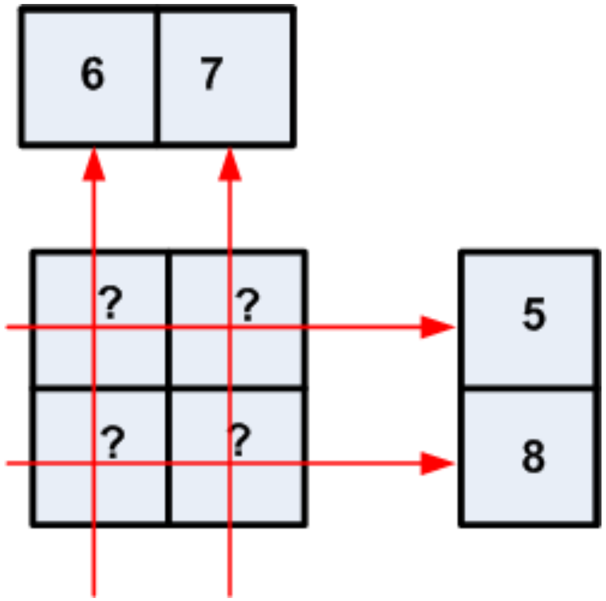
Illustrating the problem of image reconstruction on a simple 4 pixels image with 2 projections

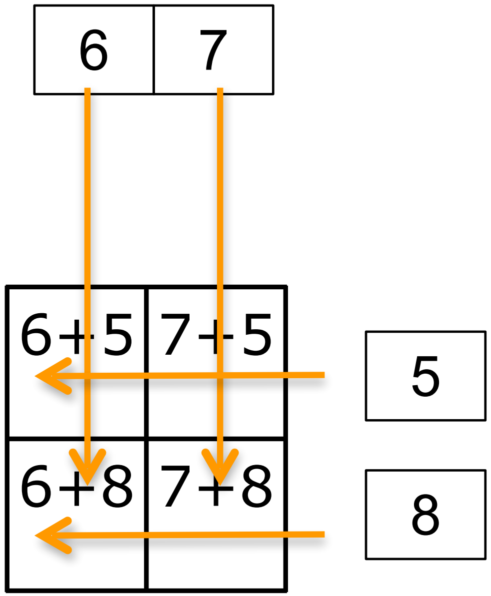


- A projection is an integral operation along the path of the ray. In other words, it sums the pixel values along its path, generating a vector of projection values.

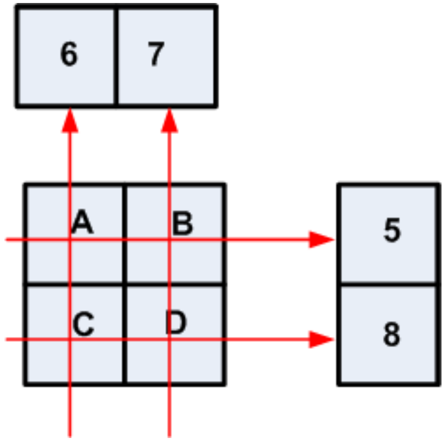
The CT Inverse problem

- Determine the original image from the
- projection data only





Solving the problem using linear algebra



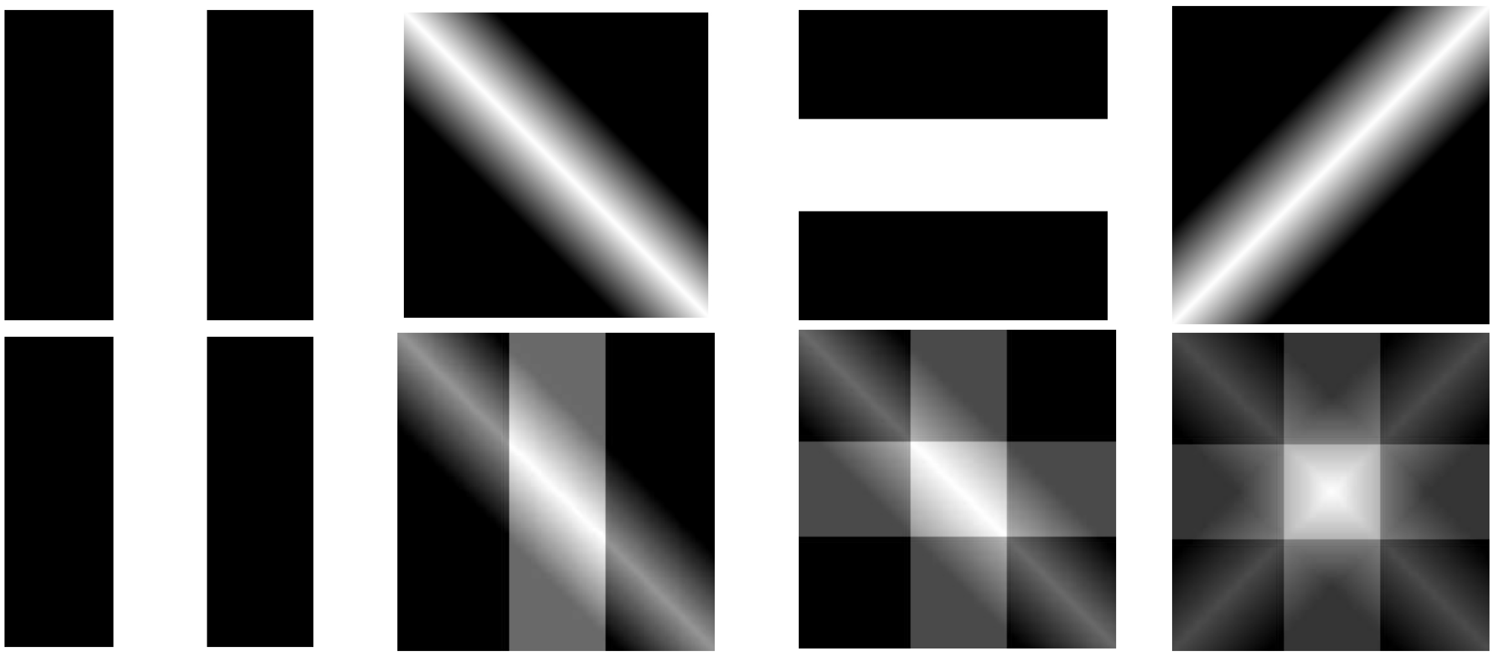
$$A + B = 5$$
$$C + D = 8$$
$$C + A = 6$$
$$D + B = 7$$

$$\underbrace{\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 5 \\ 8 \\ 6 \\ 7 \end{pmatrix}}_{b}$$

Determinant of A is zero. No unique solution exist. Infinite number of solutions. Use least square approaches.

Example

- Continue with the example of the image with a square in the center. Determine the backprojected image from each projection and the reconstruction by summing different number of backprojections.



The Fourier method

- The projection slice theorem leads to the following conceptually simple reconstruction method
  - Take 1D FT of each projection to obtain $G(\rho, \theta)$ for all θ
  - Convert $G(\rho, \theta)$ to Cartesian grid $F(u, v)$
  - Take inverse 2D FT to obtain $f(x, y)$
- Not used because
  - Difficult to interpolate polar data onto a Cartesian grid
  - Inverse 2D FT is time consuming
- But is important for conceptual understanding
  - Take inverse 2D FT on $G(\rho, \theta)$ on the polar coordinate leads to the widely used Filtered Backprojection algorithm

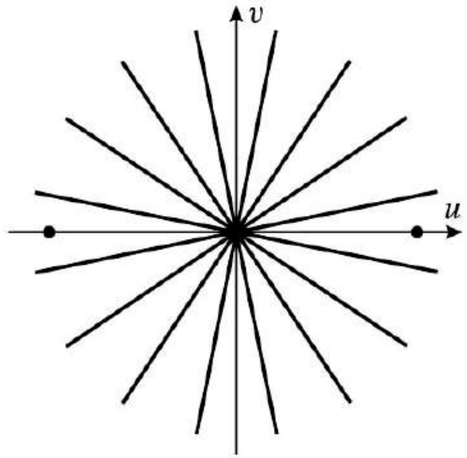
Filtered backprojection

- Algorithm:
  - For each θ
  - Take 1D FT of $g(\ell, \theta)$ for each $\theta \rightarrow G(\rho, \theta)$
  - Frequency domain filtering: $G(\rho, \theta) \rightarrow Q(\rho, \theta) = |\rho| G(\rho, \theta)$
  - Take inverse 1D FT: $G(\rho, \theta) \rightarrow g(\ell, \theta)$
  - Backprojecting $g(\ell, \theta)$ to image domain $\rightarrow b_\theta(x, y)$
- Sum of backprojected images for all θ

$$f(x, y) = \int_0^\pi \left[\int |\rho| \, G(\rho, \theta) e^{j2\pi\rho\ell} d\rho \right]_{\ell = x\cos\theta + y\sin\theta} d\theta$$

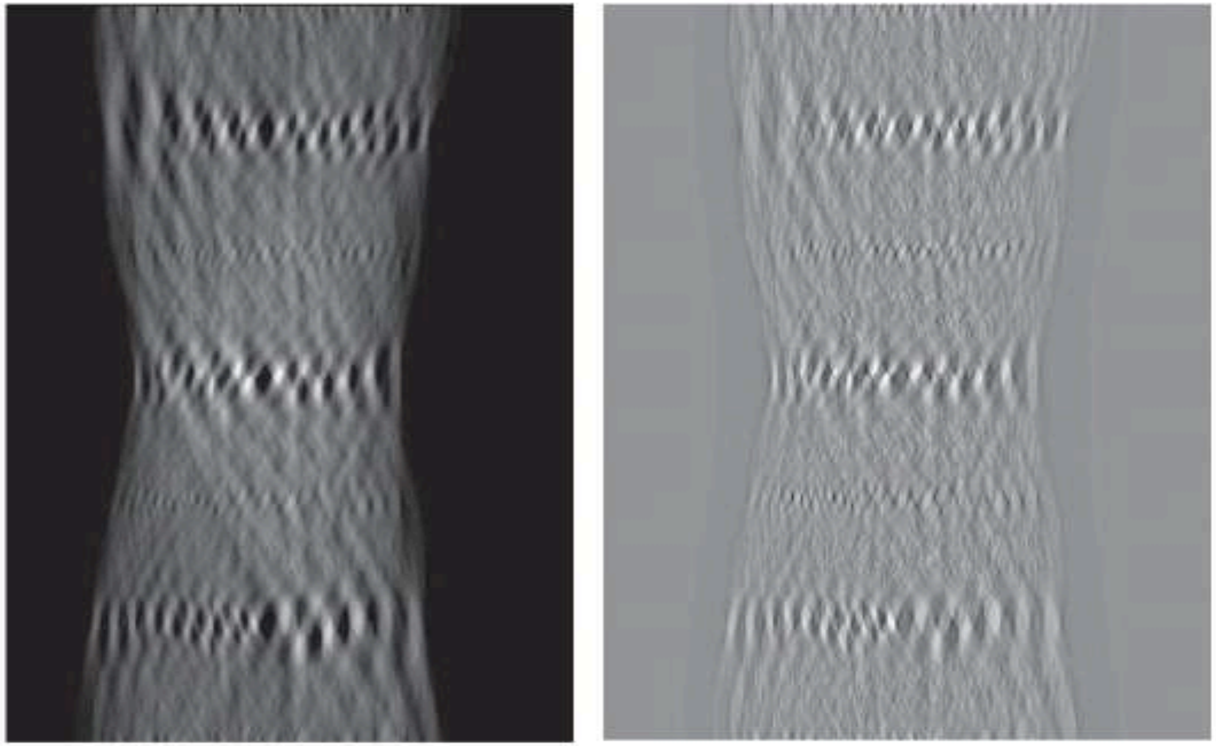
Ramp filter

- Filter response:
  - $c(\rho) = |\rho|$
  - High pass filter
- $G(\rho, \theta)$ is more densely sampled when ρ is small, and vice verse
- The ramp filter compensate for the sparser sampling at higher ρ.



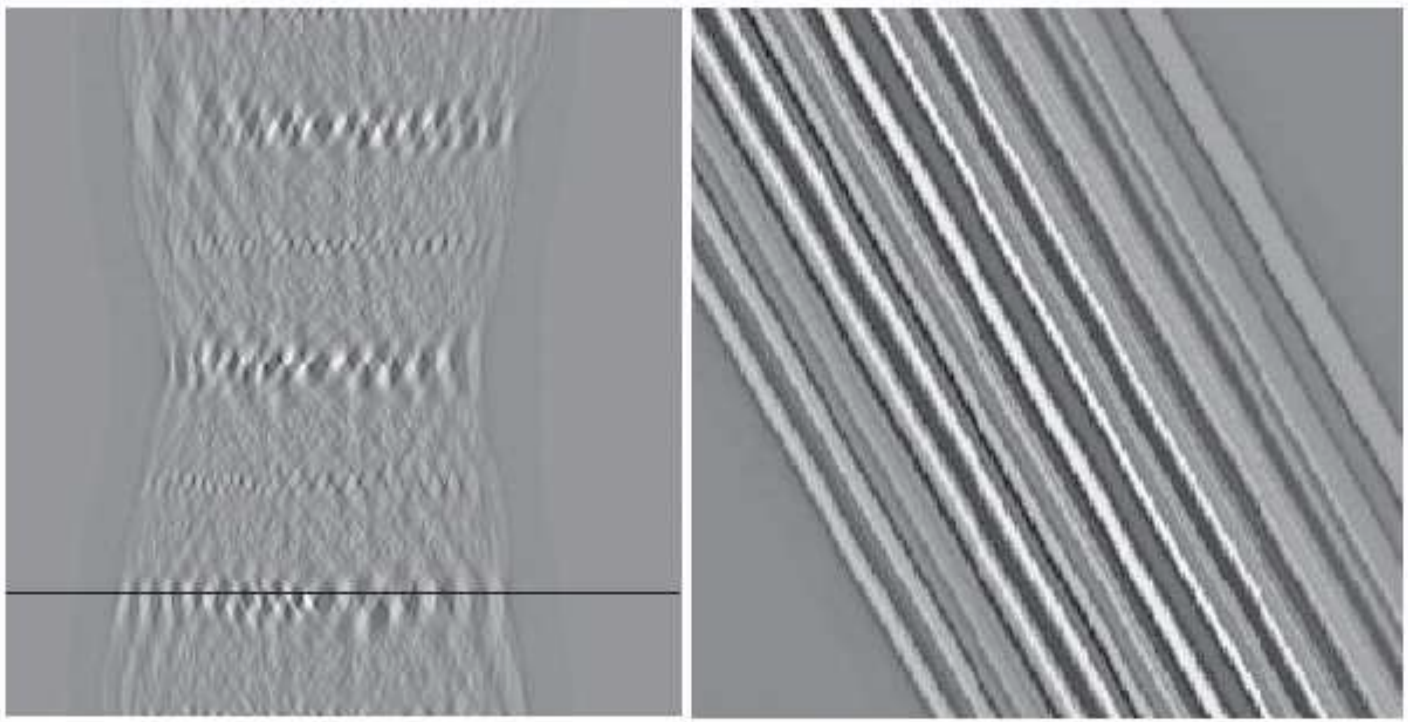
Step 1: Convolution

- Convolve every projection with $c(\ell)$
- The horizontal direction in the sinogram



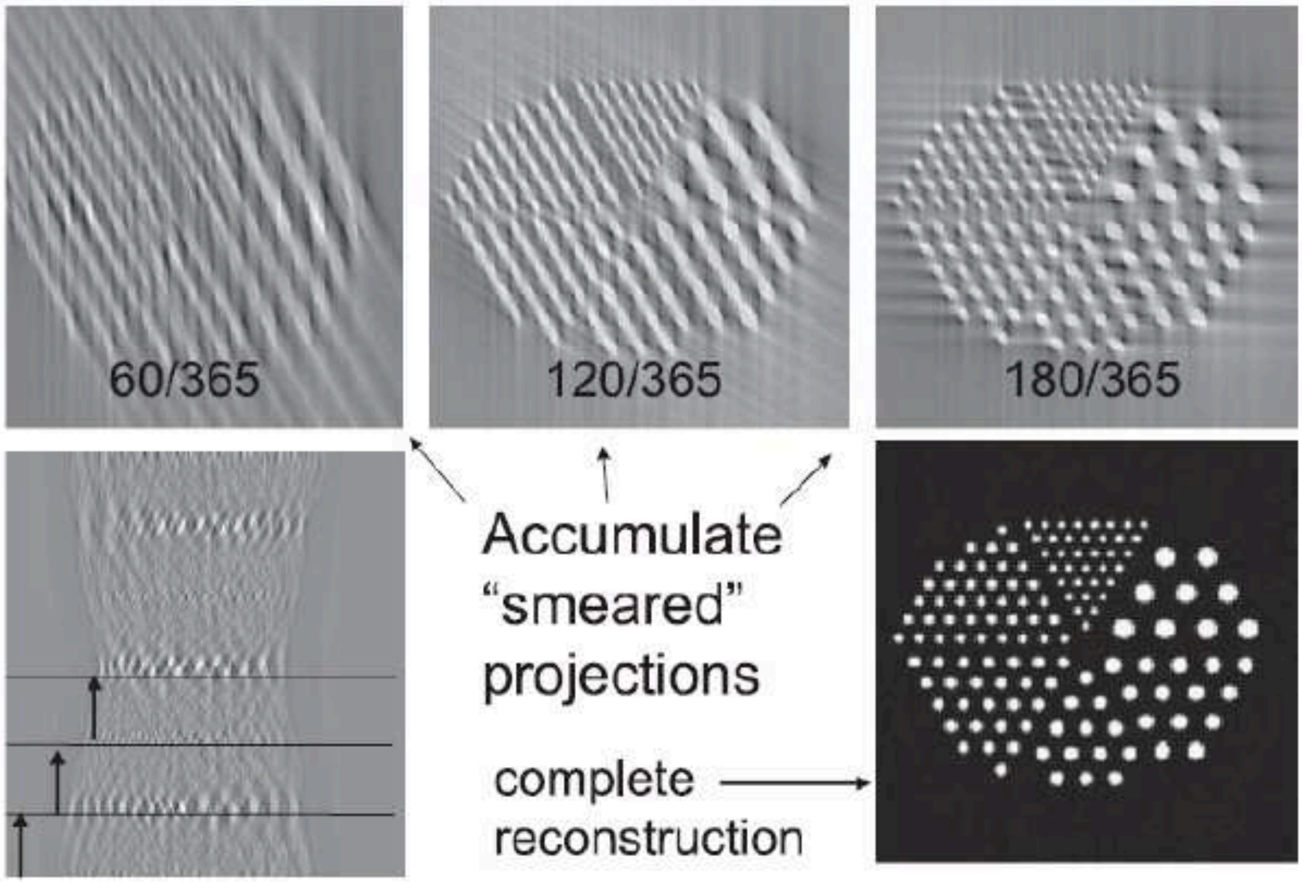
Step 2: backprojection

- 1D projection --> 2D laminar function

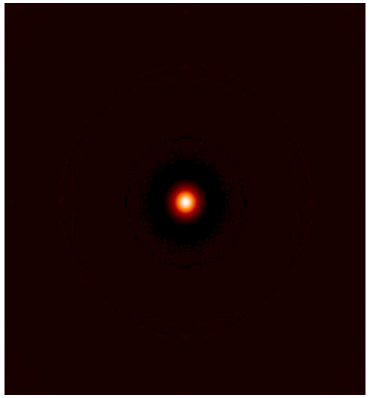


Step 3: summation

- Accumulate sum of backprojection images

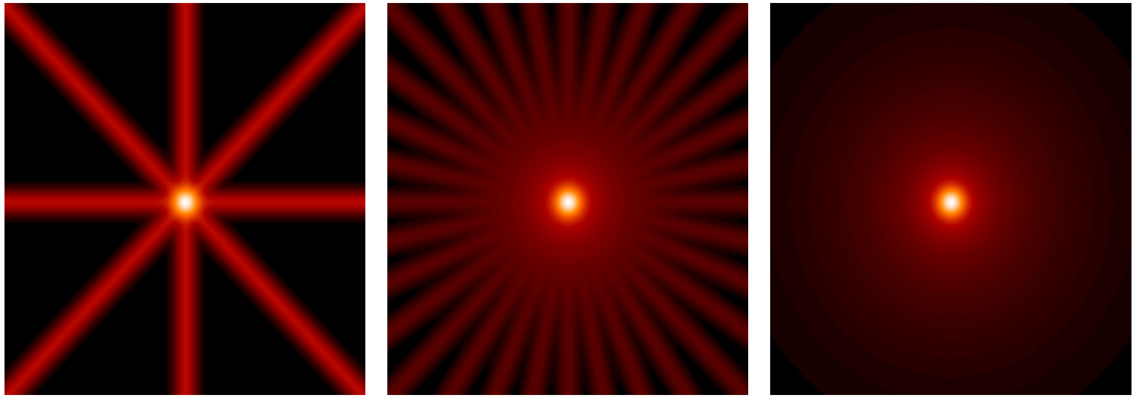


CT image reconstruction

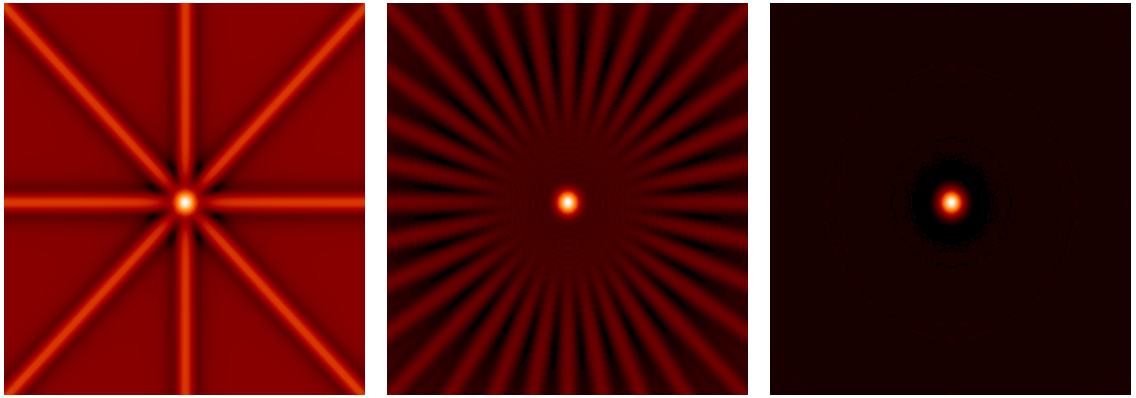


4 projections 16 projections 128 projections

Backprojection



Filtered Backprojection



Matlab implementation

- MATLAB (image toolbox) has several built-in functions:
  - phantom: create phantom images of size NxN
    - I = PHANTOM(DEF,N) DEF='Shepp-Logan',`Modified Shepp-Logan'
    - Can also construct your own phantom, or use an arbitrary image
  - radon: generate projection data from a phantom
    - Can specify sampling of θ
    - R = radon(I,theta)
    - The number of samples per projection angle = sqrt(2) N
  - iradon: reconstruct an image from measured projections
    - Can choose different filters and different interpolation methods for performing backprojection
    - [I,H]=iradon(R,theta, interpolation, filter, frequency_Scaling, OUTPUT_SIZE)
  - Use 'help radon' etc. to learn the specifics
  - Other useful commands:
    - imshow, imagesc, colormap

Radioisotope Imaging

Instructor: Cheng-Ying Chou, Ph.D.

Nuclear radiation results from unstable nuclei

- Nuclear stability is a balance between electromagnetic repulsion of protons and strong force interaction among all nucleons (protons and neutrons).
- There are ~ 2,450 isotopes of the ~ 100 elements in the Periodic Table, ~300 of which are naturally occurring, the others are human-made.
- Several mechanisms for unstable nuclei to decay to stable isotopes: fission, α-, β-, γ-emission, e- capture.
- Frequently there are multiple decay steps to reach stability.
- Each decay step is described by an exponential process with a characteristic decay time —-> Half-life of the isotope.
  - $T_{1/2} = \ln(2)/\lambda$: half-life
  - $N = N_0 e^{-\lambda t}$, where λ is the decay constant.

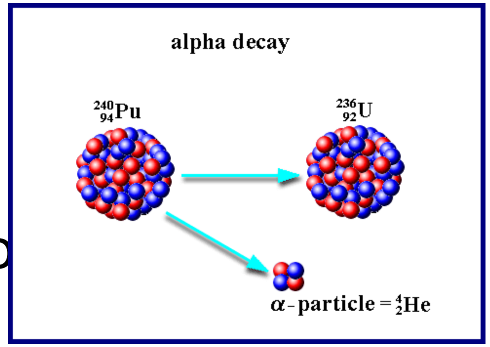
Decay Modes

- Four main modes of decay
  - alpha particles (2 protons, 2 neutrons)
  - beta particles (electrons)
  - positrons (anti-matter electrons)
  - isometric transition (gamma rays produced)

- Medical imaging is only with
  - Positrons (PET)
  - Gamma rays (scintigraphy, SPECT)
- Ionizing radiation modes
  - Particulate radiation
    - Alphas, betas, positrons
  - Electromagnetic radiation
    - Gamma rays

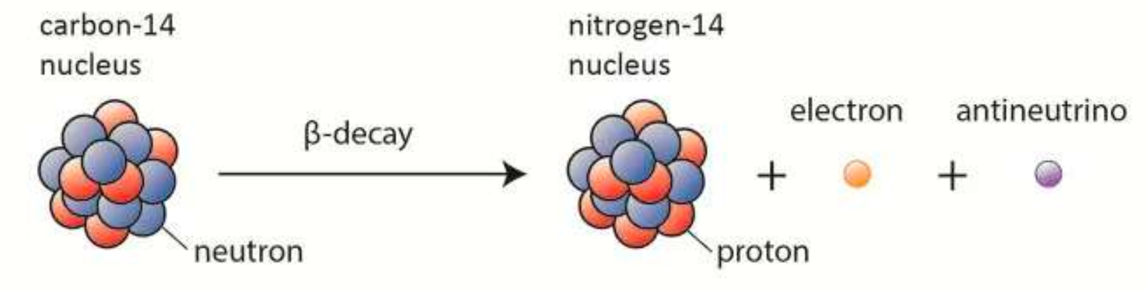
α decay (not used for imaging)

- Radioactive decay, emits an alpha particle (two protons and two neutrons bound together into a helium nucleus)
- Alpha decay occurs most often in massive nuclei that have too large a proton to neutron ratio. Alpha radiation reduces the ratio of protons to neutrons in the parent nucleus, bringing it to a more stable configuration.
- mono-energetic
- Loses energy in a more or less continuous slowing down process as it travels through matter.
- The distance it travels (range) depends only upon its initial energy and its average energy loss rate in the medium.
- The range for an α particle emitted in tissue is on the order of microns.



Beta decay

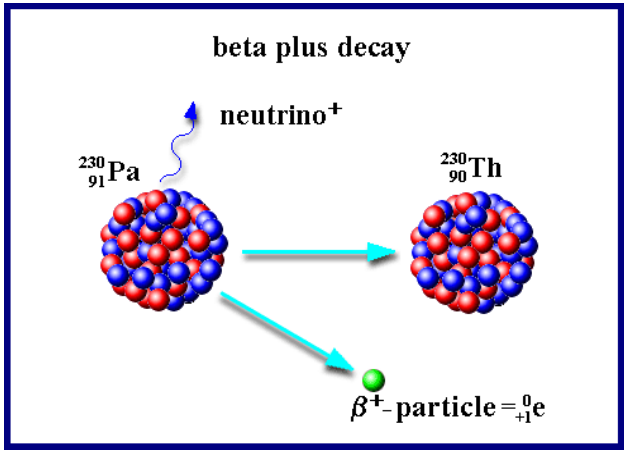
- Occurs when too many protons or too many neutrons in a nucleus, one of the protons or neutrons is transformed into the other.
- Mass number A does not change after decay, proton number Z increases or decreases.
- Beta minus decay (or simply Beta decay): A neutron changes into a proton, an electron (beta particle) and an antineutrino.



$$n \rightarrow p + e^{-} + \bar{\nu}$$

Positron decay and electron capture

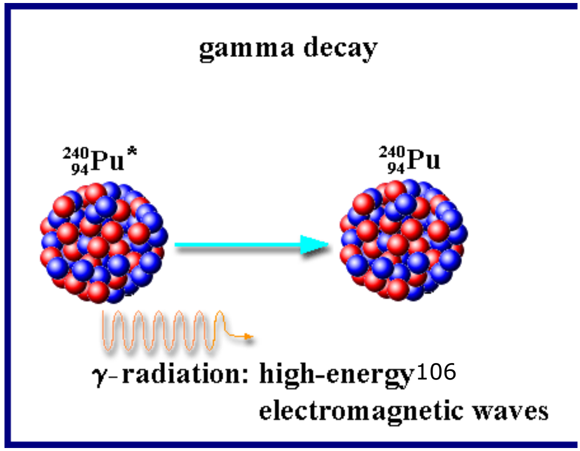
- Also known as Beta Plus decay
- A proton changes to a neutron, a positron (positive electron), and a neutrino (P -> n+β^{+}+ν)
- Mass number A does not change, proton number Z reduces.
  - Positron: antimatter electron
  - Neutrino, emitted with positron, a massless, chargeless subatomic particle.



- The positron may later annihilate a free electron, generate two gamma photons in opposite directions.
- These gamma rays are used for medical imaging (Positron Emission Tomography).
- Electron capture: P+e^{-} -> n+ν.

Gamma decay (isometric transition)

- A nucleus (which is unstable) changes from a higher energy state to a lower energy state through the emission of electromagnetic radiation (photons) (called gamma rays).
  - The daughter and parent atoms are isomers.
  - The gamma photon is used in Single photon emission computed tomography (SPECT)
- Gamma rays have the same property as X-rays, but are generated different:
  - X-ray through energetic electron interactions
  - Gamma-ray through isometric transition in nucleus.



Measurement of radioactivity

- Radioactivity, A, # disintegrations per second (dps)

1 Bq= 1 dps

$$1\ Ci = 3.7 \times 10^{10}\ \mathrm{Bq}$$

Bq=Bequerel
Ci=Curie

- The intensity of radiation incident on a detector at range r from a radioactive source is

$$I = \frac{AE}{4\pi r^2}$$

- A: radioactivity of the material; E: energy of each photon.

Radioactive decay law

- $N(t)$: the number of radioactive atoms at a given time
- $A(t)$: is proportional to $N(t)$

$$A = -\frac{dN}{dt} = \lambda N$$

$$N(t) = N_0 \mathrm{e}^{-\lambda t}$$

$$A(t) = A_0 \mathrm{e}^{-\lambda t} = \lambda N_0 \mathrm{e}^{-\lambda t}$$

- Number of photons generated (= number of disintegrations) during time T

$$\Delta N = \int_0^T A(t)dt = \int_0^T \lambda N_0 \mathrm{e}^{-\lambda t} dt = N_0(1 - \mathrm{e}^{-\lambda T})$$

Statistics of decay

- The exponential decay law only gives the expected number of atoms at a certain time t.
- The number of disintegrated atoms over a short time $\Delta t << T_{1/2}$ after time $t = 0$ with N_0 atoms follows Poisson distribution

$$\Pr\{\Delta N = k\} = \frac{a^k}{k!}\mathrm{e}^{-a}$$

- $a = \lambda N_0 \Delta t$ can be interpreted as the probabilities of having 1 disintegration from all N atoms during interval Δt.
- λN_0 is called the Poisson rate.

Radiotracers

- Decay mode:
  - Desire clean gamma decay: do not emit alpha or beta particles.
  - Positron decay: positron will annihilate with electrons to produce gamma rays.
- Energy of photon:
  - Should be high so that can leave the body w/ little attenuation.
  - Hard to detect if the energy is too high.
  - Desired energy range: 70-511 KeV.
- Half-life
  - Should not be too short (before detector can capture) or too long (longer patient scan time).
  - Minutes to hours desired.
- Monoenergetic
  - Energy sensitive detectors can discriminate the primary photons from scattered ones.

List of Nuclear Medicine Radionuclides

Isotope	Gamma energy(keV)	Half-life	
• Tc99m	140.5	6.03 hours	Kidney function
• I-131	364, 637	8.06 days	Thyroid function
• I-123	159	13.0 hours	
• I-125	35	60.2 days	
• In-111	172, 247	2.81 days	
• Tl-201	~70, 167	3.044 days	
• Ga-67	93, 185, 300	3.25 days	

Positron emitter

• Fluorine-18	202	110 mins	
• Oxygen-15	696	2 mins	Oxygen metabolism

http://en.wikipedia.org/wiki/Radiopharmaceutical

Effective half-life T_e

- T_p, physical half-life of the radionuclide
- T_b, the biological half-life of the pharmaceutical;
  - Arises from body processes that metabolize or clear the pharmaceutical.

- The effective half-life T_e combines the physical T_p and biological T_b half-lives:

$$\frac{1}{T_e} = \frac{1}{T_p} + \frac{1}{T_b}$$

Common Radionuclides in Nuclear Medicine (1 of 2)

Gamma Emitters			
Z	Nuclide	Half-life	Photon Energy (keV)
24	Chromium-51	28 d	320
31	Gallium-67	79.2 h	92, 184, 296
34	Selenium-75	120 d	265
38	Strontium-87m	2.8 h	388
43	Technetium-99m	6 h	140
49	Indium-111	2.8 d	173, 247
	Indium-113m	1.73 h	393
53	Iodine-123	13.3 h	159
	Iodine-125	60 d	35, 27
	Iodine-131	8.04 d	364
54	Xenon-133	5.3 d	81
80	Mercury-197	2.7 d	77
81	Thallium-201	73 h	135, 167

Common Radionuclides in Nuclear Medicine (2 of 2)

Positron Emitters			
Z	Nuclide	Half-life	Positron Energy (keV)
6	Carbon-11	20.3 min	326
7	Nitrogen-13	10.0 min	432
8	Oxygen-15	2.1 min	696
9	Fluorine-18	110 min	202
29	Copper-64	12.7 h	656
31	Gallium-68	68 min	1,900
33	Arsenic-72	26 h	3,340
35	Bromine-76	16.1 h	3,600
37	Rubidium-82	1.3 min	3,150
53	Iodine-122	3.5 min	3,100

Summary

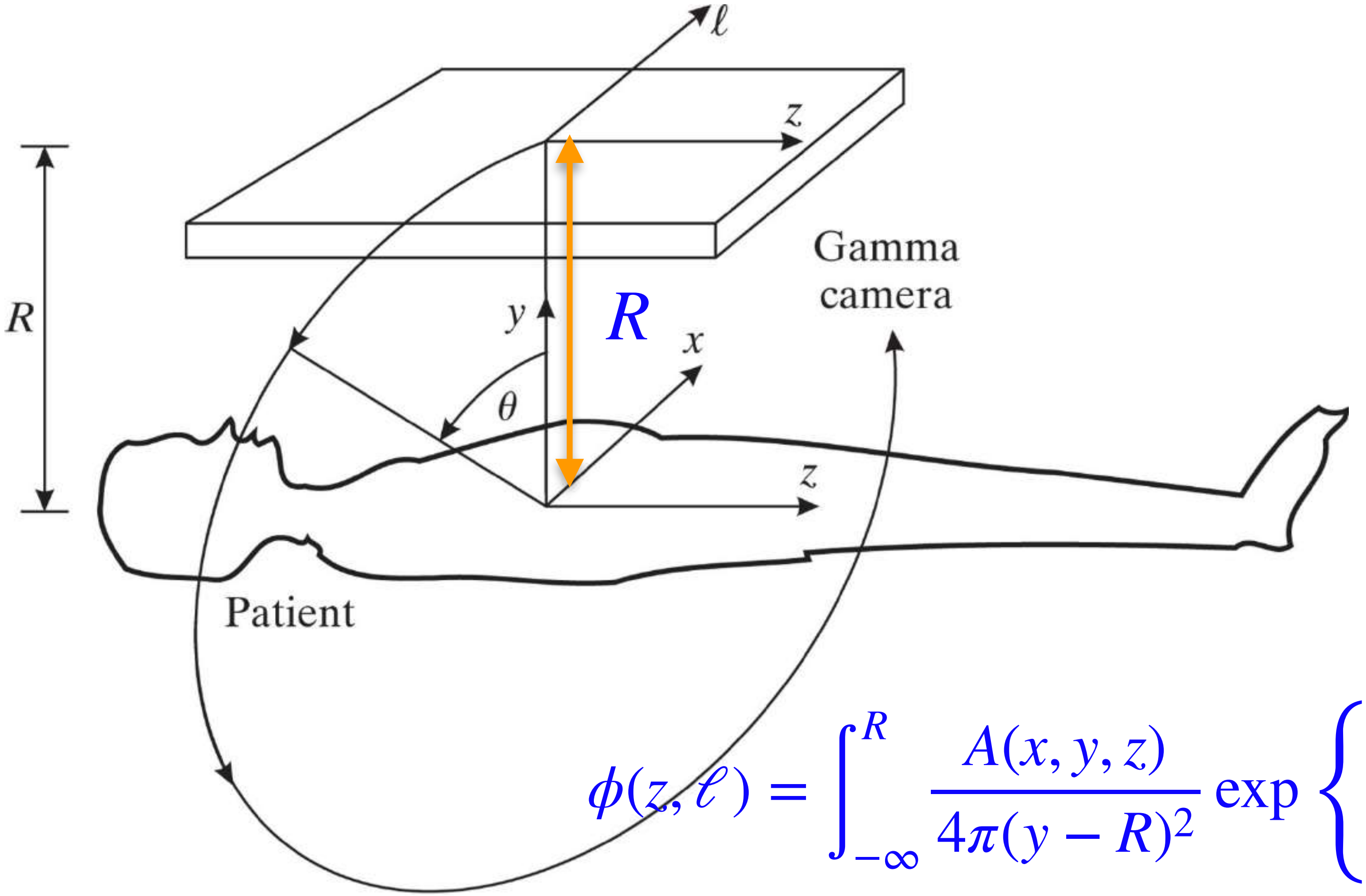
- Nuclear medicine relies on radiation (gamma rays) generated through radioactive decay.
  - Radioactive decay is the process when a unstable nuclide is changed to a more stable one.
- Four modes of decay, generating alpha particles, beta particles, positrons and gamma rays respectively.
- Radioactivity follows an exponential decay law, characterized by the decay constant or the half-life.
- Desired properties for radio tracers.
- Common radiotracers in nuclear medicine.

SINGLE PHOTON EMISSION TOMOGRAPHY (SPECT)

Single Photon Emission Tomography

CT scanner

Imaging Equation: $\theta = 0$



$$\phi(z,\ell) = \int_{-\infty}^{R} \frac{A(x,y,z)}{4\pi(y-R)^2} \exp\left\{ -\int_{y}^{R} \mu(x,y',z;E)\,dy' \right\} dy$$

Imaging Equation: General Case

- $$\phi(\ell,\theta) = \int_{-\infty}^{R} \frac{A(x(s),y(s))}{4\pi(s-R)^2} \exp\left\{-\int_{s}^{R} \mu(x(s'),y(s');E)\,ds'\right\} ds$$

- Two unknowns:
  - $A(x,y)$
  - $\mu(x,y)$
- Generally intractable
  - Ignore attenuation (often done)
  - Assume constant
  - Measure and apply attenuation correction

Approximation

- Bold approximations: ignore attenuation, inverse square law, and scale factors:

$$\phi(\ell,\theta) = \int_{-\infty}^{\infty} A(x(s), y(s))ds$$

- Using line impulse

$$\phi(\ell,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(x,y)\delta(x\cos\theta + y\sin\theta - \ell)dxdy$$

- Under this assumption, A can be reconstructed using the filtered back projection approach.
- The reconstructed signal needs to be corrected.

POSITRON EMISSION TOMOGRAPHY (PET)

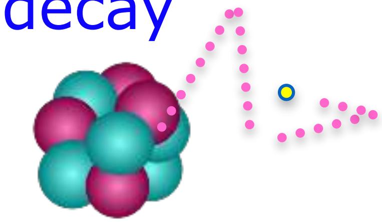
PET instrumentation

PET Principle

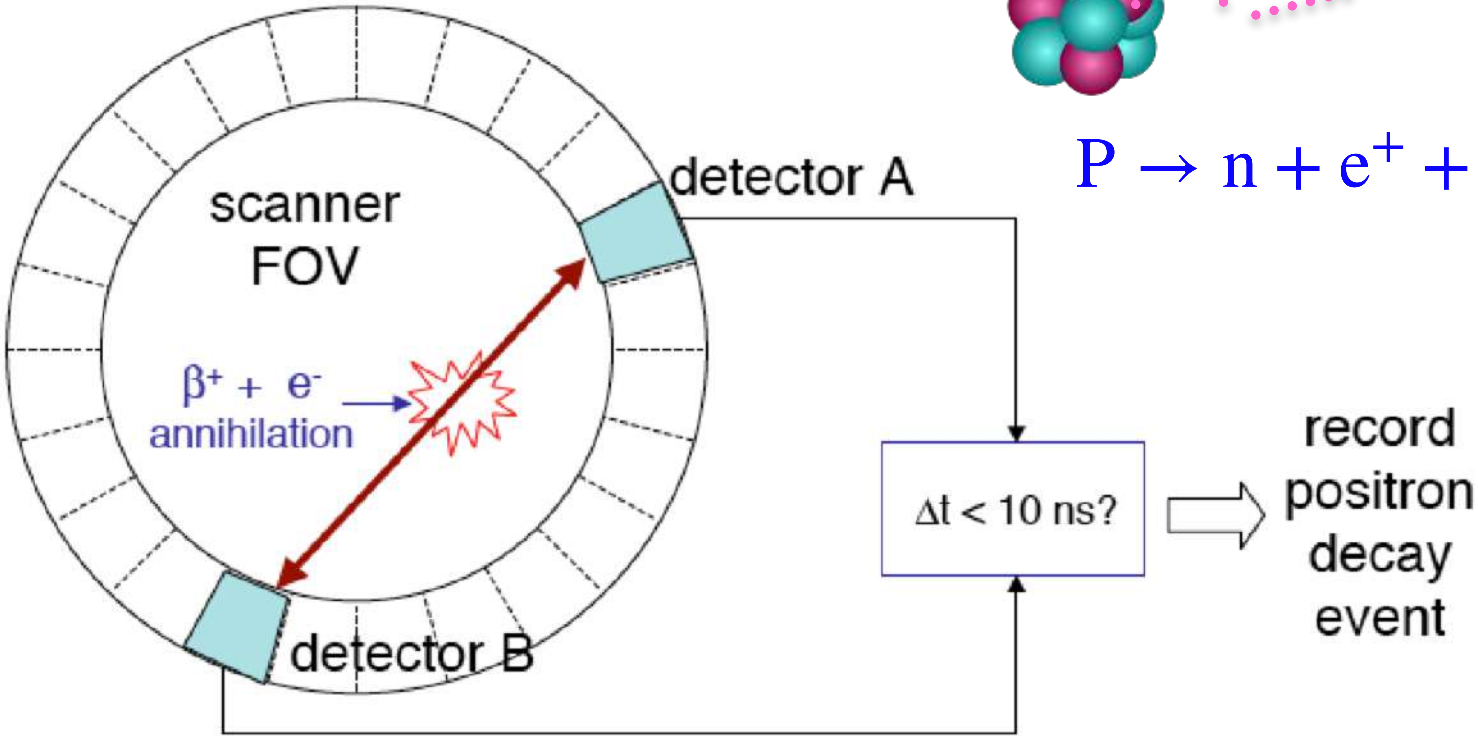
- Positron emitters
- Positron annihilation
  - Short distance from emission
  - Produces two 511 keV gamma rays
  - Gamma rays in $180°$ opposite directions
- Principle: detect coincident gamma rays

PET imaging: timing coincidence

Beta plus decay

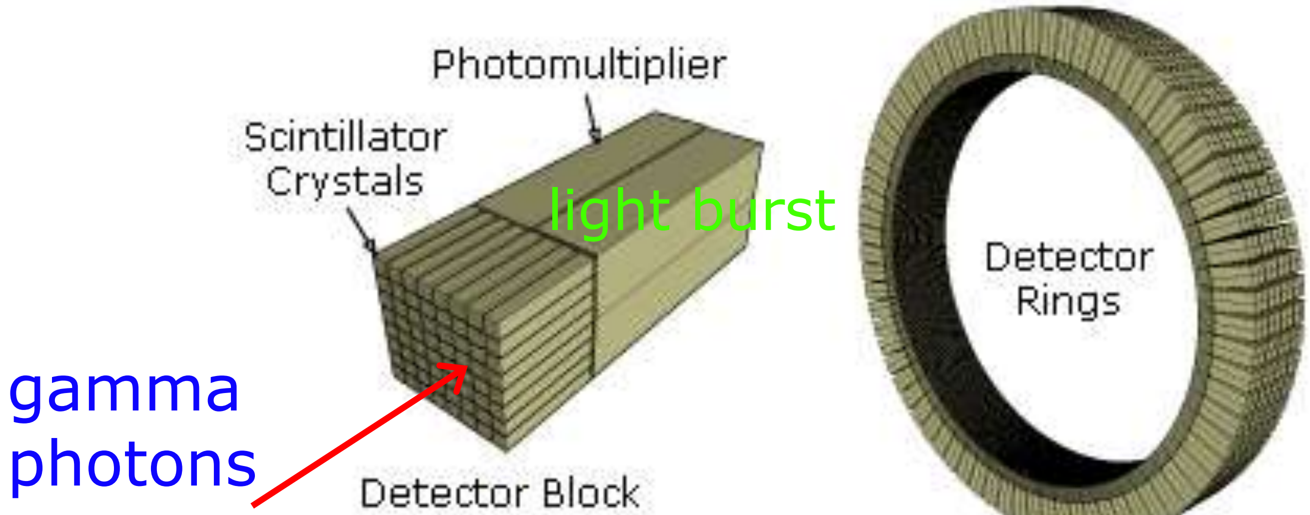


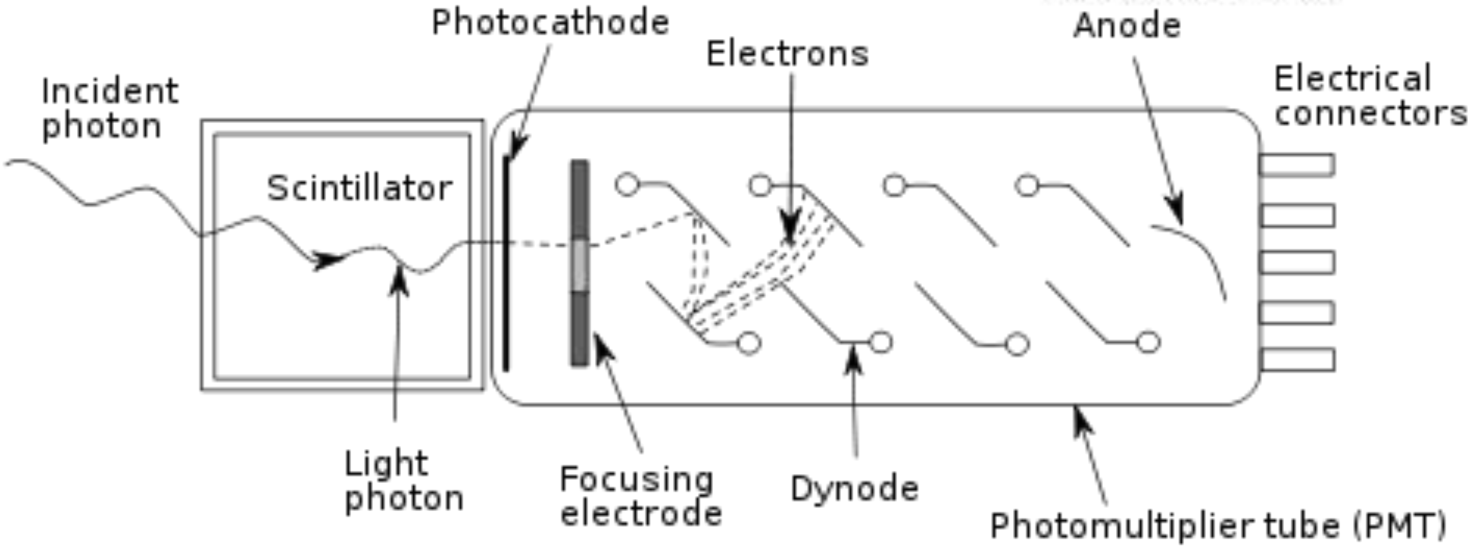
$$P \rightarrow n + e^{+} + \nu + \text{energy}$$



PET "block detector" design

- **Good performance**
- **Inexpensive**
- **Easy to pack**





Radiotracers used for PET/CT

- Isotopes such as ^{11}C, ^{15}O, ^{18}F and ^{13}N used in PET/CT undergo radioactive decay by emitting a positron, i.e. a positively charged electron (e^+), and a neutrino:

$$^{18}_{9}F \rightarrow ^{18}_{8}O + e^+ + \text{neutrino}$$

$$^{11}_{6}C \rightarrow ^{11}_{5}B + e^+ + \text{neutrino.}$$

Properties and applications of the most common PET radiotracers.

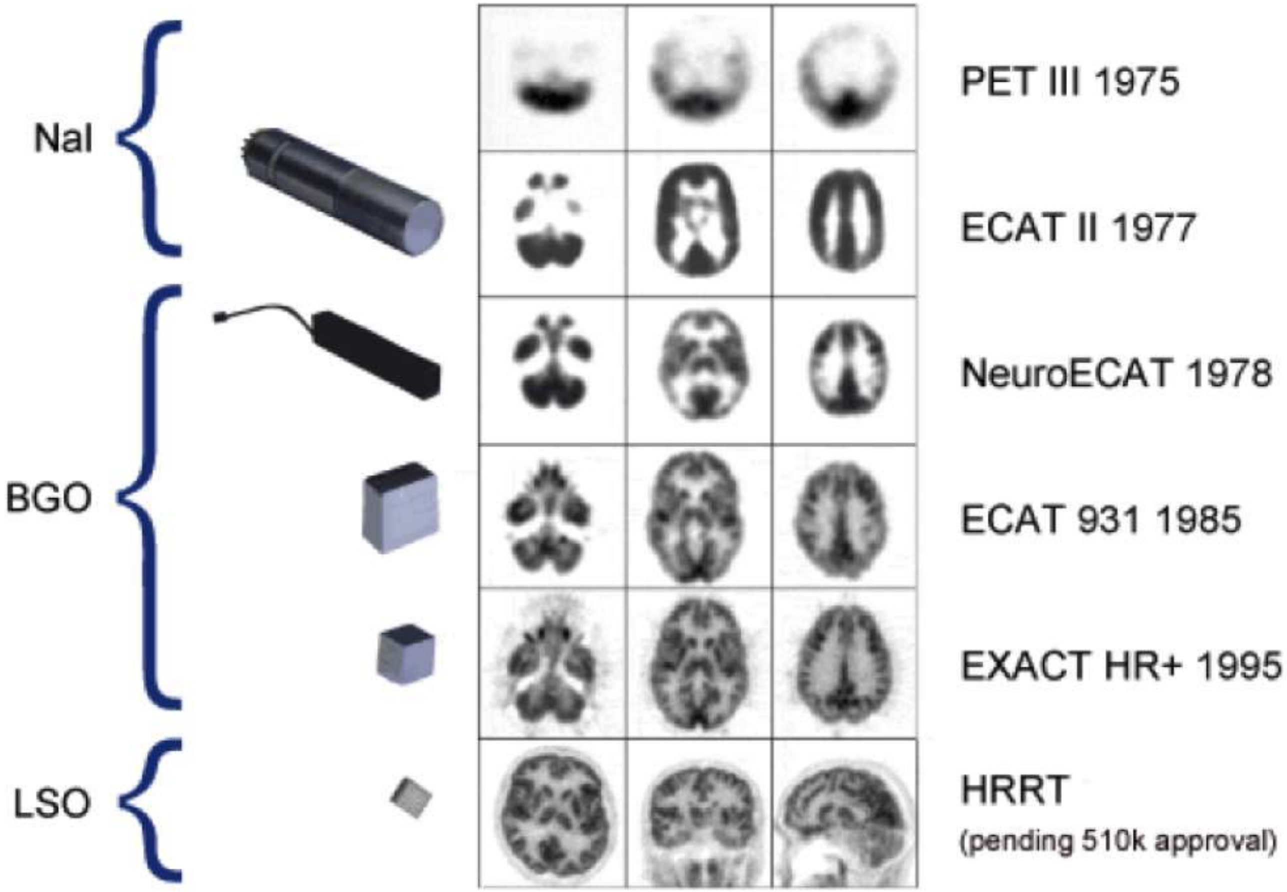
Radionuclide	Half-life (minutes)	Radiotracer	Clinical applications
^{18}F	109.7	^{18}FDG	oncology, inflammation, cardiac viability
^{11}C	20.4	^{11}C-palmitate	cardiac metabolism
^{15}O	2.07	$H_2{}^{15}O$	cerebral blood flow
^{13}N	9.96	$^{13}NH_3$	cardiac blood flow
^{82}Rb	1.27	$^{82}RbCl_2$	cardiac perfusion

Basic radiation detector systems

- Information about?
  - Energy?
  - Position (where did it come from)?
  - How many/ how much?

- Properties of radiation detectors
  - Energy resolution
  - Spatial resolution
  - Sensitivity
  - Counting speed

PET detector evolution

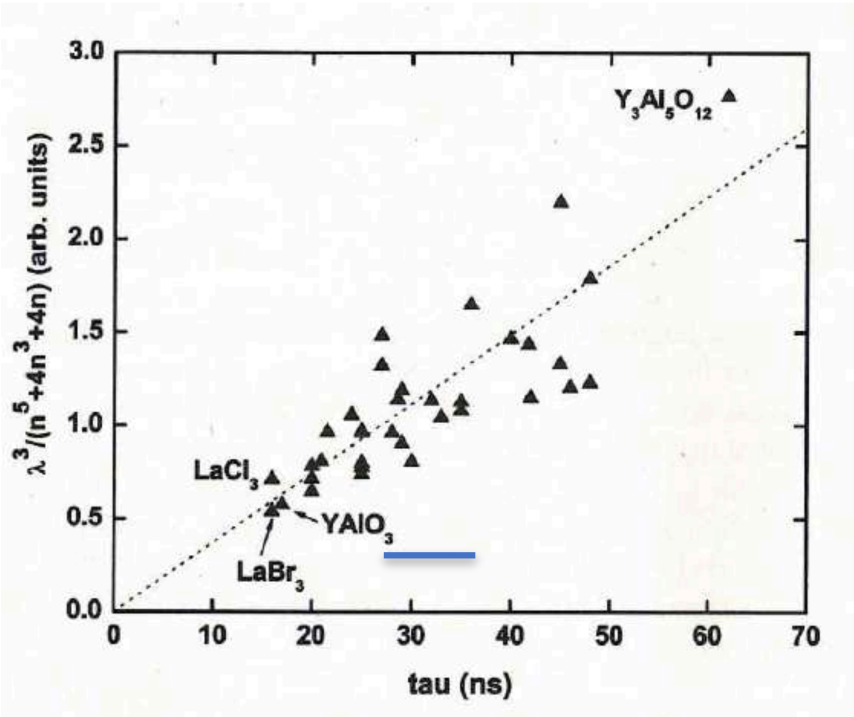


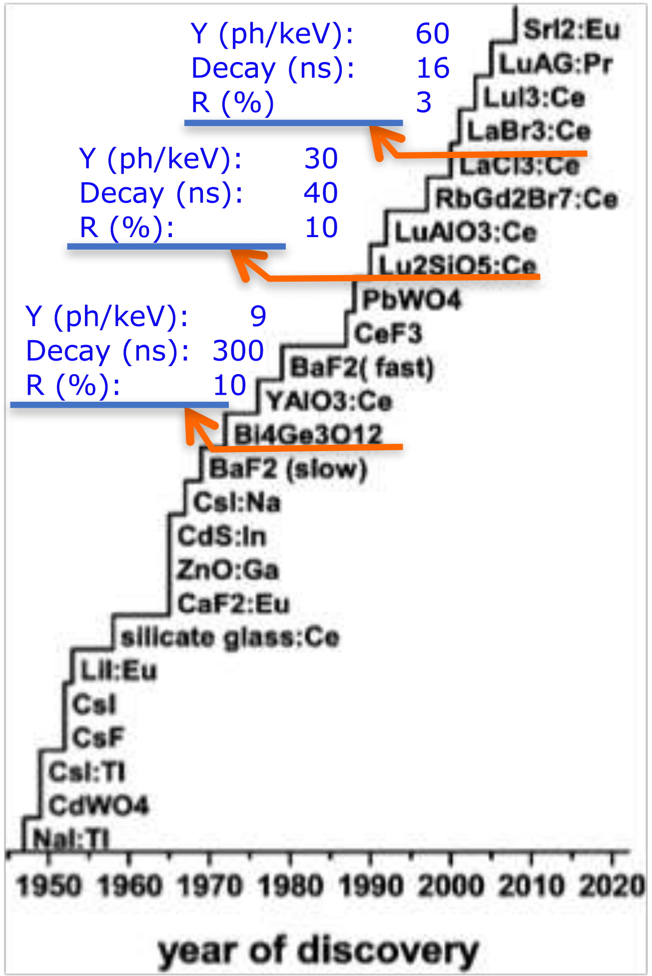
Common Inorganic Crystals

Scintillator	Wave length (nm)	Decay constant (ns)	Refraction index	Density (g/cm³)	Light yield	Attenuation coefficient (cm⁻¹)
NaI (Tl)	410	230	1.85	3.67	100	0.34
CsI (Na)	420	630	1.84	4.51	85	
CsI (Tl)	565	1000	1.80	4.51	45	
LiI (Eu)	470-485	1400	1.96	4.08	35	
CaF_4	435	900	1.44	3.19	50	
BGO	480	300	2.15	7.13	15	0.92
GSO	410	60	1.9	6.71	16	0.62
BaF_2	225/310	0.6/620	1.49	4.89	4/20	0.44
$CdWO_4$	540	5000	2.2	7.9	40	
LSO	420	**40**	1.82	7.4	75	0.87
LYSO	428	41	1.81	7.1	80	
YSO	420	70	1.80	4.54	118 ?	

How to improve the design?

- Scintillators
- Photon Detectors
- Front-End Electronics
- System Design & Integration

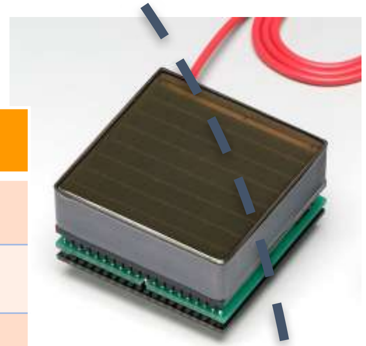




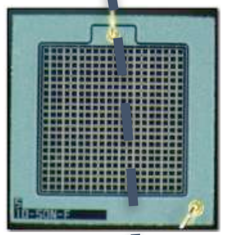
Dorenbos et col., IEEE TNS, 57, 2010 pp1162-1167

How to improve design?

- Scintillators
- **Photon Detectors**
- Front-End Electronics
- System Design & Integration



Detector	PMT	APD	SiPM	UFSD
Gain	10^5	50-1000	~10^6	5-15
Rise Time (ns)	~1	~5	~1	~0.1
QE @ 420 nm (%)	~25	~70	~25-75 (PDE)	~75
Bias (V)	>1000	300-1000	30-80	100
Temperature sensitivity (%K)	<1	~3	1-8	Negligible
Magnetic field sensitivity	Yes	No	No	No



How to Improve the Design ?

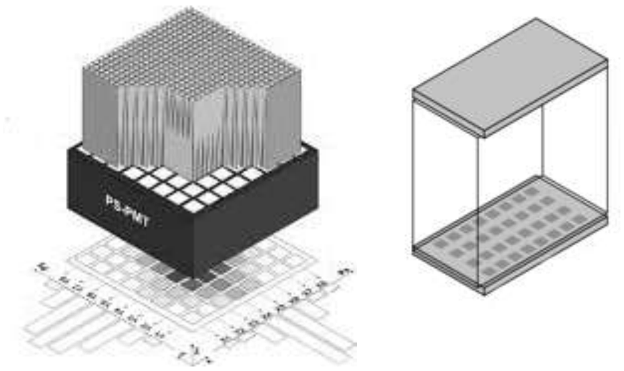
- Scintillators
- Photon Detectors
- Front-End Electronics
- System Design & Integration

- "Catch the first de-excitation photon"
  - Speed
  - Low Noise
  - Low (double) threshold
  - Low power consumption

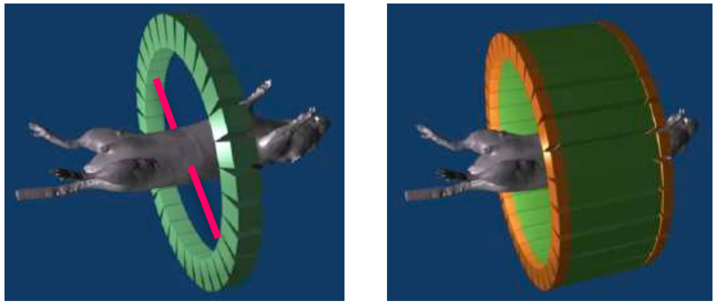

Courtesy of Christophe De La Taille, Omega

How to improve the design?

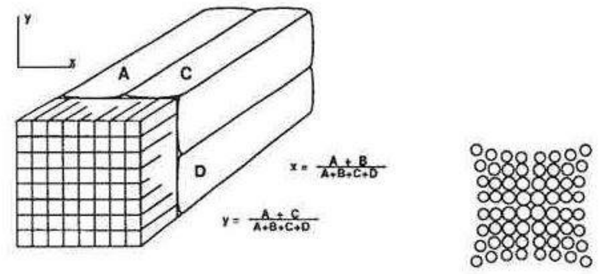
- Scintillators
- Photon Detectors
- Front-End Electronics
- System Design & Integration



- Segmented / Continuous crystal
- Radial/ axial orientation
- Block structure / 1:1 coupling



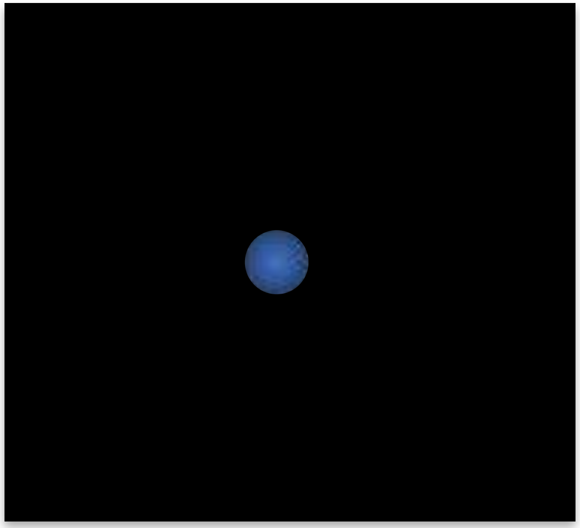
System Performances
- Spatial & timing resolutions
- Count rate capability
- Overall sensitivity
Cost/compactness/scalability



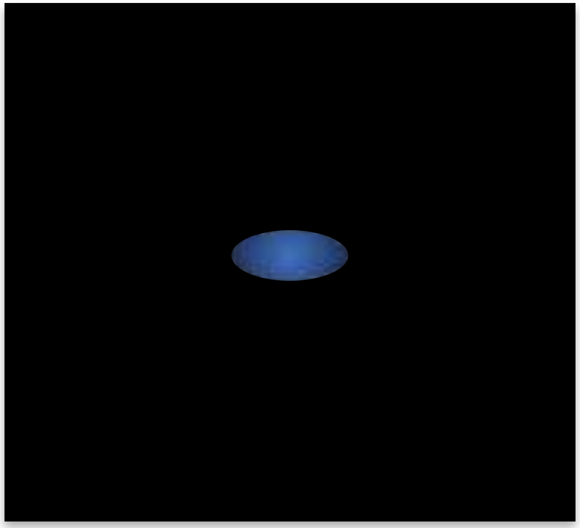
New Trends (DOI Detectors)

- Images of a point source displaced 10 cm from the center with 3mm × 3mm × 30mm crystals

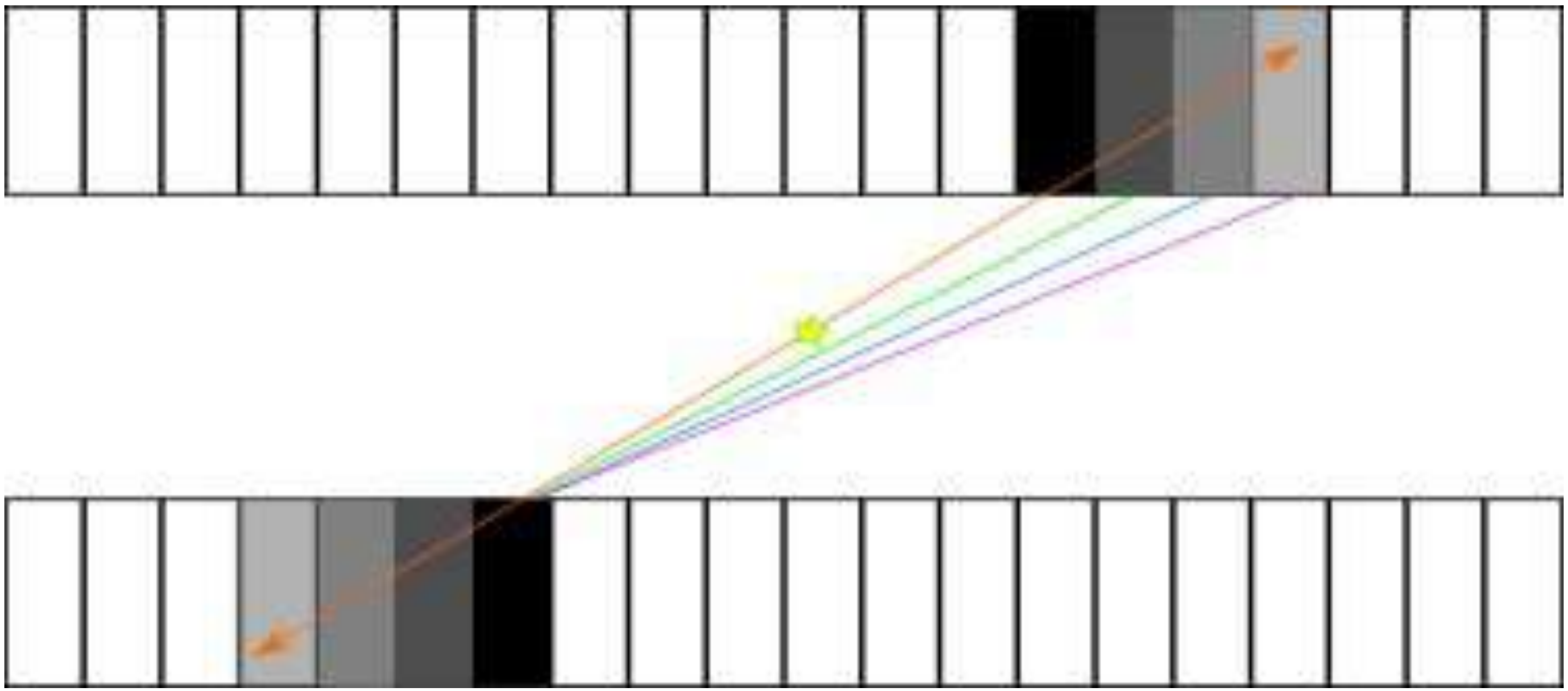
5mm DOI resolution



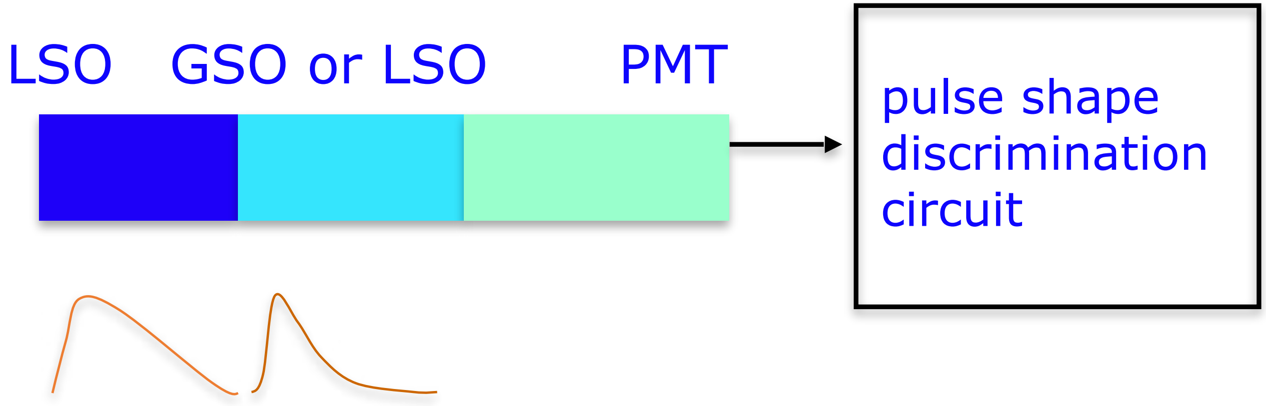
no DOI information



Depth of interaction (DOI)



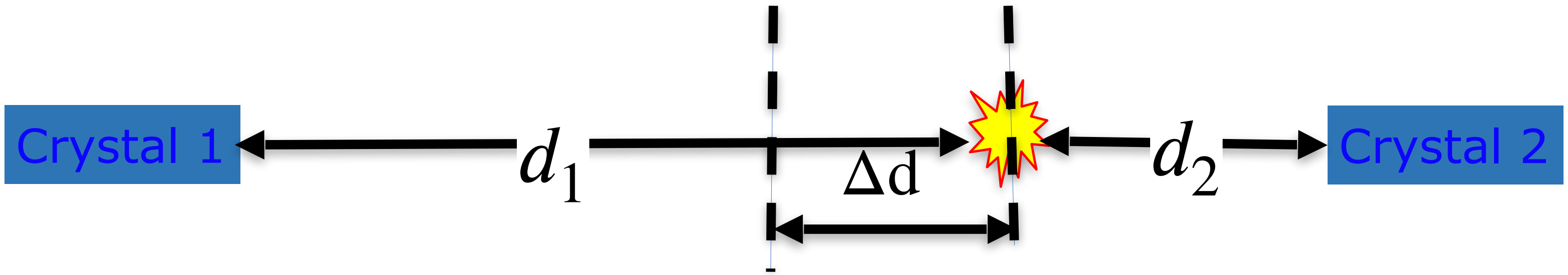
New Trends (DOI Detector)



decay constant: depends on the impurity in crystal

Phoswich detector (by CTI System)

Time of flight (TOF) concept



$$d_1 = d + \Delta d \qquad d_2 = d - \Delta d$$

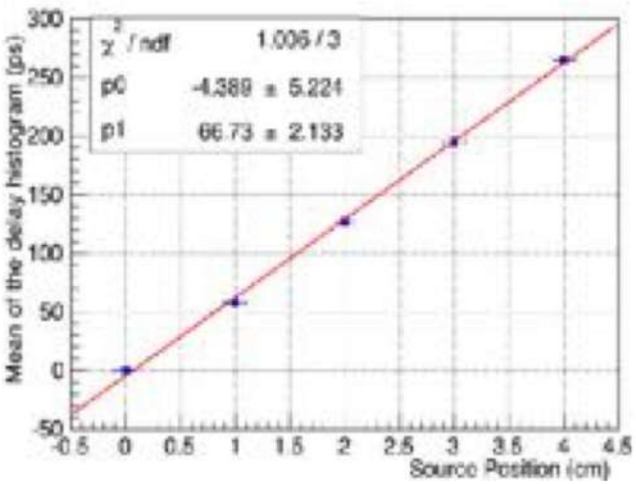
$$t_1 = \frac{d + \Delta d}{c} \qquad t_2 = \frac{d - \Delta d}{c}$$

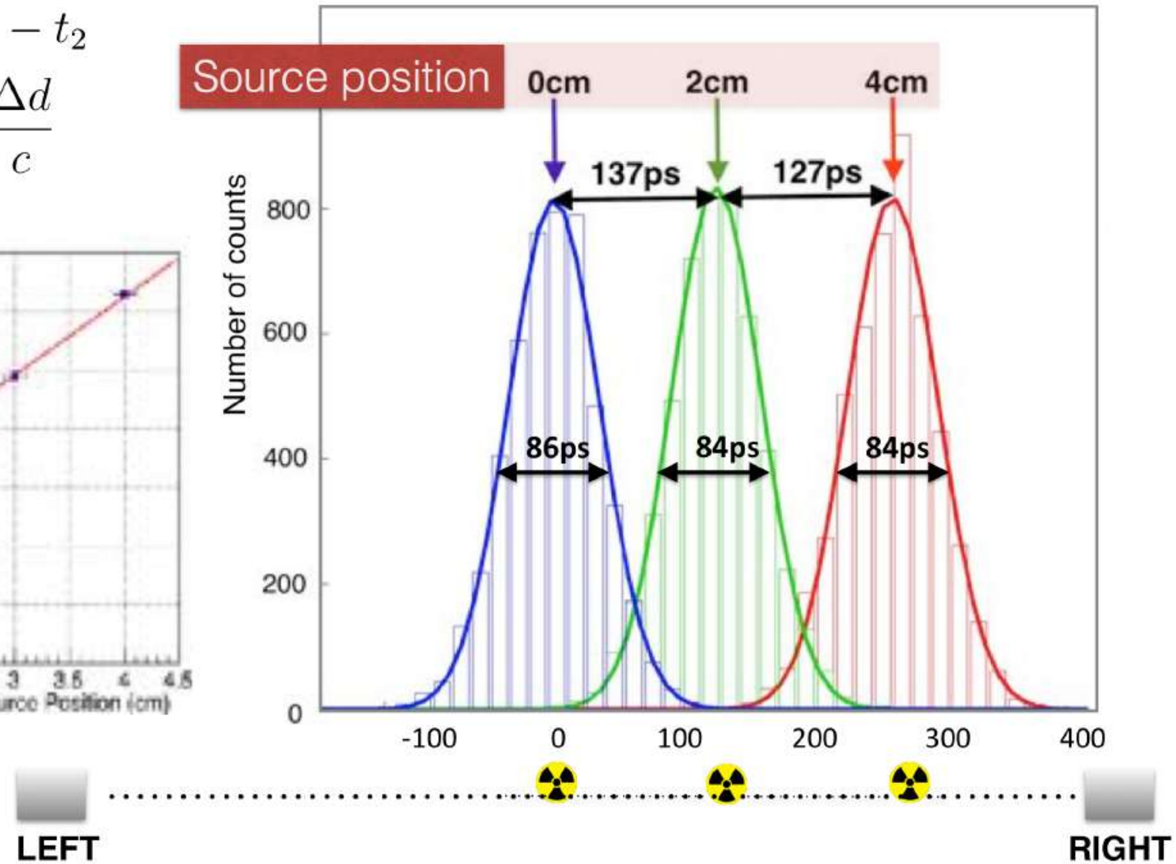
$$\Delta t = t_1 - t_2 = x = 2\frac{\Delta d}{c}$$

Coincidence time resolution

2x2x3 mm³

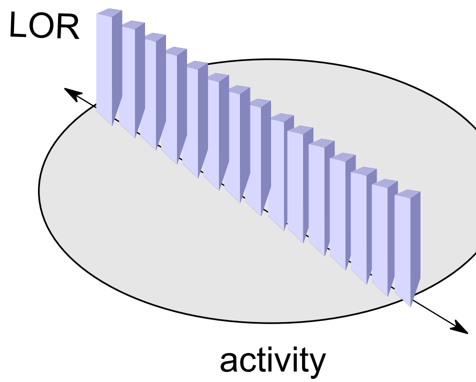
$$\Delta t = t_1 - t_2$$
$$= 2\frac{\Delta d}{c}$$





TOF gain

- Reduction of noise from other elements along the **line of response**.



Time-of-flight PET

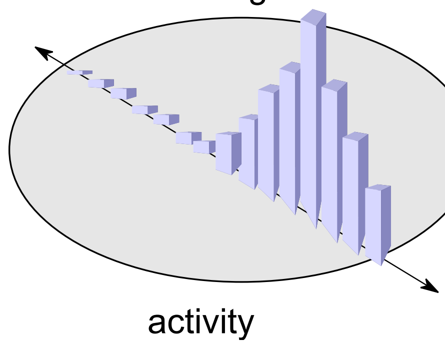
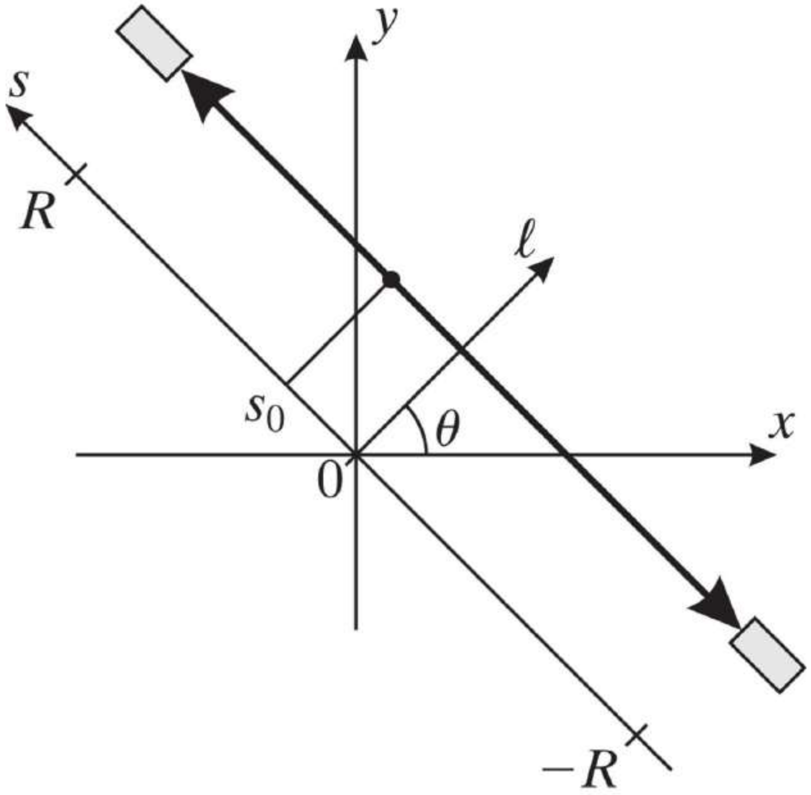


IMAGE RECONSTRUCTION

Imaging Equation



$$N^{+}(s_0) = N_0 \exp\left\{-\int_{s_0}^{R} \mu(x(s', y(s'); E)ds'\right\}$$

$$N^{-}(s_0) = N_0 \exp\left\{-\int_{-R}^{s_0} \mu(x(s', y(s'); E)ds'\right\}$$

$$N_c(s_0) = N_0 \exp\left\{-\int_{s_0}^{R} \mu(x(s', y(s'); E)ds'\right\}$$

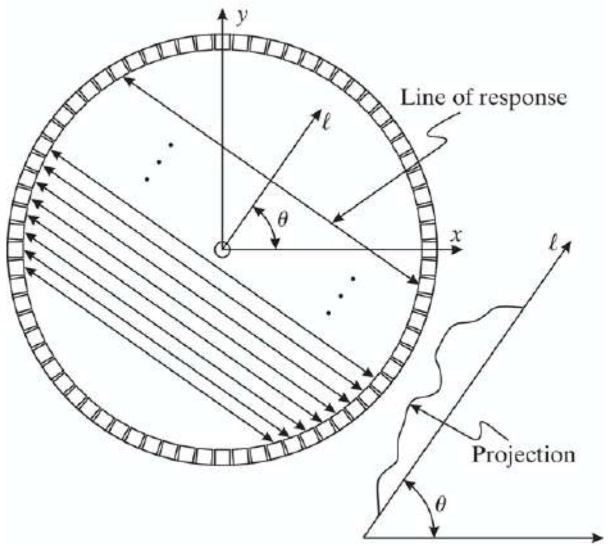
$$\times \exp\left\{-\int_{-R}^{s_0} \mu(x(s', y(s'); E)ds'\right\}$$

$$= N_0 \exp\left\{-\int_{-R}^{R} \mu(x(s', y(s'); E)ds'\right\}$$

Photon fluence

- The photon fluence rate at a point (x_d, y_d) on either detector due to this differential source $dA = f(x,y,z)dx\,dy\,dz$ at origin is

$$d\phi(x_d, y_d) = \frac{dA}{4\pi R^2}\exp\left\{-\int_{-R}^{R}\mu(s;E)ds\right\}$$



- $dA(s) = f(x(s), y(s), z)\,\tilde{A}_b ds$ effective activity off the central axis
- The total fluence

$$\phi(x_d, y_d) = \tilde{A}_b\int_{-R}^{R}\frac{f(x(s),y(s))}{4\pi(R+|s|)^2}\exp\left\{-\int_{-R}^{R}\mu(s';E)ds'\right\}ds$$

- Expected number of coincidence for LOR

$$\bar{n}(\ell,\theta) = \epsilon T A_b\tilde{A}_b\int_{-R}^{R}\frac{f(x(s),y(s))}{4\pi(R+|s|)^2}\exp\left\{-\int_{-R}^{R}\mu(s';E)ds'\right\}ds$$

Image reconstruction

$$\bar{n}(\ell,\theta) \approx K \int_{-R}^{R} f(x(s), y(s))ds$$

$$K = \frac{\epsilon T A_b \tilde{A}_b}{4\pi R^2} \exp\left\{ -\int_{-R}^{R} \mu(x(s'), y(s'); E)ds' \right\}$$

$$g(\ell,\theta) = \frac{\bar{n}(\ell,\theta)}{K}$$

- PET scanner measures the Radon transform of the radiotracer.

$$g(\ell,\theta) \approx \int_{-R}^{R} f(x(s), y(s))ds$$

Attenuation Correction

- Corrected sinogram

$$\phi_c(\ell,\theta) = \frac{\phi(\ell,\theta)}{K\exp\left\{-\int_{-R}^{R}\mu(x(s),y(s);E)ds\right\}}$$

- $\mu(x,y)$ from CT (transmission PET).

- One can apply filtered backprojection algorithm to reconstruct $A(x,y)$ from the corrected sinogram.

Summary

- Emission computed tomography
  - Single photon emission computed tomography (SPECT)
  - Positron emission tomography (PET)
- SPECT to planar scintigraphy is analogous to CT to projection radiography
  - With rotating Anger scintillation camera
  - No closed-form solution for attenuation correction
- PET based on coincidence detection or paired gamma rays
  - No projection analog
  - Dedicated PET scanner
  - Has closed-form solution for attenuation correction
- Imaging principle of PET:
  - Coincidence detection: detect two photons reaching two opposite detectors simultaneously (within a short time window)
  - Detected signal is the product of two terms, depending on the radioactivity A and attenuation μ separately
  - Can reconstruct radioactivity more accurately if μ can be measured simultaneously

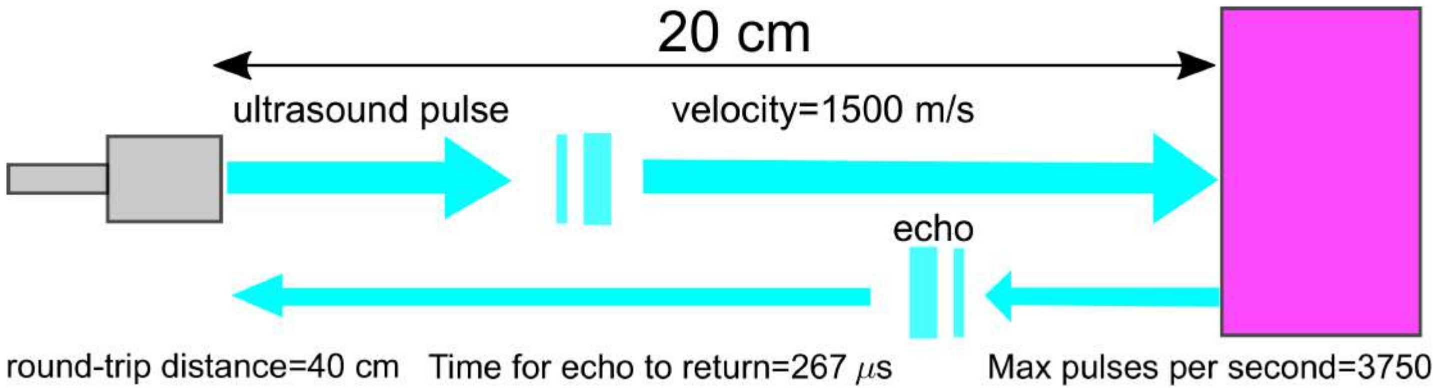
THE PHYSICS OF ULTRASOUND

Introduction to Ultrasound Imaging

- What is ultrasound?
- Ultrasound - sound have a frequency > 20,000 Hz
- This is above the audible range for humans (hence "ultra")
- Medical ultrasound imaging: > 2 MHz
- Propagation: affected by mechanical properties

Introduction to Ultrasound Imaging

- Ultrasound image formation:



- Transducer produces ultrasound (pressure) wave
- The returned echoes are processed - depth information
- Image is built up by sending waves in different directions

What is acoustic wave

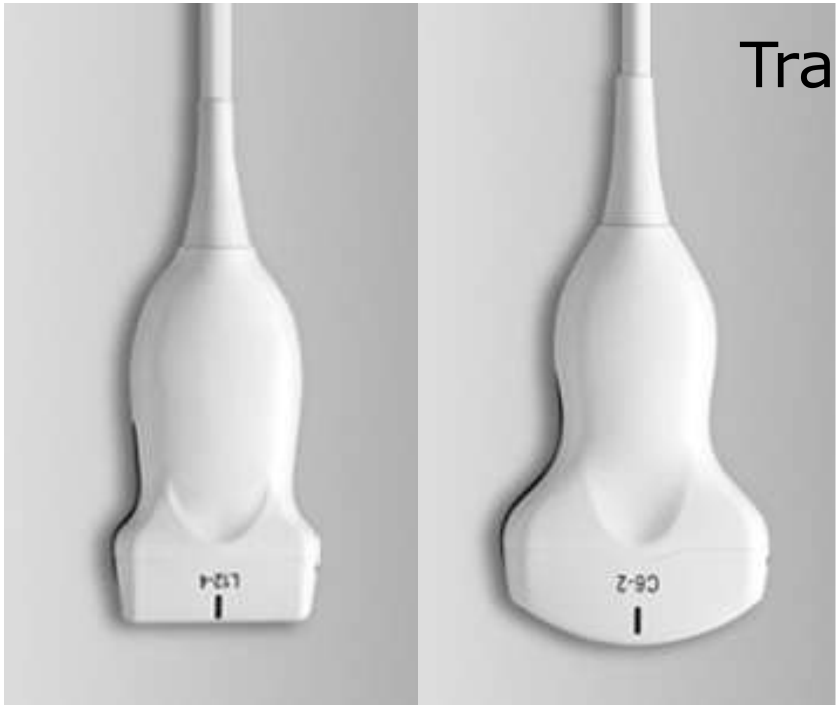
- Pressure waves that propagate through matter via compression and expansion of the material
  - Generated by compressing and releasing a small volume of tissue
- Longitudinal wave
  - Particles in the medium move back and force in the same direction that the wave is traveling
- Shear Wave
  - Particles move at right angles to the direction of the wave
  - Not used for medical ultrasound imaging

Ultrasound image formation - Basic principle:

- Insonify tissue with sound wave.
- The sound wave is reflected by interfaces in the tissue.
  - Echoes are produced
- The time it takes the echoes to return is measured.
- The depth of the interfaces is computed and displayed.

Ultrasound

System



Transducers

Facts:
1-20MHz
Tomographic view
Mechanical vibrations, no ionizing radiation

What is Ultrasound?

- General medical ultrasound: 2 ~ 18 MHz
  - 7 ~ 18 MHz: superficial tissue
    - Muscle, tendon, neonatal brain
  - 1 ~ 6 MHz: deep tissue
    - Liver, kidney...

Introduction to Ultrasound Imaging

- Speed of sound in a medium depends on the medium property

$$c = \sqrt{\frac{1}{\kappa\rho}}$$

- *Compressibility* κ and *density* ρ of tissue; $B = 1/\kappa$ is called the *bulk modulus* of a material.
- Speed of sound higher in "stiff" tissues.
  - In tissue: ~1540 m/s;
  - Air: ~ 330 m/s
  - water: 1480 m/s
  - bone: 4080 m/s

Introduction to Ultrasound Imaging

- Acoustic pressure $p = Zv$
  - v: particle displacement velocity, different from sound speed c.
  - Analogy: p (voltage), v (current), Z (impedance)

- Exactly what material properties are conveyed?
  - Answer: Interfaces in acoustic impedance Z:

$$Z = \rho c$$

- units: $\mathrm{kg/m^2 s}$ or rails (after Rayleigh); c: speed of sound in tissue

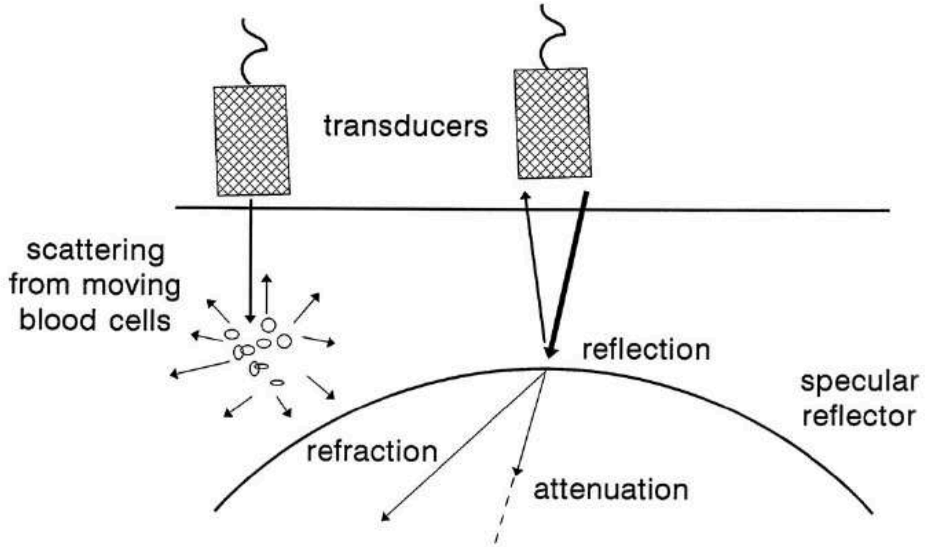
- A measure of mechanical tissue properties.
- Different tissues may have different $Z = \rho c = \sqrt{\rho/\kappa}$.

Acoustic properties vary between different biological tissues

Material	ρ Density (kg m^{-3})	c Speed (m s^{-1})	Z Impedance (Mrayl)
Perspex	1180	2680	3.16
Air	1.2	330	0.004
Bone	1912	4080	7.8
Water	1000	1480	1.48
Lung	400	650	0.26
Fat	952	1459	1.38
Soft Tissue	1060	1540	1.63

Ultrasound can undergo a range of interactions in soft tissue

- Reflection
- Scattering
- Refraction
- Absorption



Lawrence (2007) *Crit Care Med*

Energy attenuation due to absorption and scattering

- The pressure of an acoustic wave decreases as the wave propagates due to
  - Absorption
    - The wave energy is converted to thermal energy
    - Two forms
    - Classical: due to frictions between particles as the wave propagates
    - Relaxation: due to particle motion to return (relax) to original position after displacement by the wave pressure
  - Scattering
    - When the sound wave hits an object much larger than its wavelength, reflection occurs.
    - When the object size <= wavelength, scattering occur (reflection in all directions)

Absorption occurs when mechanical energy of the ultrasound beam is converted to heat energy

- Absorption in tissues is strong, accounts for 80 – 90 % of all energy loss by an ultrasound beam
- Depends on:
  - Frequency
  - Viscosity of the medium
  - Relaxation time of the medium
- Relaxation:
  - At low frequencies the particles move easily with the passing pressure wave and return to equilibrium before the next disturbance so all energy is transmitted
  - At higher frequencies, particles are unable to keep up so do not pass all energy

Attenuation: the loss of intensity

- Attenuation includes both scattering and absorption

$$I = I_0 e^{-afl}$$

where I is intensity, $a \sim 0.5\ \mathrm{dB\,cm^{-1}\,MHz^{-1}}$ in soft tissue, f is frequency (MHz) and l is thickness of tissue (cm).

- Analogy to X-ray half-value thickness (HVT):

Material	HVT (cm) @ 2MHz	HVT (cm) @ 5MHz
Air	0.06	0.01
Bone	0.1	0.04
Liver	1.5	0.5
Blood	8.5	3.0
Water	344	54

Decibel scale

- To avoid the exponential in attenuation calculations, the decibel scale is used

$$\frac{I}{I_0} = e^{-afl}$$

I/I_0	dB
1,000,000	60
100	20
10	10
2	3
1	0
0.01	-20

Echo pressure amplitudes vary by a factor of 10^5 or greater, so a logarithmic scale helps:

$$\text{Intensity ratio (dB)} = 10\log_{10}\left(\frac{I}{I_0}\right)$$

$$\text{Amplitude ratio (dB)} = 20\log_{10}\left(\frac{A}{A_0}\right)$$

(Factor of 2, since intensity is proportional to square of amplitude)

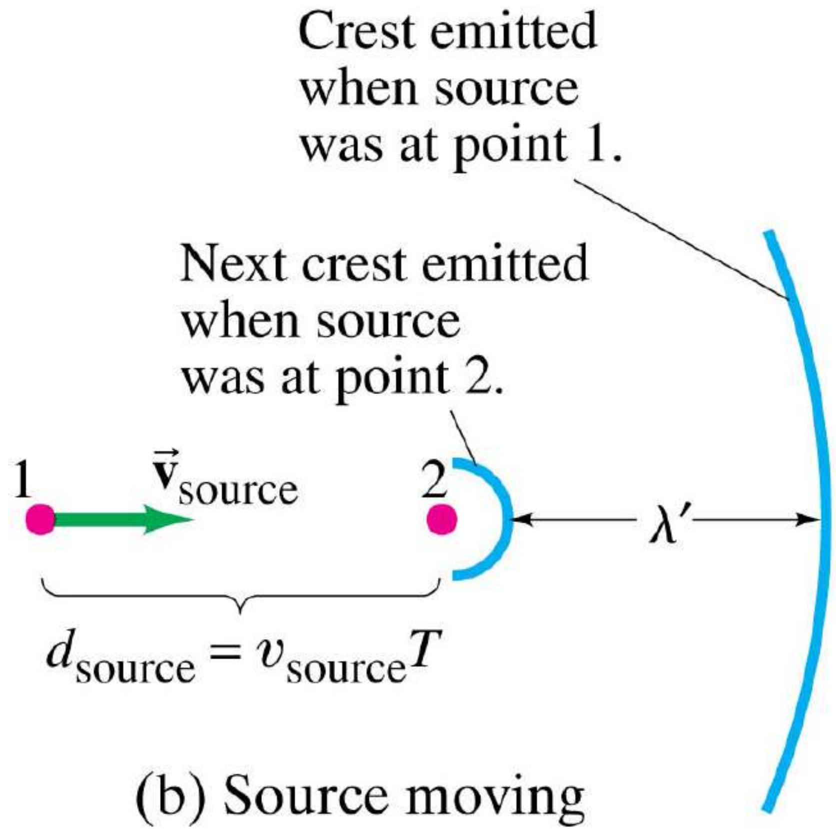
Example: Working with decibels

- Consider two consecutive regions of tissue with the same acoustic impedance but different attenuation coefficients:
- The ultrasound frequency is 5 MHz.

Ultrasound in -> 10 Wcm^{-2}	$a = 0.5\ \mathrm{dBcm^{-1}MHz^{-1}}$ $l = 6$ cm	$a = 0.8\ \mathrm{dBcm^{-1}MHz^{-1}}$ $l = 5$ cm	Ultrasound out?

DOPPLER EFFECT

Doppler Effect



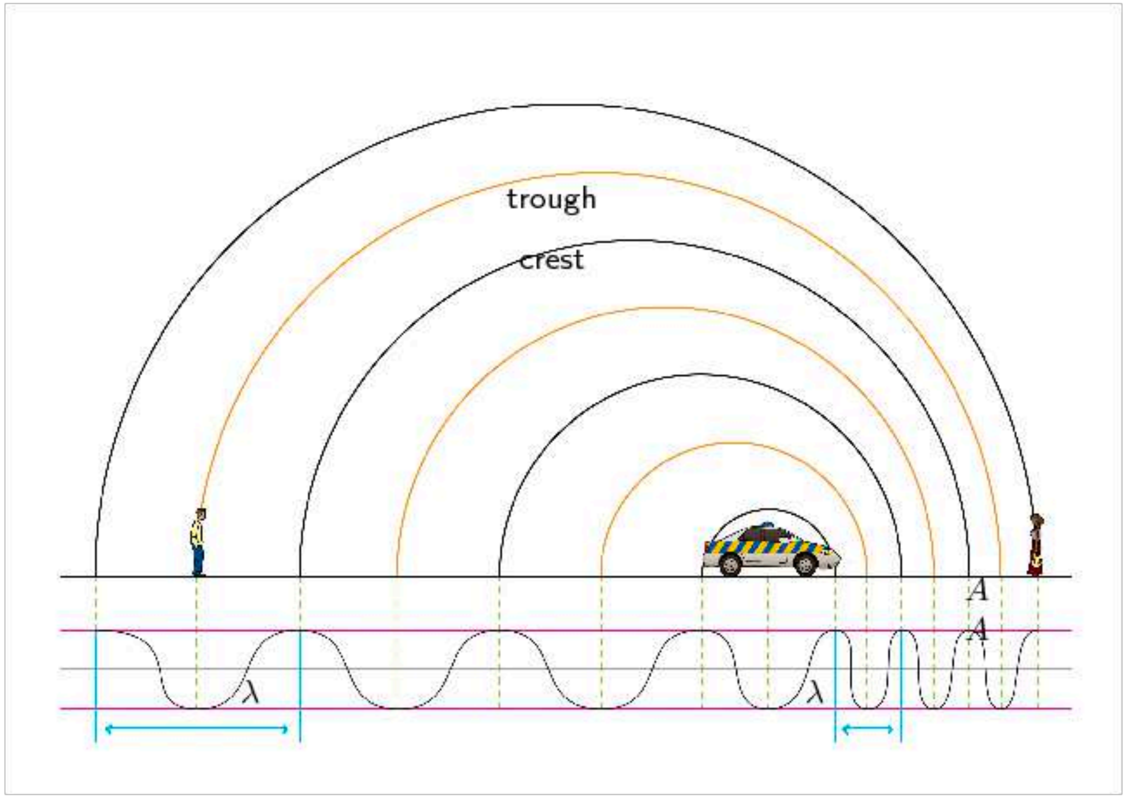
(b) Source moving

If we can figure out what the change in the wavelength is, we also know the change in the frequency.

Doppler Effect

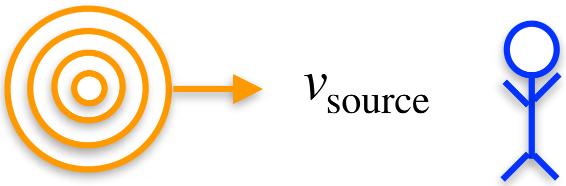
The change in the wavelength is given by:

$$
\begin{aligned}
\lambda' &= d - d_{\text{source}} \\
&= \lambda - v_{\text{source}} T \\
&= \lambda - v_{\text{source}} \frac{\lambda}{v_{\text{snd}}} \\
&= \lambda \left(1 - \frac{v_{\text{source}}}{v_{\text{snd}}} \right).
\end{aligned}
$$



Doppler Effect

And the change in the frequency:



$$f' = \frac{f}{(1 - \frac{v}{c})} = \frac{c}{c - v} f$$

source moving forward stationary observer

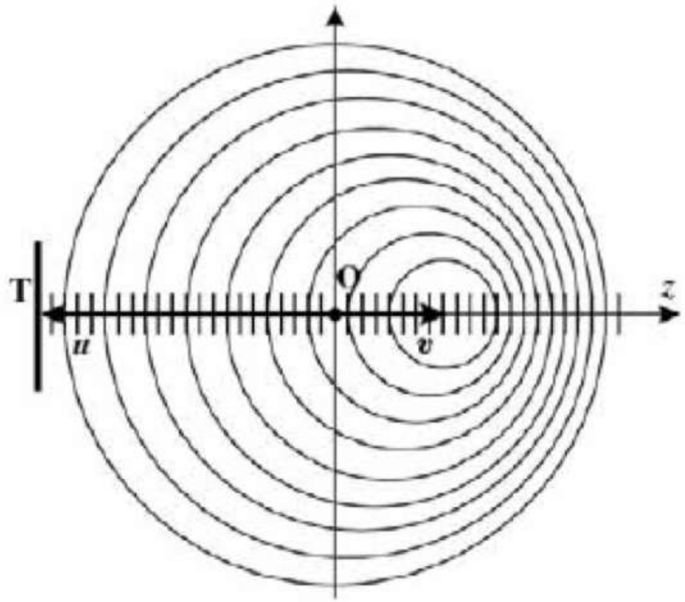
If the source is moving away from the observer:

$$f' = \frac{f}{(1 + \frac{v}{c})} = \frac{c}{c + v} f$$

source moving away from stationary observer

Doppler effects: moving source

- Doppler effect: change in frequency of sound due to the relative motion of the source and receiver
- Case 1: moving source (scatterer), stationary receiver (transducer)
  - Source moving away, $\lambda \nearrow, f \searrow$ wavelength longer, lower freq
  - Source moving closer, $\lambda \searrow, f \nearrow$ wavelength shorter, higher freq.



Doppler effect: moving receiver

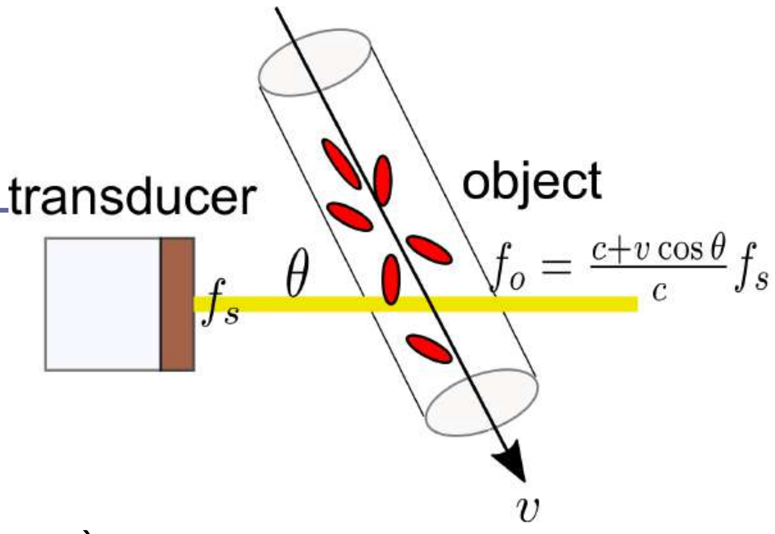
- Case 2: stationary source (transducer), moving receiver (target)
  - Transducer transmitting a wave at freq f_S, wavelength $= c/f_S$
  - Object is a moving receiver with speed v, with angle θ.
  - Target moving away ($\theta \leq = 0$, sound moves slower
  - Target moving closer ($\theta \geq 0$), sound moves faster

  - $f_o = \dfrac{c + v\cos\theta}{c} f_s$
  - $f_D = f_o - f_s = \dfrac{v\cos\theta}{c} f_s$

$\theta < 0$: source moving away from receiver, $f_D < 0$

$\theta \geq 0$: source moving towards receiver, $f_D > 0$

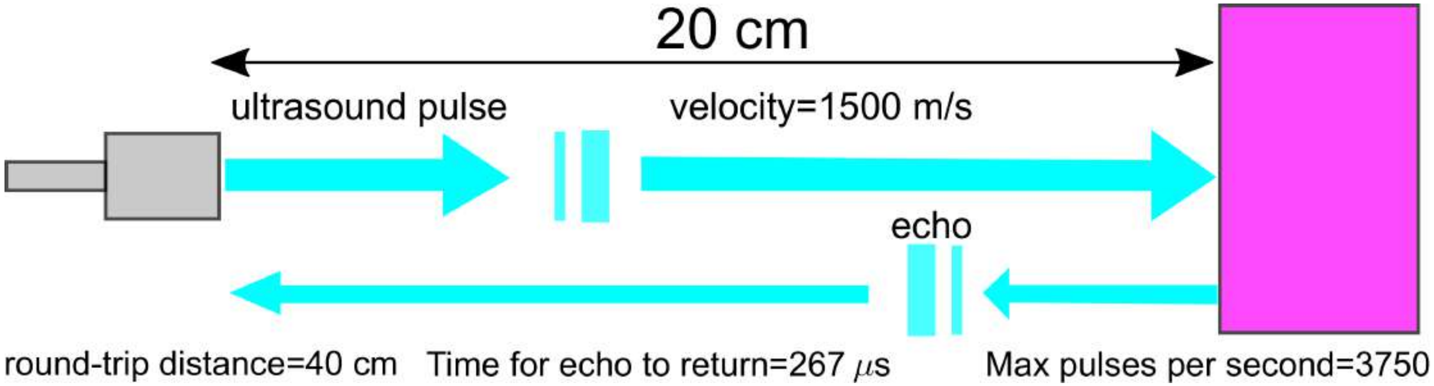
Doppler effect for transducer



- Transducer:
  - Transmit wave at freq f_s to object
  - Object moves with velocity v at angle θ
  - Object receives a wave with freq $f_o = \dfrac{c + v\cos\theta}{c} f_s$
  - The object (scatterer) reflects this wave (acting as a moving source)
  - Transducer receives this wave with freq
  - Doppler freq $f_T = \dfrac{c}{c - v\cos\theta} f_o = \dfrac{c + v\cos\theta}{c - v\cos\theta} f_s$.
- $f_D = f_T - f_s = \dfrac{2v\cos\theta}{c - v\cos\theta} f_s \approx \dfrac{2v\cos\theta}{c} f_s$
- Doppler-shift velocimeter: as long as $\theta \neq 90^\circ$, can recover object speed from doppler frequency.
- Doppler imaging: display f_D in space and time.

IMAGING MODES

Ultrasound image formation

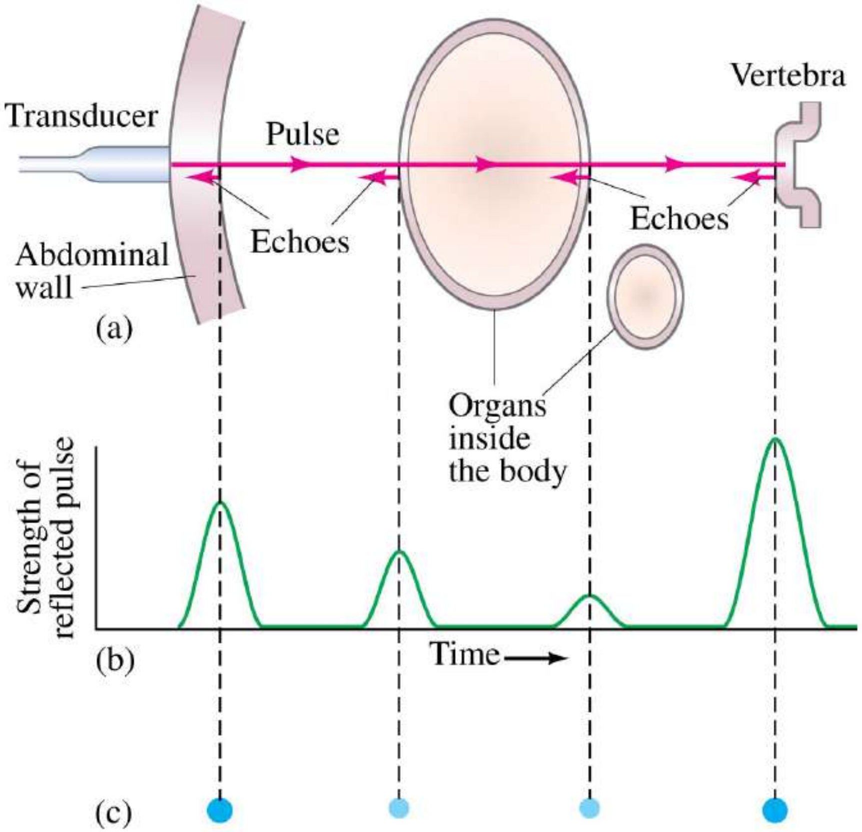


- Transducer produces ultrasound (pressure) wave.
- The returned echoes are processed - depth information.
- Image is built up by sending waves in different directions.

Ultrasonic imaging modes

- Echo Display Modes:
- A-mode (amplitude):
  - Display of processed information from the receiver versus time
  - Speed of sound equates to depth
  - (only used in ophthalmology applications now)
- B-mode (brightness):
  - Conversion of A-mode information into brightness-modulated dots.
- M-mode (motion):
  - Uses B-mode information to display the echoes from a moving organ.

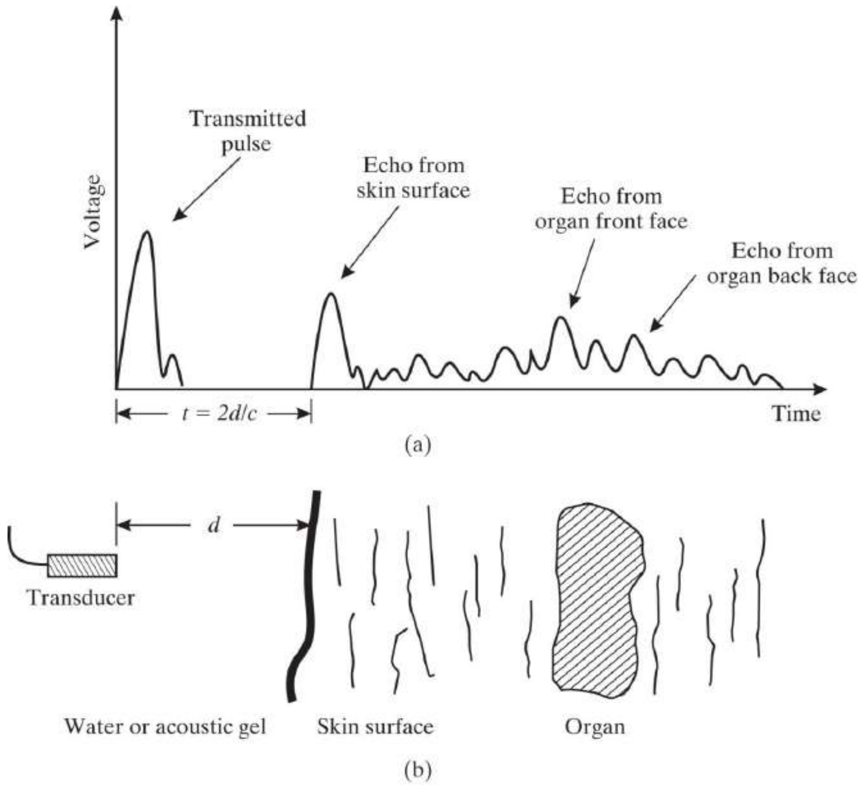
Applications: Sonar, Ultrasound, and Medical Imaging



Ultrasound is also used for medical imaging. Repeated traces are made as the transducer is moved, and a complete picture is built.

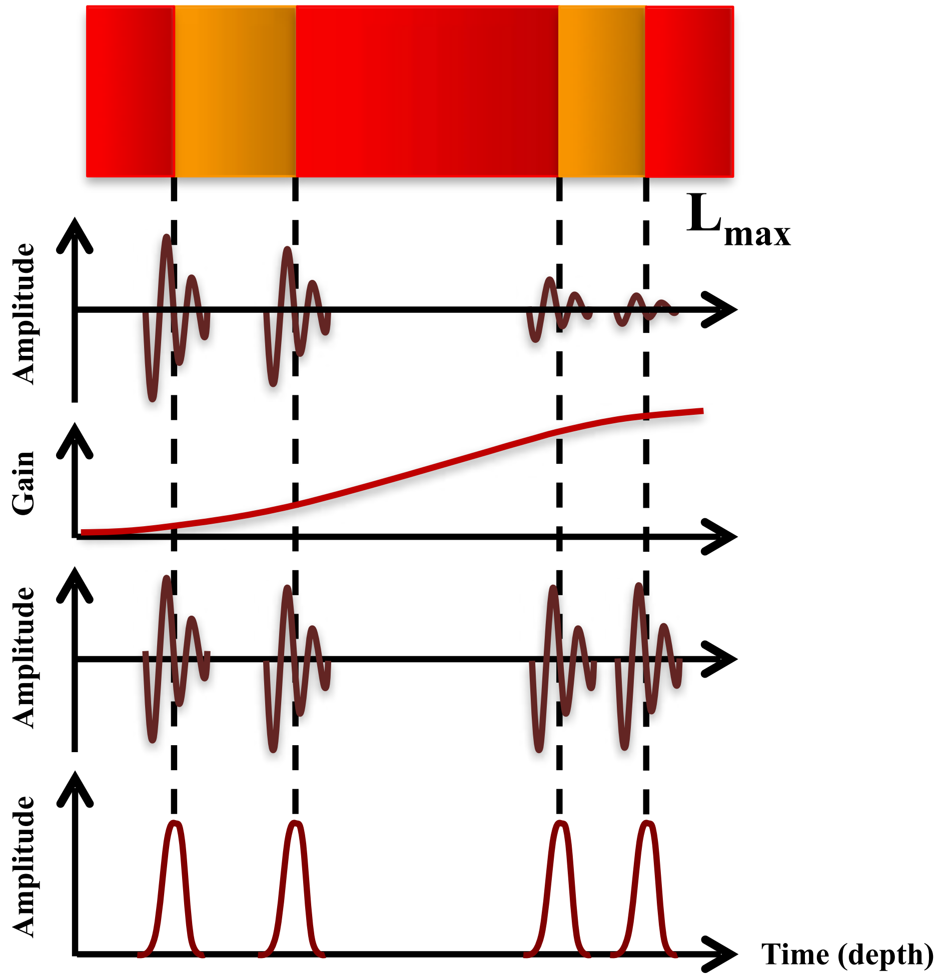
A-mode display

- Oldest, simplest type
- Display of the envelope of pulse-echoes vs. time, depth $d = ct/2$.
- Measure the reflectivity at different depth below the transducer position



The horizontal axis can be interpreted as z, with $z=t\ c/2$
Along each line we transmit a pulse and plot the reflections that come back vs time

Amplitude (A) mode ultrasound displays the ultrasound echoes along one beam, or 'A Line'

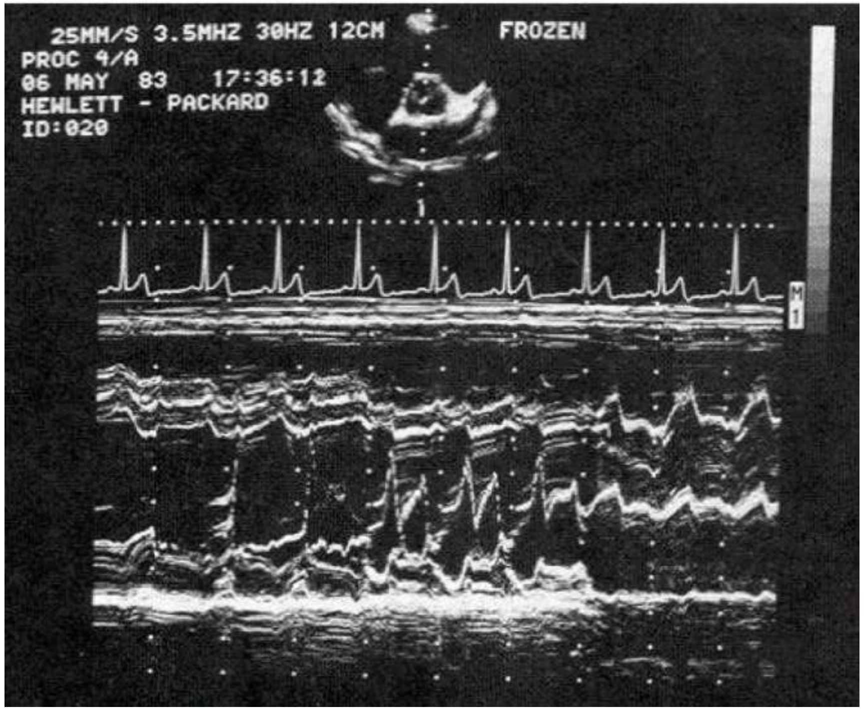


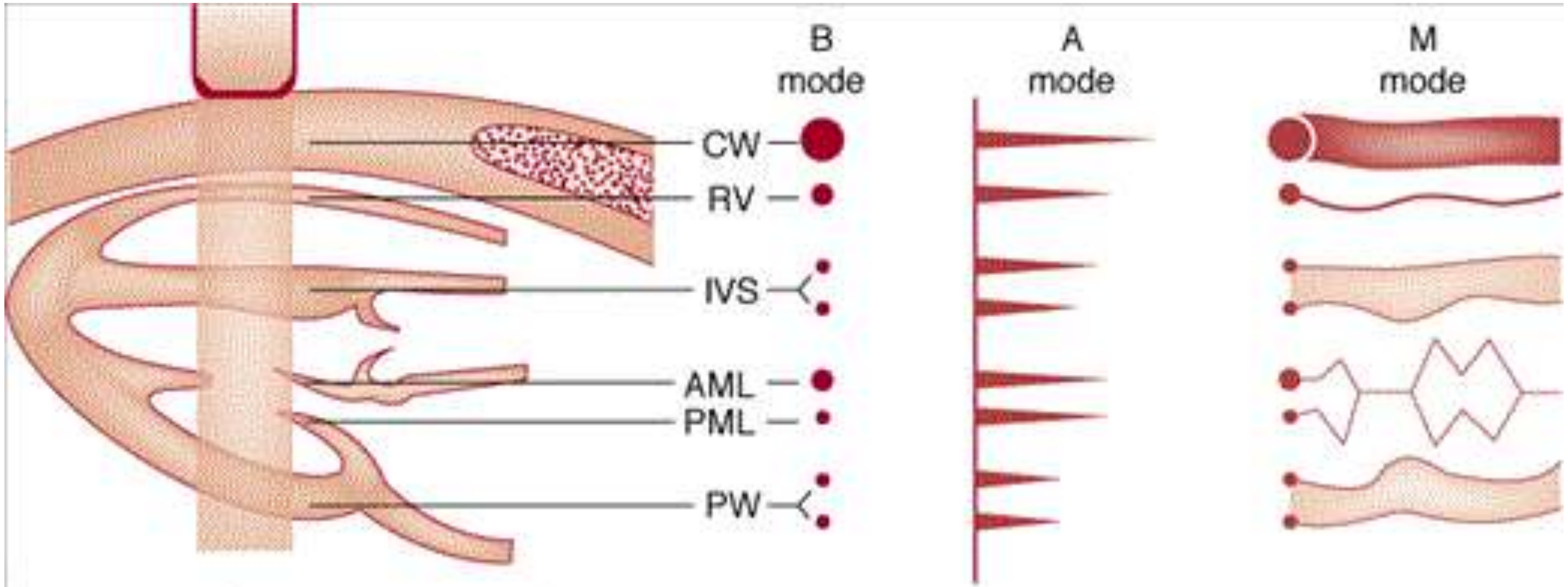
Application of A-mode

- Applications: ophthalmology (eye length, tumors), localization of brain midline, liver cirrhosis, myocardium infarction.
- Frequencies:
  - 2-5 MHz for abdominal, cardiac, brain;
  - 5-15 MHz for ophthalmology, pediatrics, peripheral blood vessels
- Used in ophthalmology to determine the relative distances between different regions of the eye and can be used to detect corneal detachment.
  - High freq is used to produce very high axial resolution.
  - Attenuation due to high freq is not a problem as the desired imaging depth is small.

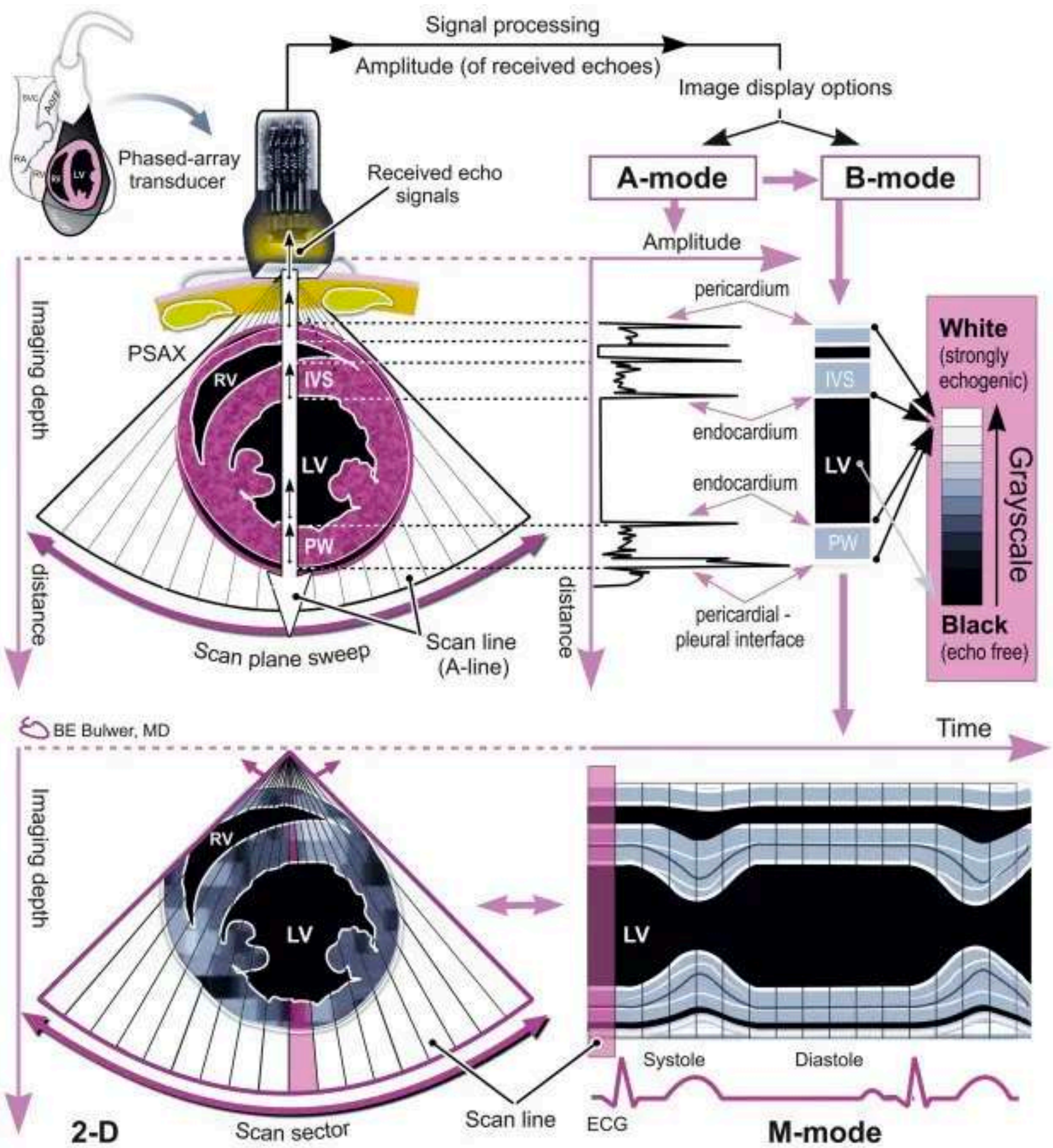
M-mode display

- Display the A-mode signal corresponding to repeated input pulses in separate column of a 2D image, for a fixed transducer position.
- Motion of an object point along the transducer axis (z) is revealed by a bright trace moving up and down across the image.
- Often used to image motion of the heart valves, in conjunction with the ECG.





Formation of A-mode, B-mode, and M-mode echocardiograms.


Signal processing
Amplitude (of received echoes)
Image display options
Phased-array transducer
Received echo signals
A-mode
B-mode
Amplitude
pericardium
PSAX
RV
IVS
LV
PW
endocardium
endocardium
pericardial - pleural interface
White
(strongly echogenic)
Grayscale
Black
(echo free)
Imaging depth
distance
distance
Scan plane sweep
Scan line (A-line)
BE Bulwer, MD
Time
Imaging depth
RV
LV
LV
Systole
Diastole
2-D
Scan sector
Scan line
ECG
M-mode

Summary

- Characterization of wave:
  - Material properties: k, ρ, c, Z
  - Wave properties: particle velocity v, pressure p, intensity I

- Reflection and refraction of wave at an interface
- Scattering of wave
- Different US scanning mode
  - A-mode (reflectivity in z for fixed (x,y) position)
  - B-mode (reflectivity in one cross section)
  - M-mode (motion trace in z for fixed (x,y))
  - 3D imaging
  - Doppler imaging

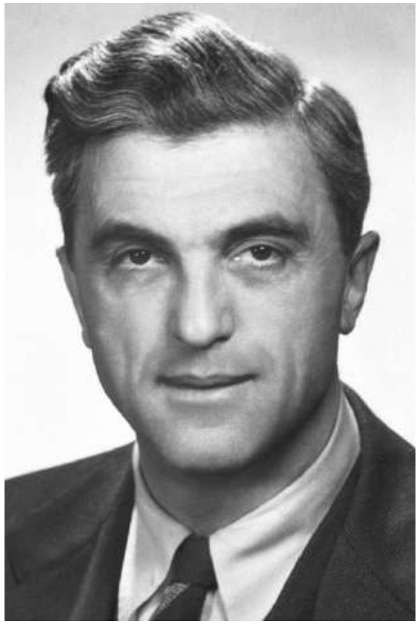
MAGNETIC RESONANCE IMAGING (MRI)

Magnetic Resonance Imaging

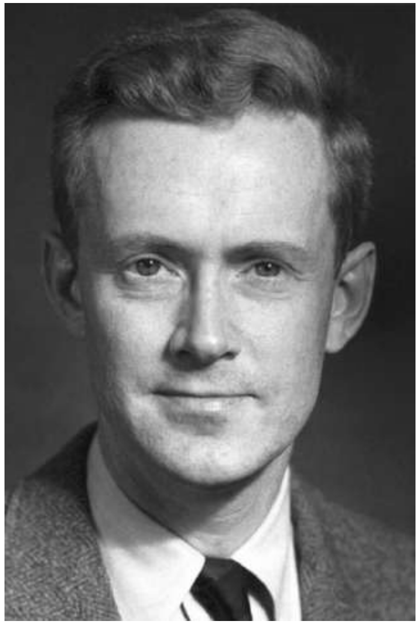
- Provide high resolution anatomic structure (as X-ray CT)
- Provide high contrast between different soft tissues (X-ray CT cannot)
- No exposure to radiation and hence safe
- More complicated instrumentation
- Takes longer to acquire a scan than CT, more susceptible to patient motion

The Discovery of NMR

- 1952 Nobel Prize for Physics



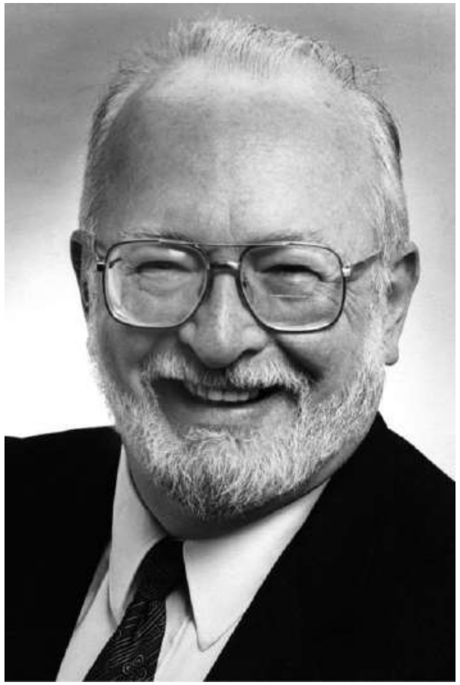
Felix Bloch



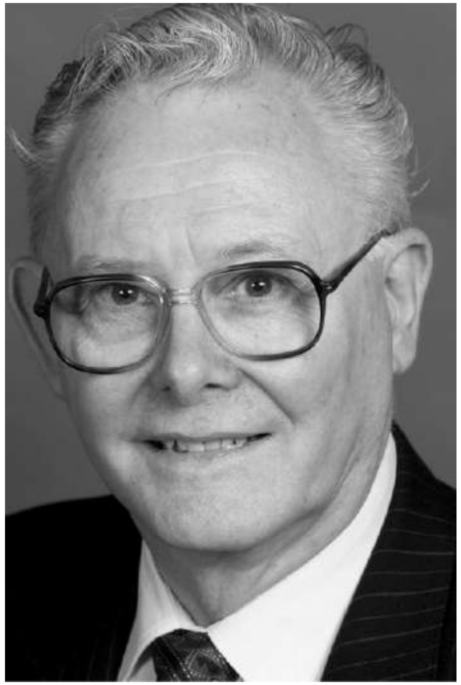
Edward Mills Purcell

The Discovery of MRI

- 2003 Nobel Prize in Physiology or Medicine



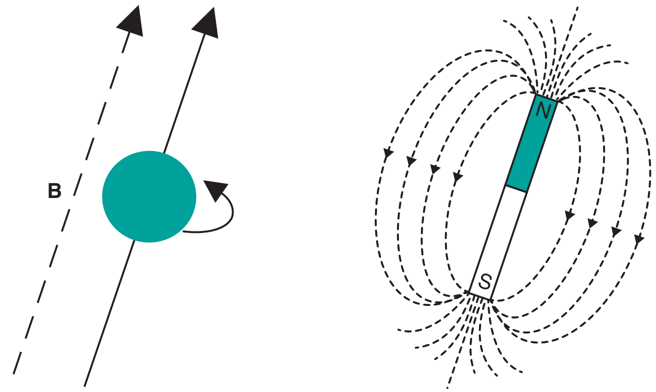
Paul C. Lauterbur



Sir Peter Mansfield

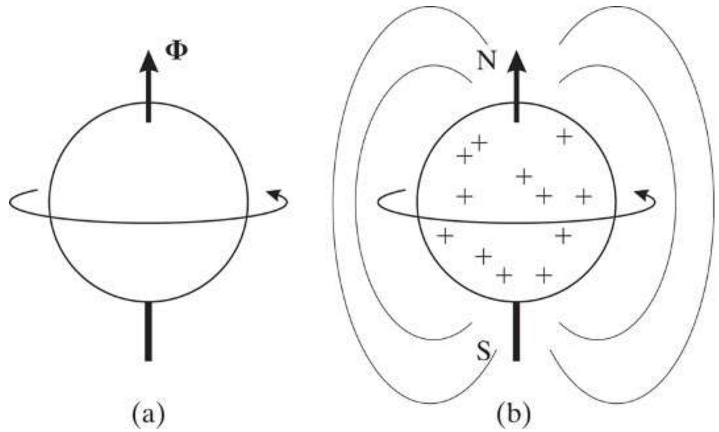
What kinds of nuclei can be used for NMR?

- Nucleus needs to have 2 properties:
  - Spin
  - Charge
- Nuclei are made of protons and neutrons
  - Both have spin ½
  - Protons have charge
- Pairs of spins tend to cancel, so only atoms with an odd number of protons or neutrons have spin
  - Good MR nuclei are ^{1}H, ^{13}C, ^{19}F, ^{23}Na, ^{31}P
  - Biological tissues are predominantly ^{12}C, ^{16}O, ^{1}H, and ^{14}N



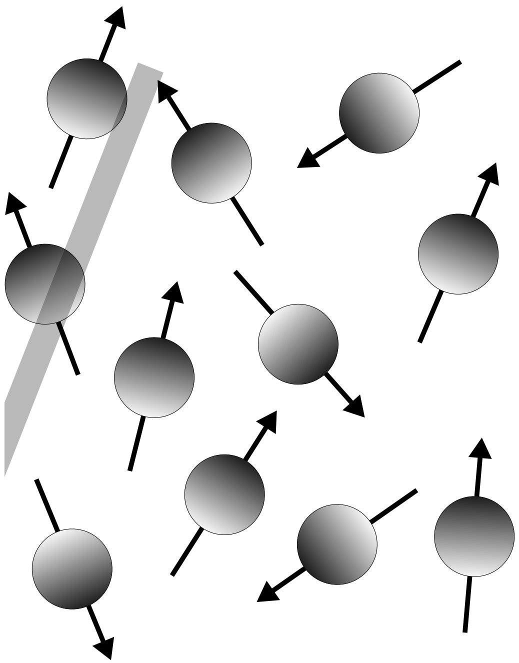
Nuclear Spin

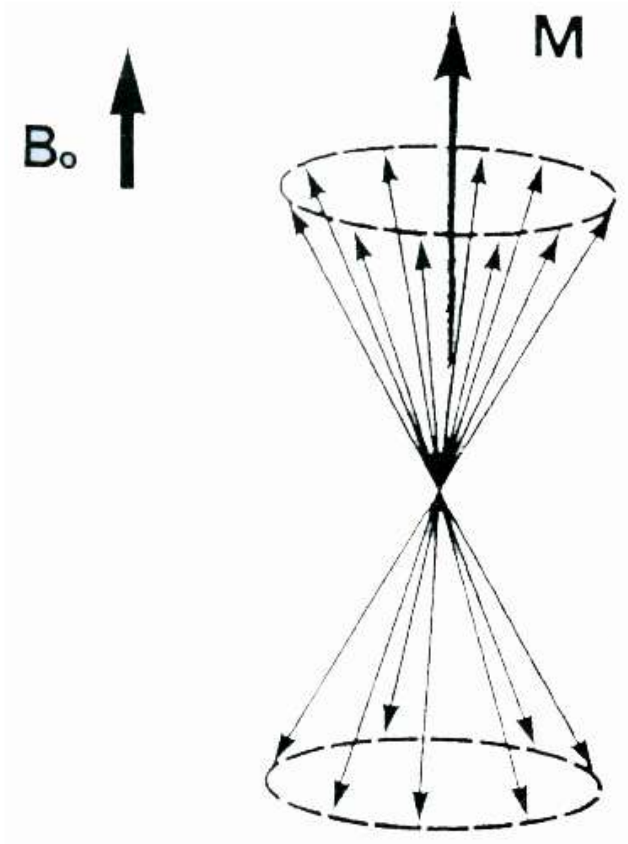
- A nucleus consists of protons and neutrons
- When the total number of protons and neutrons (=mass number A) is odd or the total number of protons is odd, a nucleus has an angular momentum (Φ) and hence spin
  - Ex. Hydrogen (^{1}H) (1 proton), ^{13}C.
- The spin of a nucleus generates a magnetic filed, which has a magnetic moment (μ).
- The spin causes the nucleus behave like a tiny magnet with a north and south pole.



Before and After Applying Magnetic Field

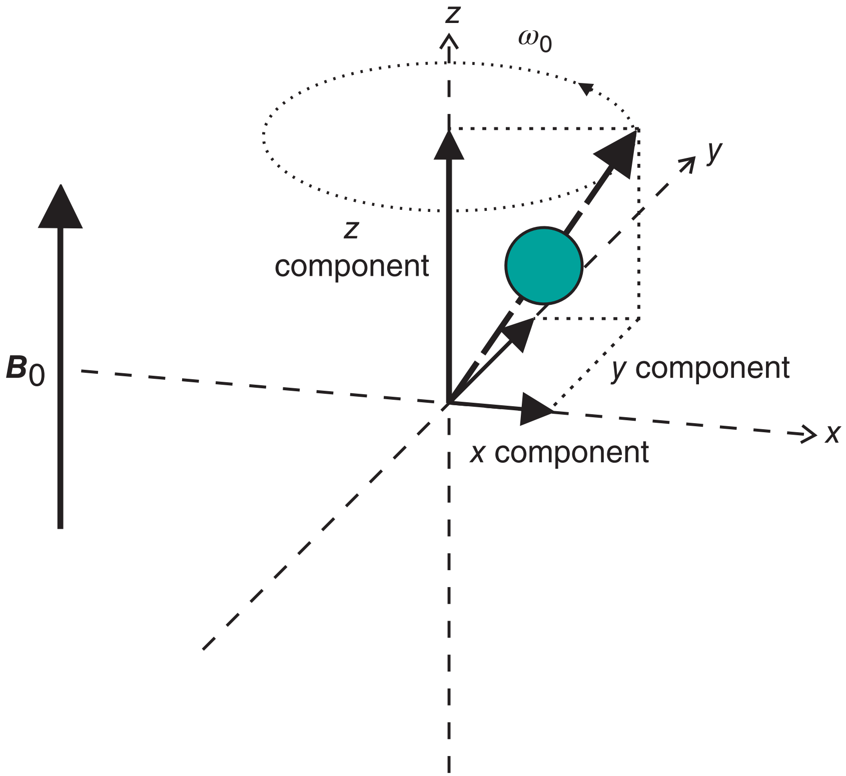
- Spins PRECESS incoherently

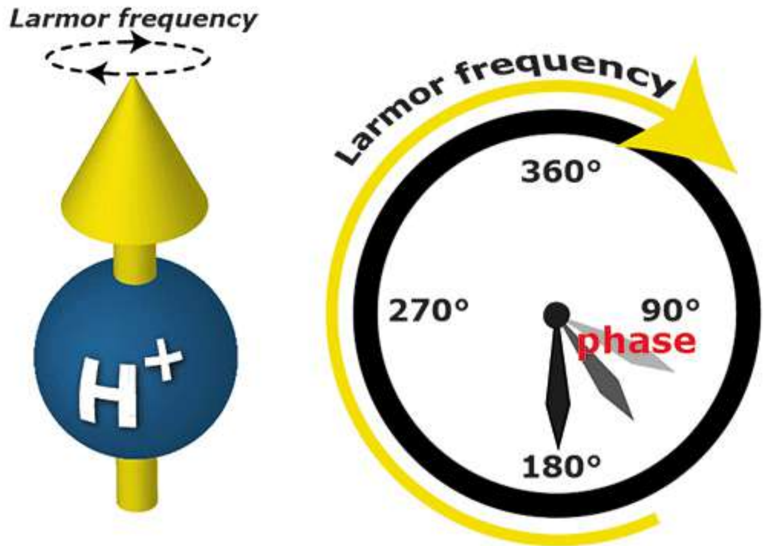




$$\omega_0 = \gamma B_0$$

Proton precession inside a magnetic field

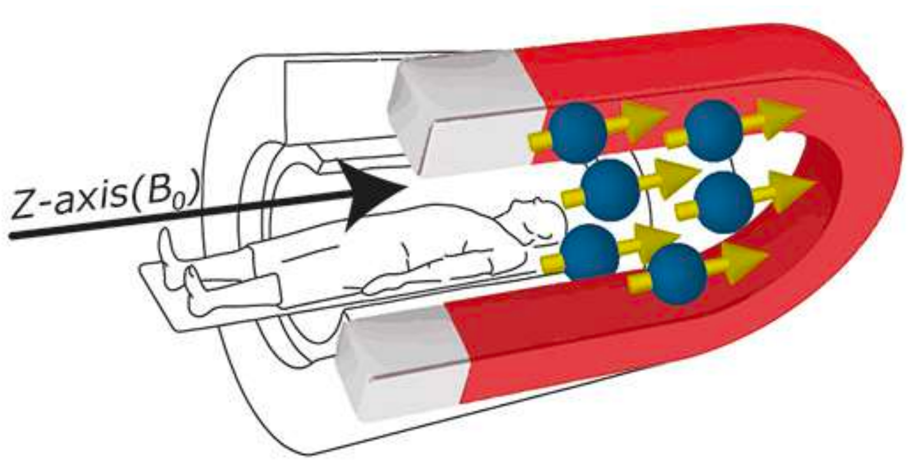




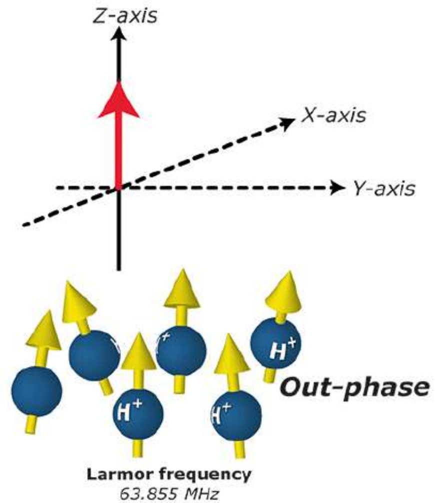
https://www.startradiology.com/the-basics/mri-technique/index.html

Synopsis of MRI

A. Put subject in big magnetic field
B. Transmit radio waves into subject [2~10 ms]
C. Turn off radio wave transmitter
D. Receive radio waves re-transmitted by subject
E. Convert measured RF data to image



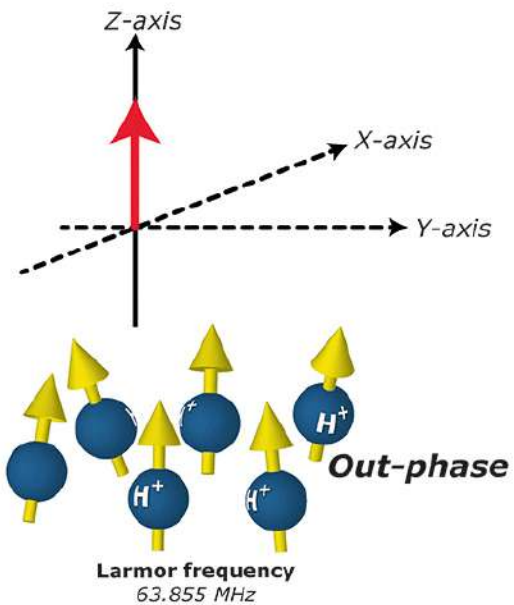
A

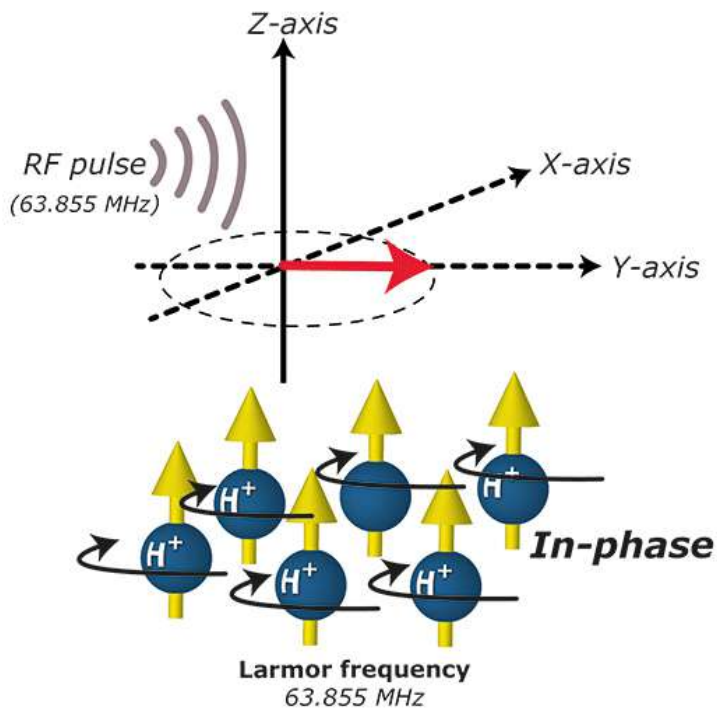


B

Radiofrequency (RF) excitation

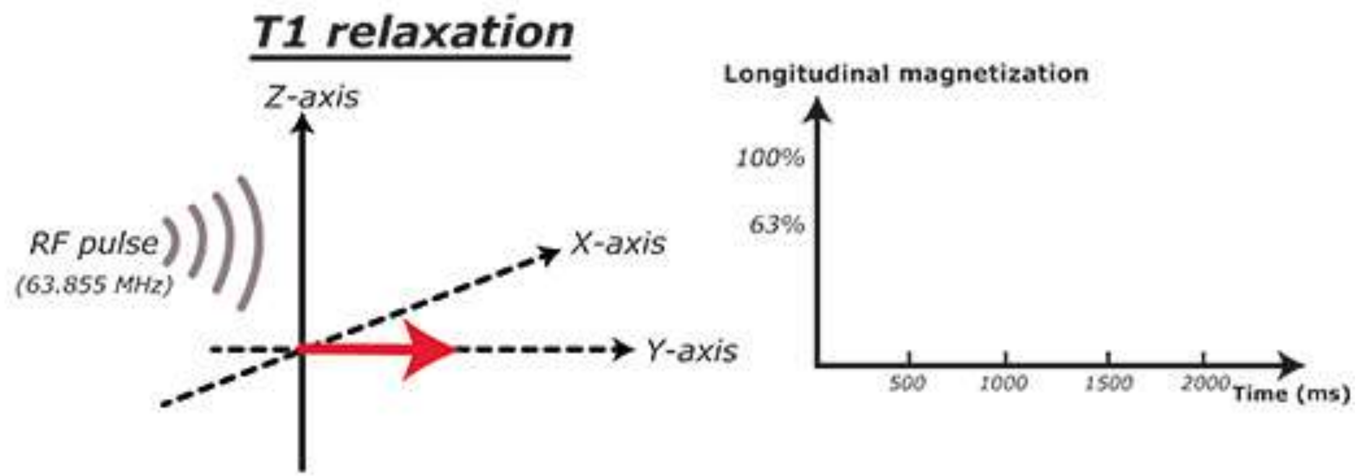
- In magnetic field   Excitation





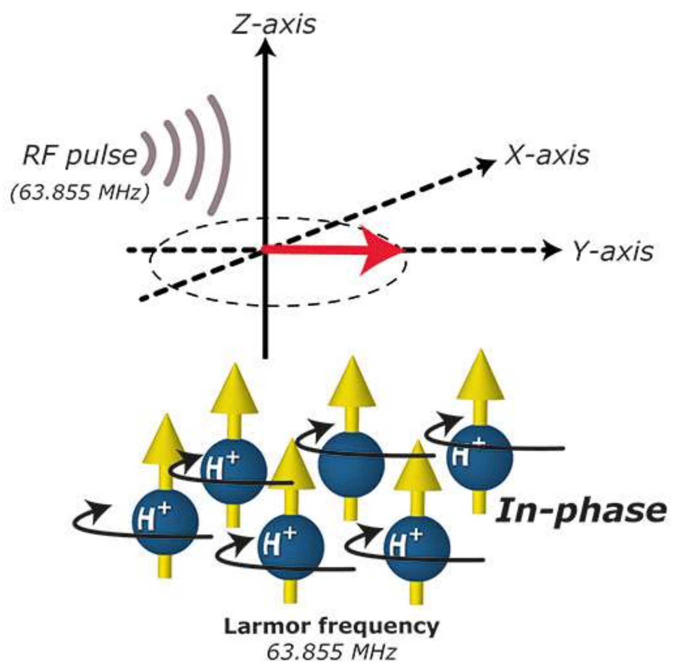
T1 relaxation

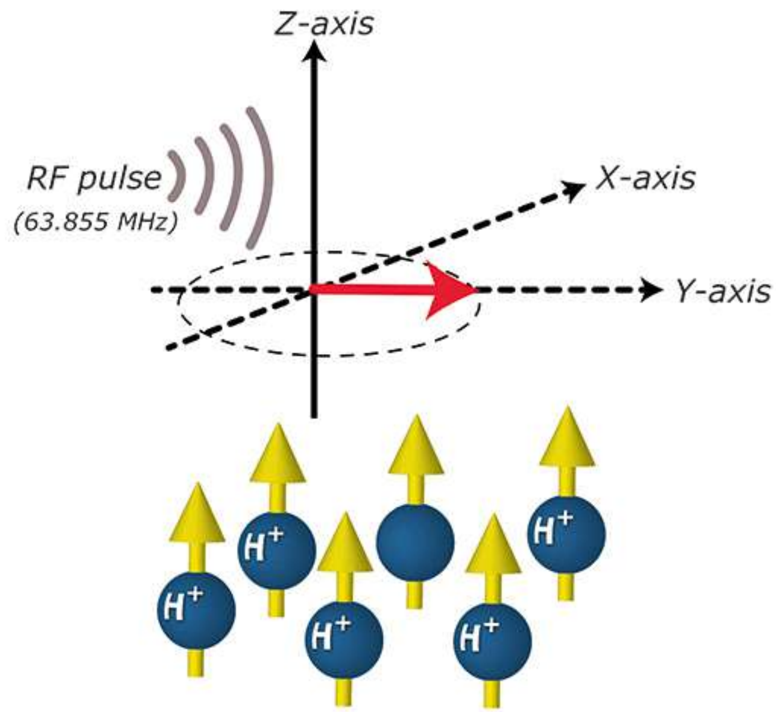
- Relaxation time T1
  - Time required for the z component of M to return to 63% of its original value following an excitation pulse
  - In T1 relaxation, protons will return to their original position and the energy received from the radiofrequent pulse is transferred to their surroundings.



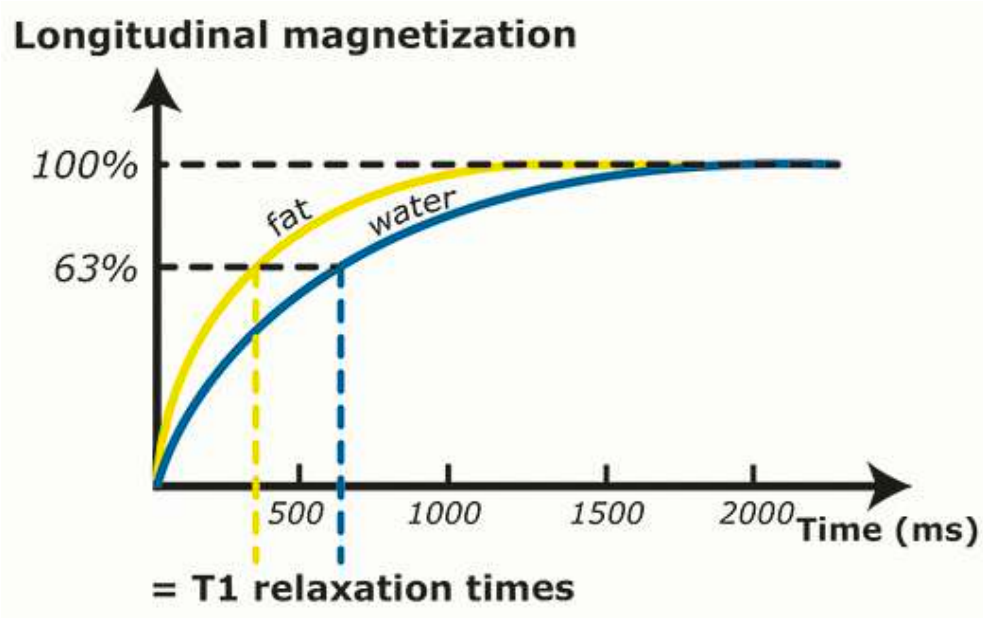
T2 relaxation

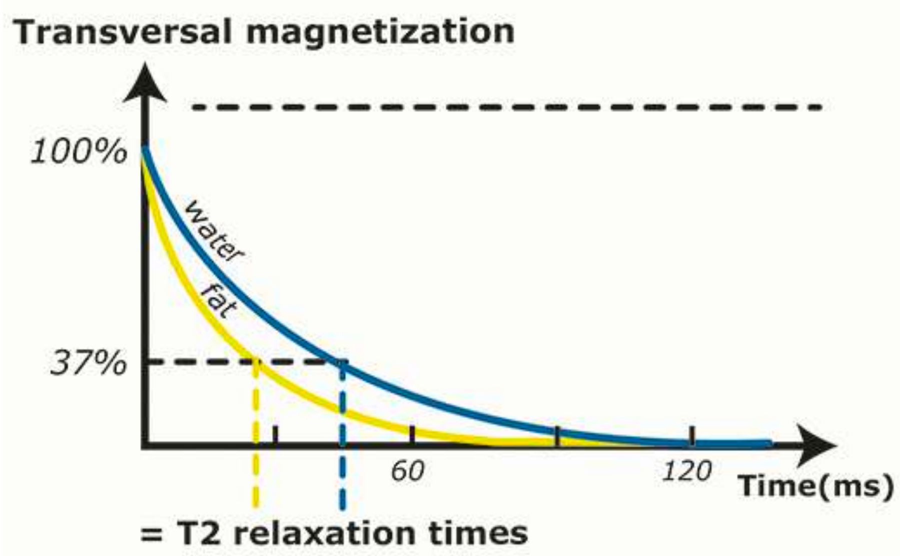
- T2 relaxation time is defined as the time needed to dephase up to 37% (1/e) of the original value.





T1 >> T2

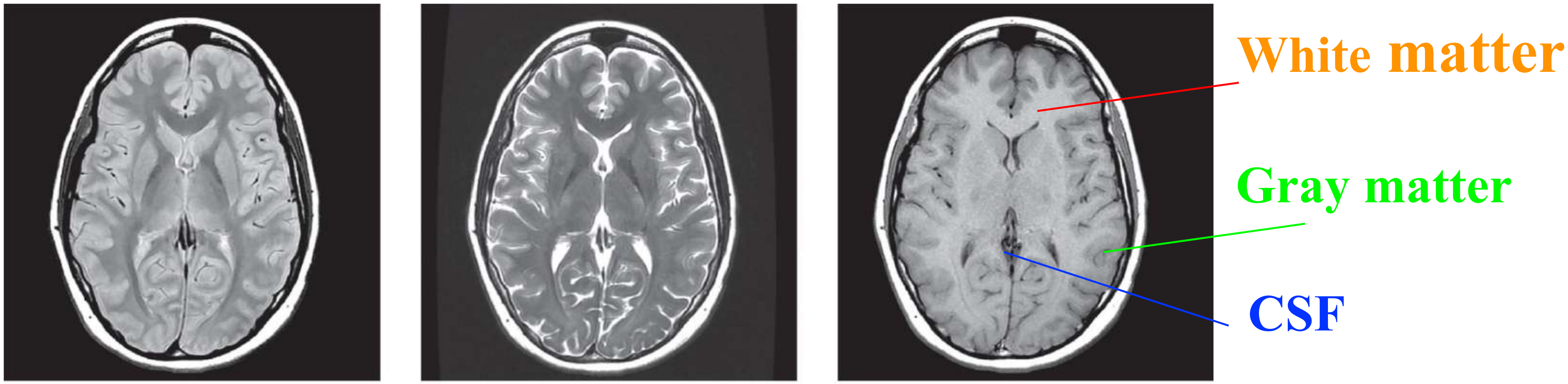

Longitudinal magnetization
100%
63%
fat
water
500
1000
1500
2000
Time (ms)
= T1 relaxation times


Transversal magnetization
100%
37%
water
fat
60
120
Time(ms)
= T2 relaxation times

ECHO TIME & REPETITION TIME

Typical brain tissue parameters

Tissue Type	Relative P_D	T_2 (ms)	T_1 (ms)
White matter	0.61	67	510
Gray matter	0.69	77	760
Cerebrospinal fluid	1.00	280	2,650

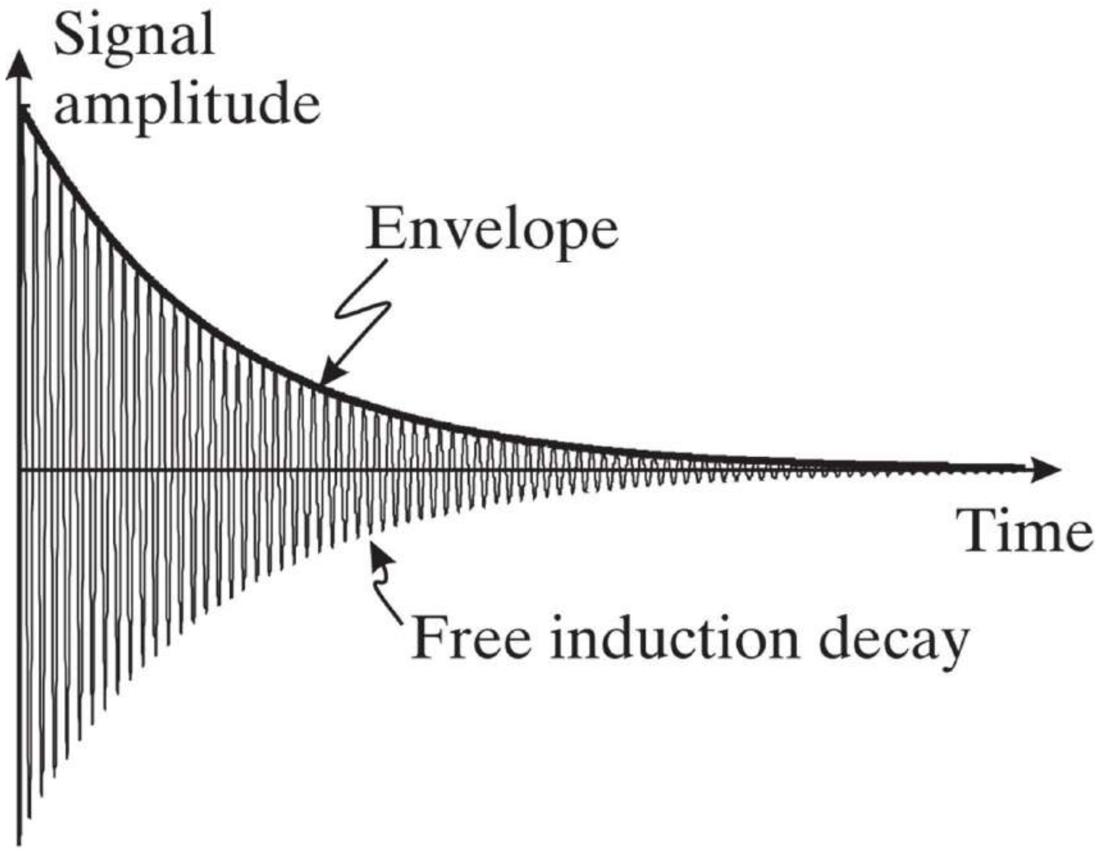


(a) (b) (c)

PD weighted T2- weighted T1- weighted

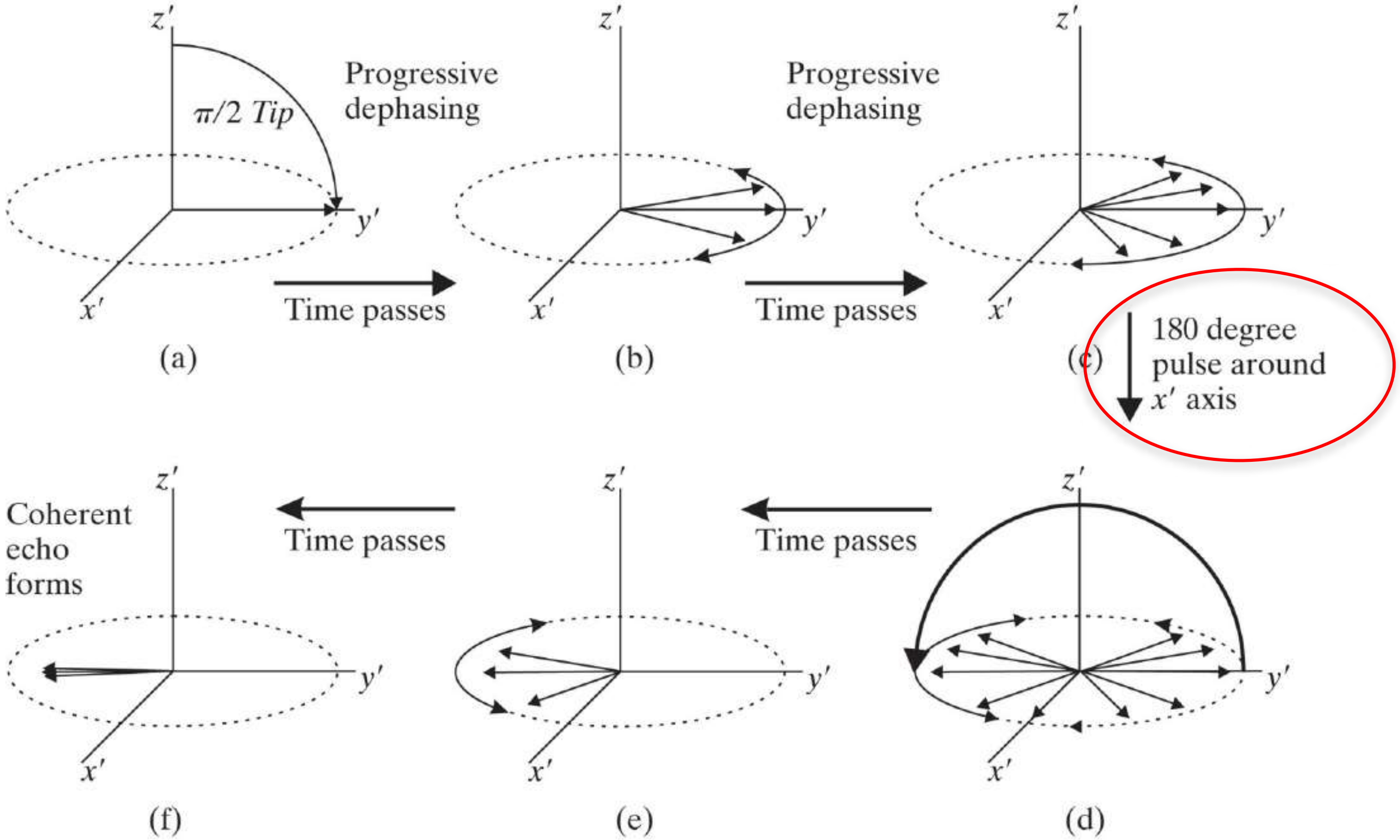
A Free Induction Decay

- The signal decay is caused by transverse relaxation.

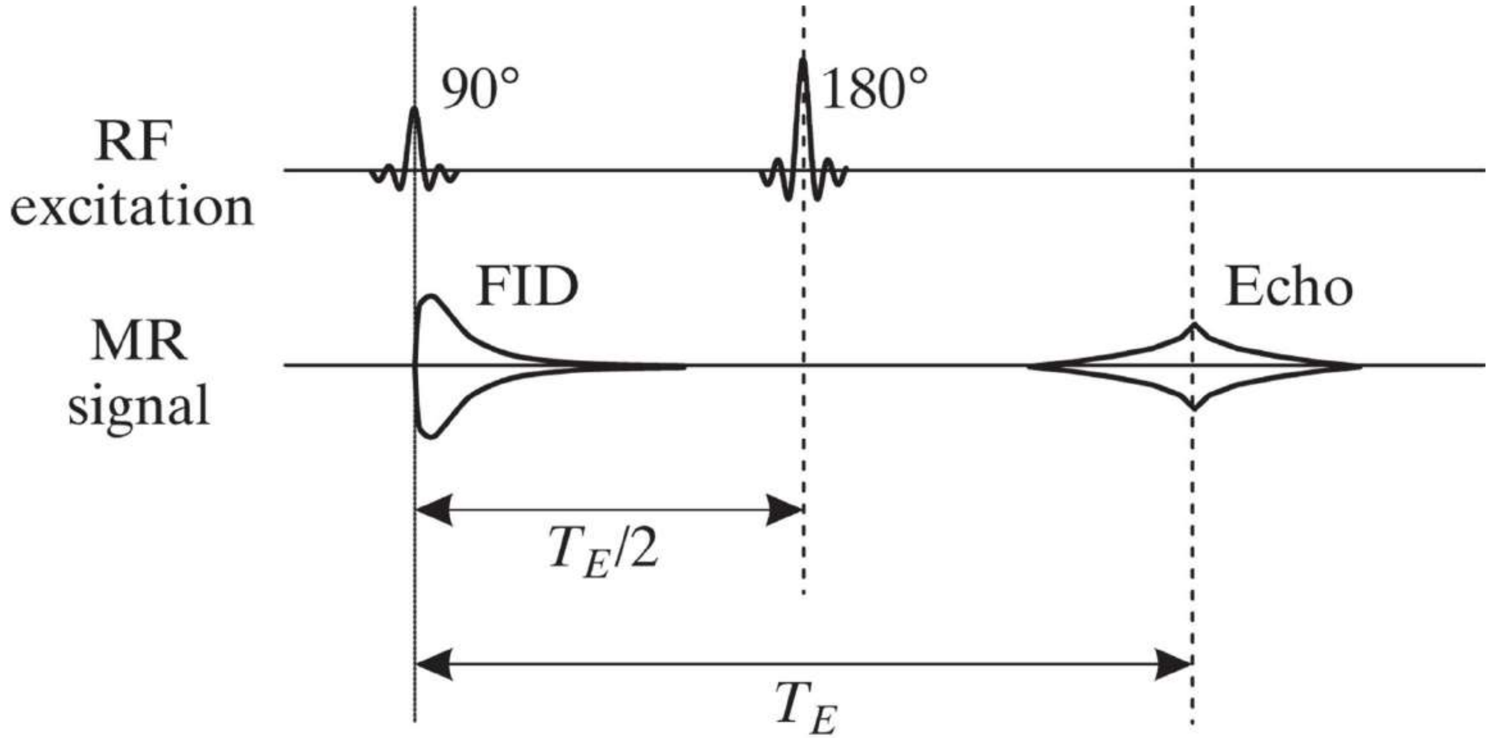


Formation of Spin Echo

- By applying a 180 degree pulse, the rephrased spins can recover their coherence and form an echo signal.

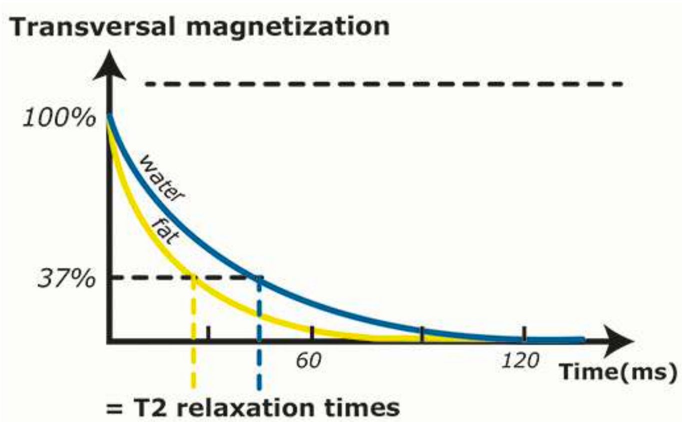


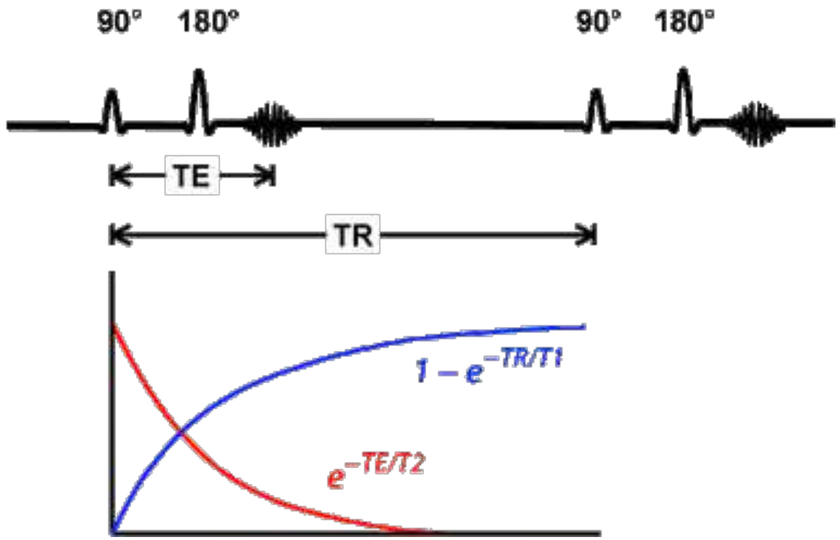
RF Pulse Sequence and Corresponding NMR Signal



Echo Time (TE)

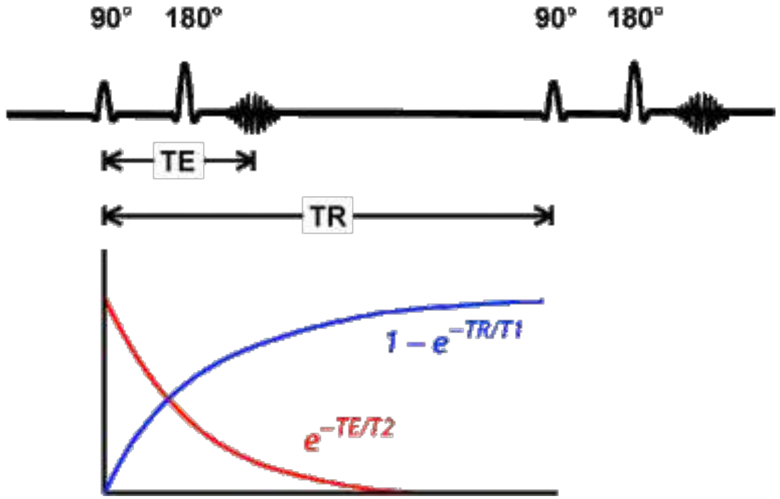
- The time delay after an emitted RF pulse until the RF signal is measured.
- Transversal magnetization decay and signal loss occur due to T2 relaxation, which means that the TE determines the T2 weighting of images.
- Ex: A long TE compared to the T2
  - Strong T2 contrast, but only little signal

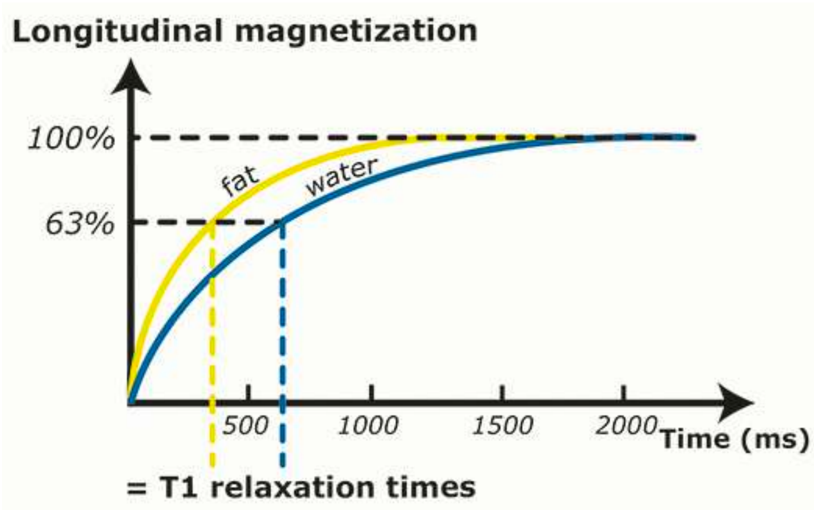




Repetition time (TR)

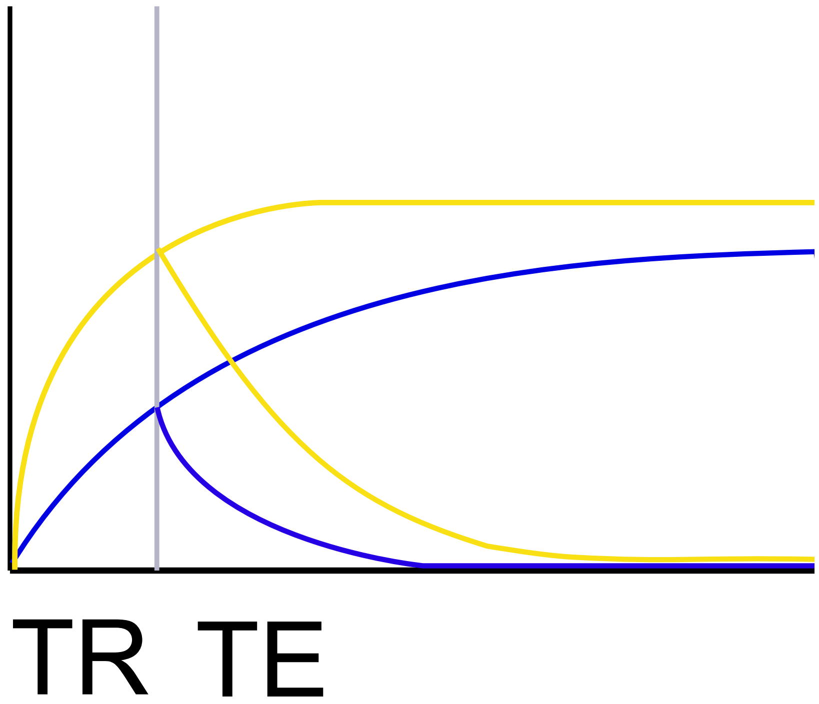
- Repetition time (TR)
  - The period of time between successive RF pulses.
  - An RF pulse in succession will flip parts of the available longitudinal magnetization into the transversal plane.
  - Short TR
    - The available magnetization cannot recover to equilibrium, yielding a relatively small signal per repetition.
  - A long TR
    - Produces a stronger signal as most of the longitudinal magnetization will have recovered by then.





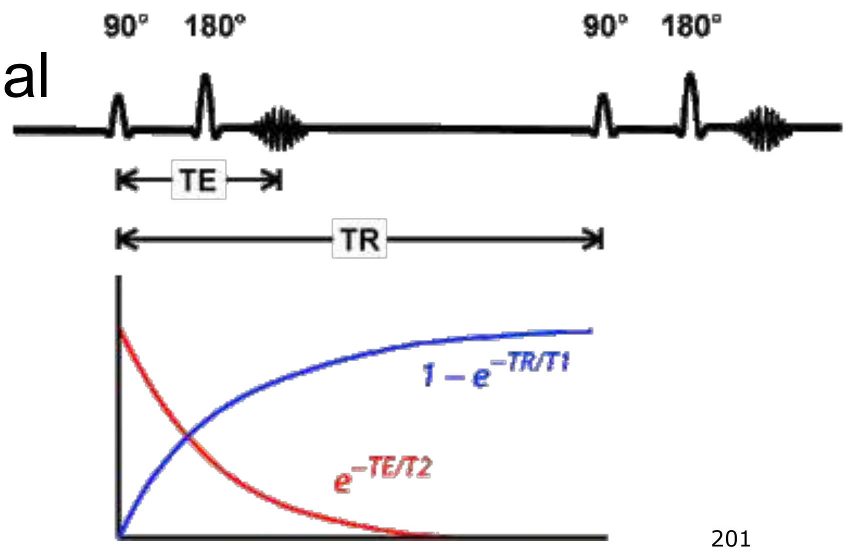
TR, TE

	TR~T1	Long TR
TE<<T2		
Long TE		



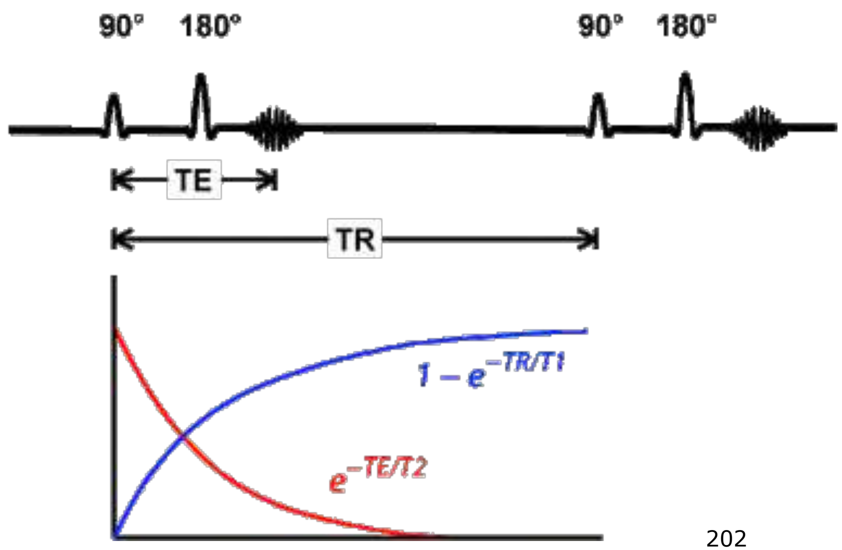
T1 weighted sequence

- Short TR (450－750 ms)
  - Maximizes T1 contrast due to different degrees of saturation
  - If TR too long, tissues with different T1 all return equilibrium already
- Short TE (echo time)
  - Minimizes T2 influence, maximizes signal



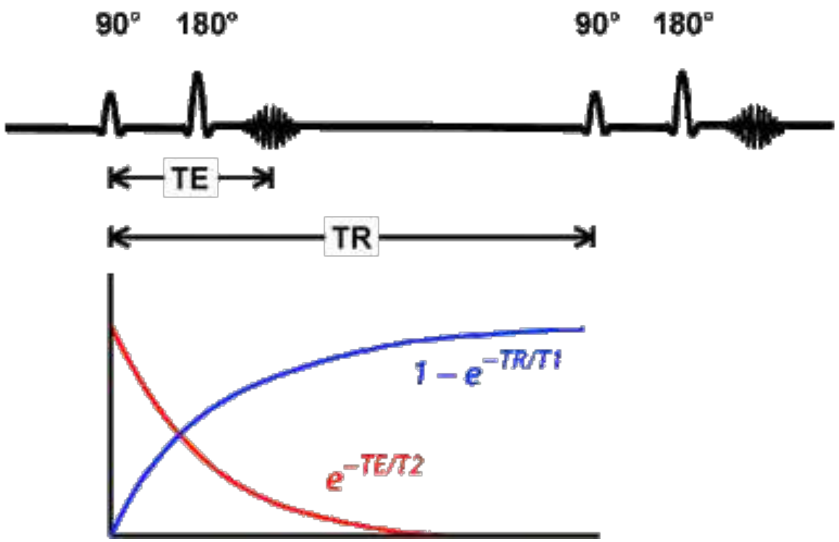
Spin Density Weighting

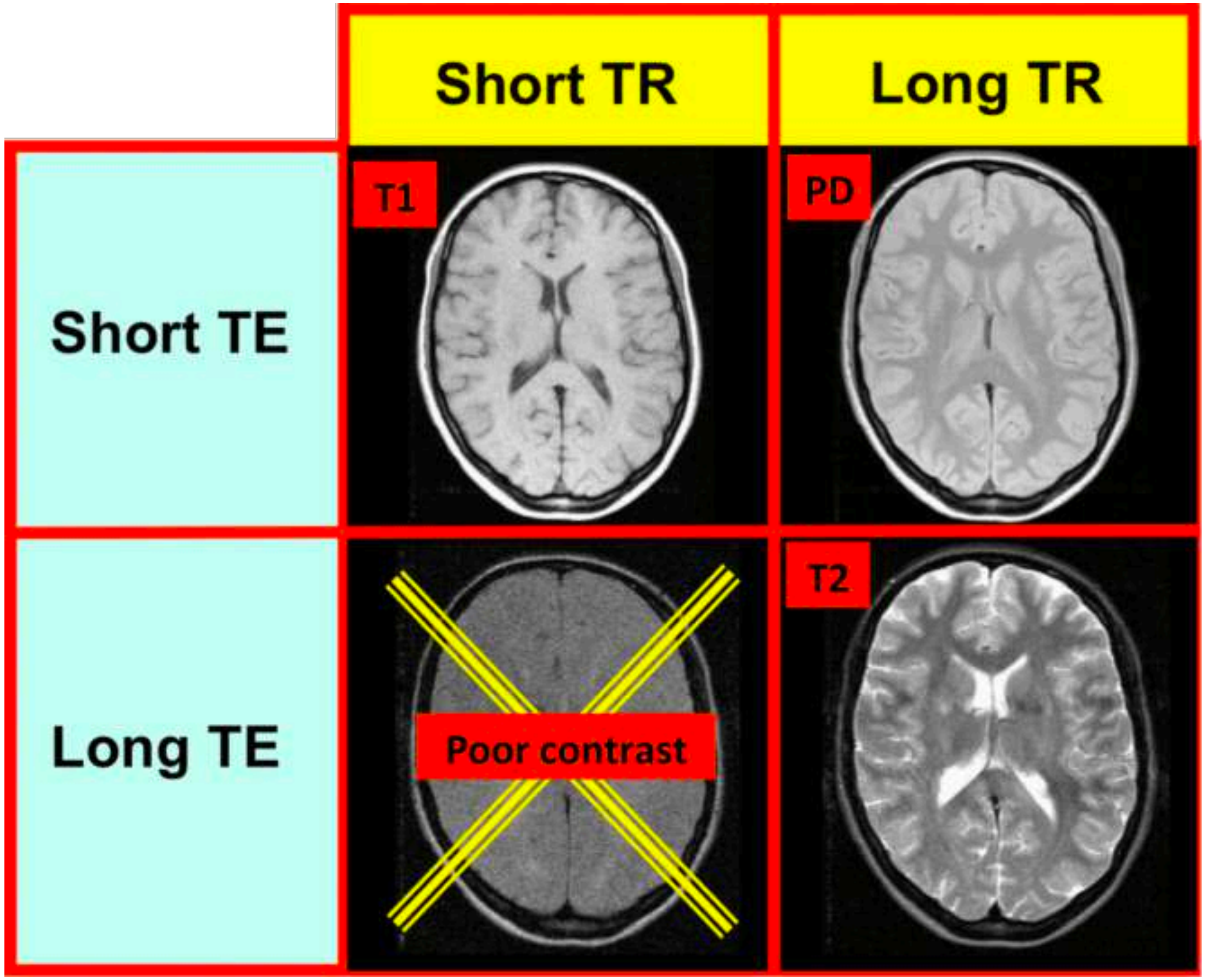
- Signal at equilibrium proportional to PD
- Long TR:
  - Minimizes effects of different degrees of saturation (T1 contrast)
  - Maximizes signal (all return to equilibrium)
- Short TE:
  - Minimizes T2 contrast
  - Maximizes signal



T2 weighting

- Long TR:
  - Minimizes influence of different T1
- Long TE:
  - Maximizes T2 contrast
  - Relatively poor SNR





Minimize T2 effect

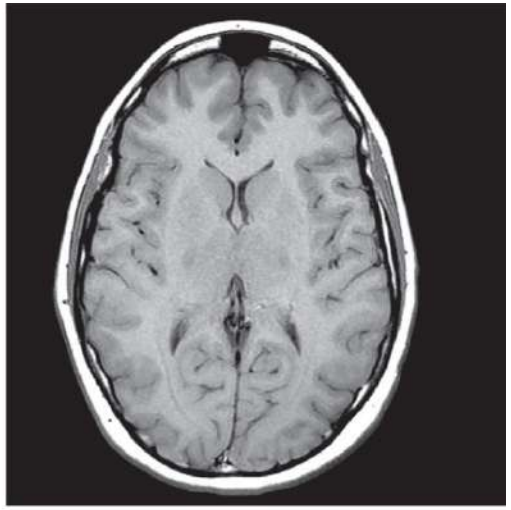
http://mriquestions.com/image-contrast-trte.html

MRI CONTRASTS

T1-weighted image

- Fat is bright
- Water is dark
- New blood is bright

- Useful for anatomic details, vascular changes

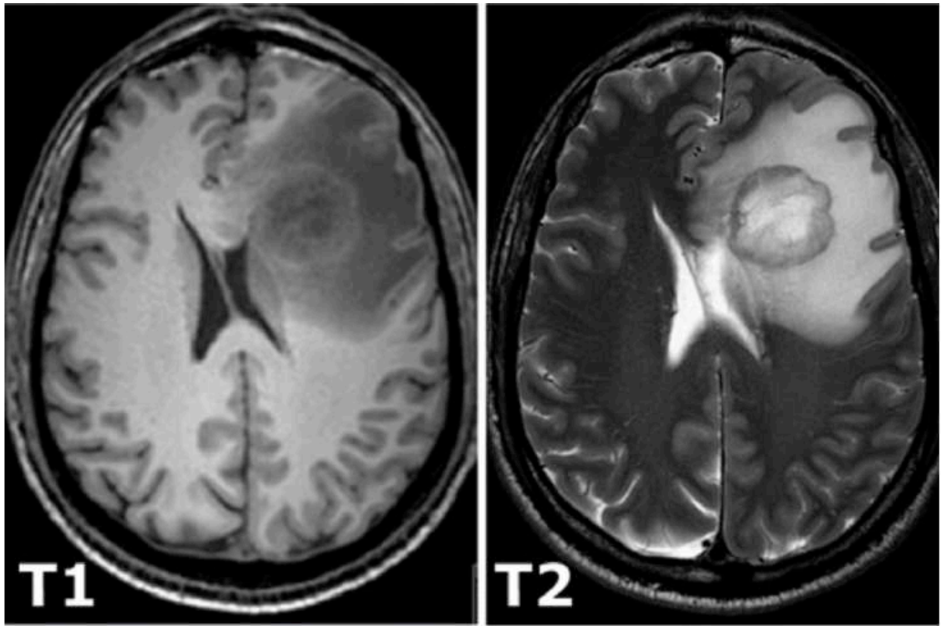


Air, calcium, cortical bone, rapid flowing blood	Fluid, ligaments/ muscles/tendons, abdominal organs, cartilage	High protein tissue (abscess, complex cysts, synovial fluid)	Fat, blood, gadolinium (contrast), melanin, protein

T2-weighted image

- Fat is dark
- Water is bright
  - High signal intensity of water
- Flow is dark (blood vessel)

- Useful for pathology
  - Often associated with edema/fluid



https://www.startradiology.com/the-basics/mri-technique/index.html

PD-weighted Contrast

- Image intensity proportional to the number of hydrogen nuclei in the sample.
    1. Start with the sample in equilibrium
    2. Apply an excitation RF pulse
    3. Image quickly, before the signal decays from T2 effects.
- Long TR (which allows the tissues to be in equilibrium) and either no echo or a short TE (in order to minimize T2 decay).
- Useful in brain MRI to evaluate gray/white matter pathology

T2*- weighted image

- T2* Relaxation
  - The decay time constant of the transverse (xy-) magnetic moment
  - Affected by homogeneity of the magnetic field

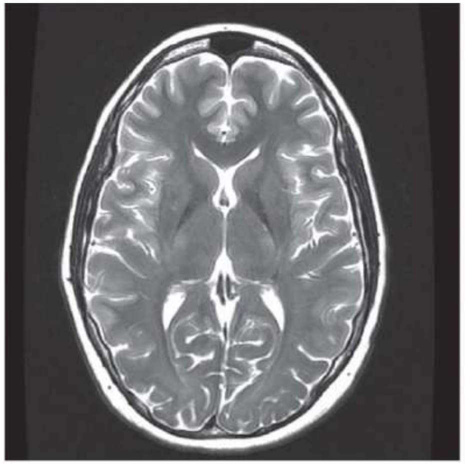
- Low intensity signal
  - Short T2 or inhomogeneous magnetic field

- High intensity signal
  - Long T2 or homogeneous magnetic field

Cerebrospinal Fluid (CSF) contrast

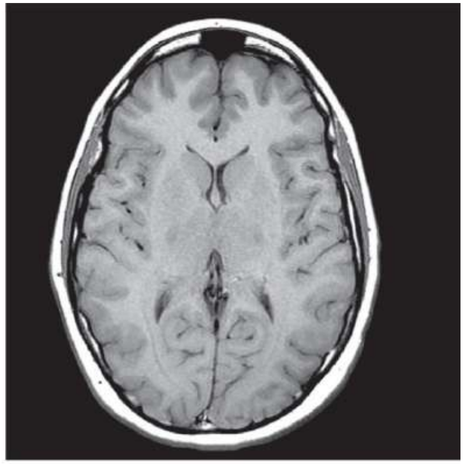
- T1-weighted: dark
- T2-weighted: bright

- Lesions near ventricle
  - T1-weighted: weak contrast with normal tissue
  - T2-weighted: weak contrast with CSF



(b)

T2

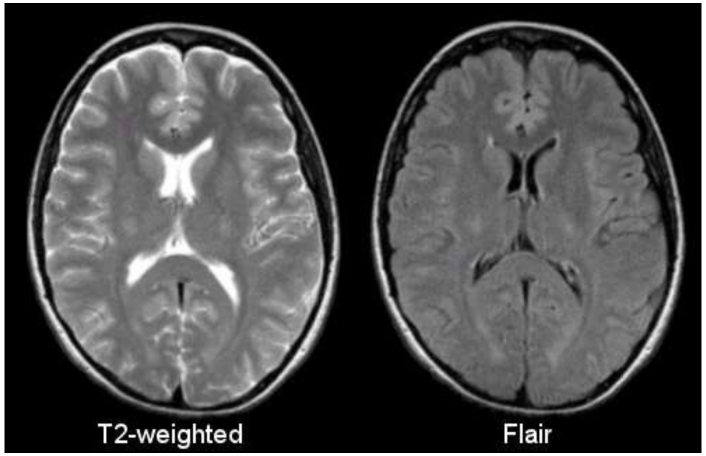


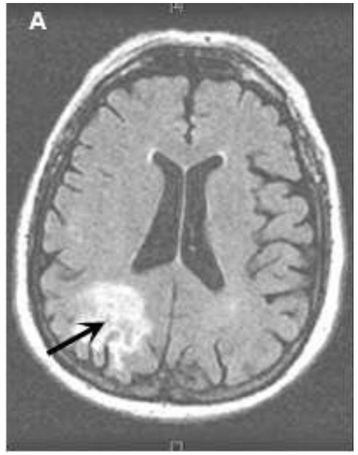
(c)

T1

Why do we need FLAIR (Fluid Attenuated Inversion Recovery)?

- T2 + free flowing water (CSF) is dark
- Non free flowing water is bright
  - Edema
- Highlight lesions





- Clinical MR images (usually)
  - T1-weighted: anatomy
  - T2-weighted: pathology

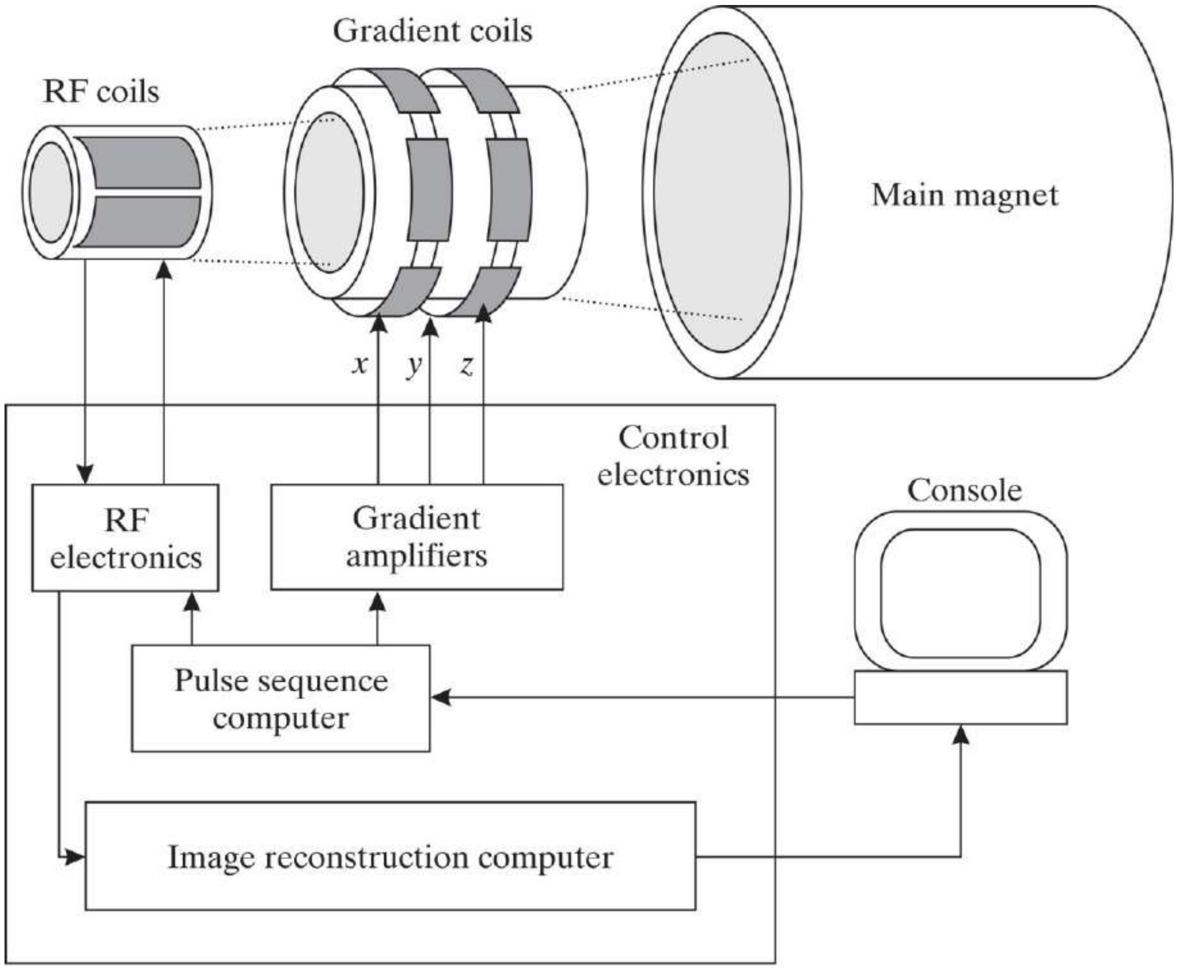
https://case.edu/med/neurology/NR/MRI%20Basics.htm

http://www.stritch.luc.edu/lumen/MedEd/Radio/curriculum/Neurology/edema_2013.htm

Source of MR contrast

- Different tissues have different relaxation times. These relaxation time differences can be used to generate image contrast.
  - T1 - Gray/White matter
  - T2 - Tissue/CSF
  - T2* - Susceptibility (functional MRI)
- The pulse sequence parameters can be designed so that the captured signal magnitude is mainly influenced by one of these parameters
- Pulse sequence parameters
  - Tip angle α
  - Echo time T_E
  - Pulse repetition time T_R

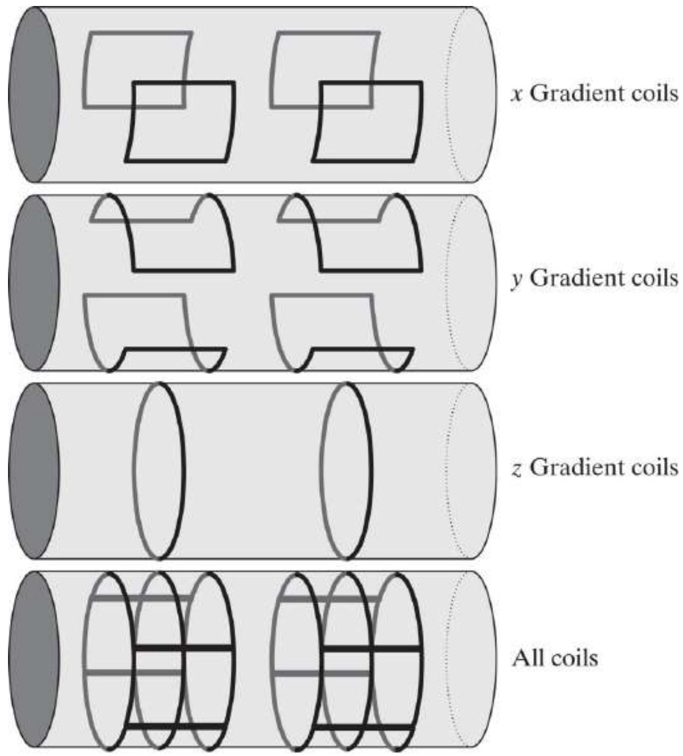
MRI Scanner Components



Gradient Coils

- Used to produce linear variations in the main magnetic field B_0.

- 3 sets of gradient coils
  - x and y are saddle coils
  - z is opposing coils
  - Allows localization of image slices, phase encoding, and frequency encoding

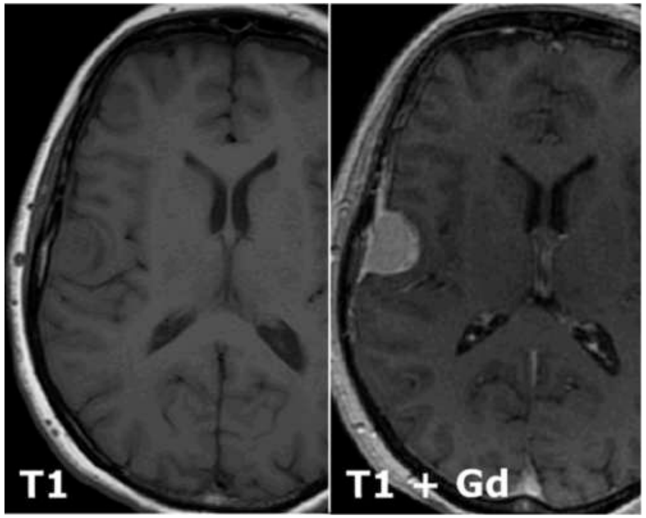


RF Coils

- RF coils create the B1 field which rotates the net magnetization in a pulse sequence. (transmission mode)
- They also detect the transverse magnetization as it precesses in the xy plane. (receive mode)
- Three general categories;
  1. Transmit and receive coils,
  2. Receive only coils
  3. Transmit only coils
- Coils are resonant circuits, tuned w/ capacitors for efficient transmitting and receiving at Larmor frequency .
- Safety: limit absorbed power to prevent heating in excess of 1°C

MRI Contrast

- A contrast series is generally combined with a T1 weighted
- Detect lesions
  - Tumor/metastasis, abscess
- Characterization of lesions
  - Hepatic lesions
- Imaging of vessels/vascular pathology
  - MR angiography



- As pathology is often associated with fluid, the combination of contrast and a T2 weighted image has little value (Note: both fluid and contrast have high signal intensity).
- Gadolinium (Gd)
  - Gadolinium has paramagnetic properties to reduces the T1 relaxation time of the protons.
  - Higher signal intensity (whiter)

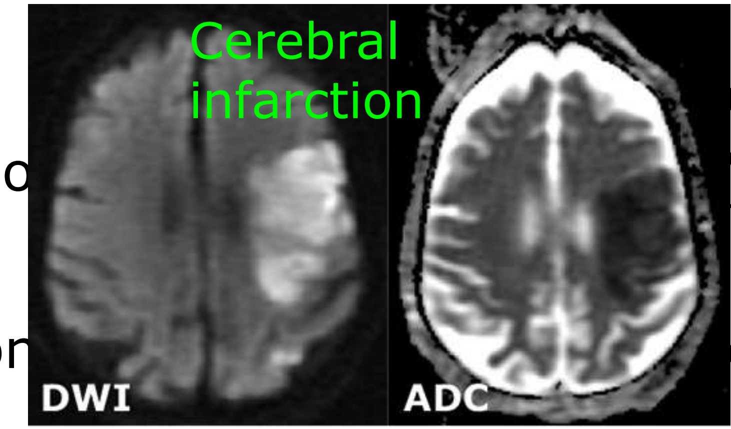
Diffusion Weighted Imaging

- Diffusion: the Brownian motion of molecules in a substance.
- Diffusion behavior of hydrogen molecules is determined under different field strengths.
- The diffusion images obtained are T2 weighted images.
  - Cellularity of the tissue;
    - Many vs few cells —> lower diffusion vs. faster diffusion.
  - Integrity of the cellular membrane
    - In an infarction, the ion pump of the cell membrane will break down and ions & water will stay in the cell (= cytotoxic edema). This will increase intracellular pressure, leading to reduced intracellular diffusion.
  - Blockage of fluid; large vs small molecules.
    - Tissues with large molecules have relatively lower diffusion.

DWI vs. ADC (apparent diffusion coefficient)

- Diffusion weighted images
  - Faster diffusion, signal loss occurs in DWI.
    - Ex: Cerebrospinal fluid (CSF)
  - Reduced diffusion, a high signal intensity on DWI due to 2nd pulse.
    - Ex: cytotoxic edema, inflammation

Diffusion restrictio



otoxic edema,
cess/inflammation,
te demyelination

Increased diffusion

ebrospinal fluid (CSF)

MRI sequence	Property	Characteristic/practice
T1W	Fat high, water low	Normal anatomy
T2W	Fat low, water high	Pathology
PD	Number of protons per volume	Menisci, gray/white matter
STIR	Selective suppression of fat signal	Suppression of intra-abdominal fat, evaluation of bone marrow edema
FLAIR	T2W with selective suppression of CSF signal	Detection of white matter abnormality
Gadolinum (Gd)	Reduced T1 relaxation time	Detect & characterize lesions, MR angiography
DWI & ADC	Motion of protons	Acute ischemia, abscess/ infection, cell-rich tissue
In-out-phase	Detection of microscopic fat	Characterize adrenal lesion
Gradient echo (GE)	FLASH (Siemens), FISP (Siemens), THRIVE (Philips), FFE (Philips), FE (Toshiba), FIESTA (GE healthcare)	Fast sequence, detect blood products
Spin echo (SE)	Turbo SE (Siemens/ Philips), HASTE (Siemens), FAST SE (Toshiba/ GE healthcare)	Fewer susceptibility artifacts

THANK YOU FOR YOUR ATTENTION.