

# Set Theory

# A Set

- ◆ Zermelo-Fraenkel set definition
- ◆ Informally: “any collection of things”

# Formally

- ◆ Sets based on axioms of ZF set theory
  - ◆ 9 (major) axioms
- ◆ Axiomatic logic

Citing Cummings *Real Analysis*

# ZFC axioms

- ◆ Axiom of extensionality
- ◆ Axiom of regularity
- ◆ Axiom schema of specification
- ◆ Axiom of pairing
- ◆ Axiom of union

# ZFC continued

- ◆ Axiom of infinity
- ◆ Axiom of power set
- ◆ Axiom of choice

[You ask a question, I said, to which a reply can only be given in a parable.]

“Yes, Socrates; and that is a way of speaking to which you are not at all accustomed, I suppose.”

—Plato (Glaucón, Republic)

# why

- ◆ Technique for easily dealing with complex data

# Naïve sets

- ◆ Sets are values

$S \leftarrow \{ \dots \}$

- ◆ Sets have elements

$\{0,1,2,3,4,squid\}$

# Inside a set

- ◆ Sets are unordered  $\{0,2,3,9\} = \{9,3,0,2\}$
- ◆ Sets can identify elements  $0 \in \mathbb{M}$
- ◆ A set can have a set as an element  $\{0\} \in \mathbb{M}$

Working with sets