

Set Theory

A Set

- ◆ Zermelo-Fraenkel set definition
- ◆ Informally: “any collection of things”

Formally

- ◆ Sets based on axioms of ZF set theory
 - ◆ 9 (major) axioms
- ◆ Axiomatic logic

ZFC axioms

- ◆ Axiom of extensionality
- ◆ Axiom of regularity
- ◆ Axiom schema of specification
- ◆ Axiom of pairing
- ◆ Axiom of union

ZFC continued

- ◆ Axiom of infinity
- ◆ Axiom of power set
- ◆ Axiom of choice

[You ask a question, I said, to which a reply can only be given in a parable.]

“Yes, Socrates; and that is a way of speaking to which you are not at all accustomed, I suppose.”

—Plato (Glaucon, Republic)

why

- ◆ Technique for easily dealing with complex data

Naïve sets

- ◆ Sets are values
- ◆ Sets have elements

$\mathbb{S} \leftarrow \{ \dots \}$

$\{0, 1, 2, 3, 4, \text{squid}\}$

Inside a set

- ◆ Sets are unordered $\{0,2,3,9\} = \{9,3,0,2\}$
- ◆ Sets can identify elements $0 \in M$
- ◆ A set can have a set as an element $\{0\} \in M$

Working with sets