W1 1-4,5 Discrete Probability

1、一些概念

U: finite set (e.g. $U = \{0,1\}^n$)

Def: Probability distribution P over U is a function $P: U \longrightarrow [0,1]$ such that $\Sigma\ P(x) = 1$

Uniform distribution: for all $x \in U$: $P(x) = 1/|U|$

Point distribution at x0 : $P(x0) = 1$, $\forall x \neq x0 : P(x) = 0$

Distribution vector: (P(000), P(001), P(010), … , P(111))

2、Events

For a set $A \subseteq U$: $Pr[A] = \Sigma\ P(x) \in [0,1]$ （Pr[U]=1）

The set A is called an event

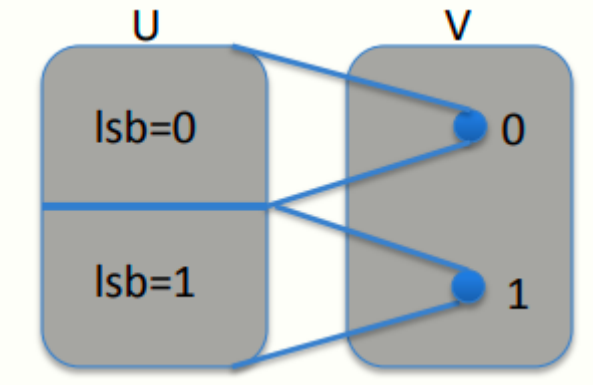
3、The union bound

For events $A_1$and A_2，$Pr[A_1 \cup A_2] \leq Pr[A_1] + Pr[A_2]$

不等式恒成立，当事件A1和A2相互独立时取等号

4、Random Variables

Def: a random variable X is a function X:U⟶V Example: X: {0,1}n^^ ⟶ {0,1} ; X(y) = lsb(y) ∈{0,1}



More generally: rand. var. X induces a distribution on V: $Pr[X=v] := Pr[X^{-1}(v)]$

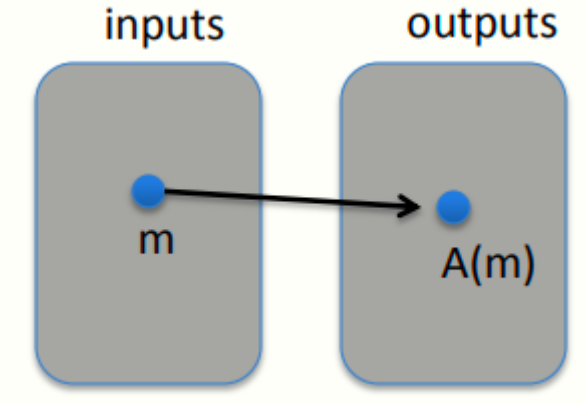
5、The uniform random variable

Let U be some set, e.g. $U = \{0,1\}^n$

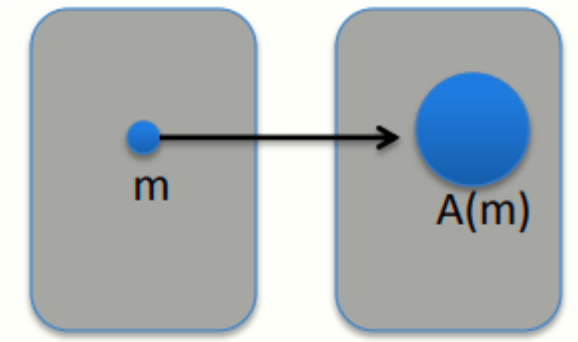
We write $r \longleftarrow U$ to denote a uniform random variable over U for all $a \in U$: $Pr[r = a] = 1/|U|$

6、Randomized algorithms

Deterministic algorithm: $y \longleftarrow A(m)$，对于每次相同的输入，确定性算法总能得到相同的输出



Randomized algorithm y ⟵ A(m ; r) where r ⟵ $\{0,1\}^n$， output is a random variable，随机化算法每一次输入往往得到不同的输出



7、Independence

Def: events A and B are independent if Pr[A and B] = Pr[A] · Pr[B]

random variables X,Y taking values in V are independent if ∀a,b∈V: Pr[X=a and Y=b] = Pr[X=a] · Pr[Y=b]

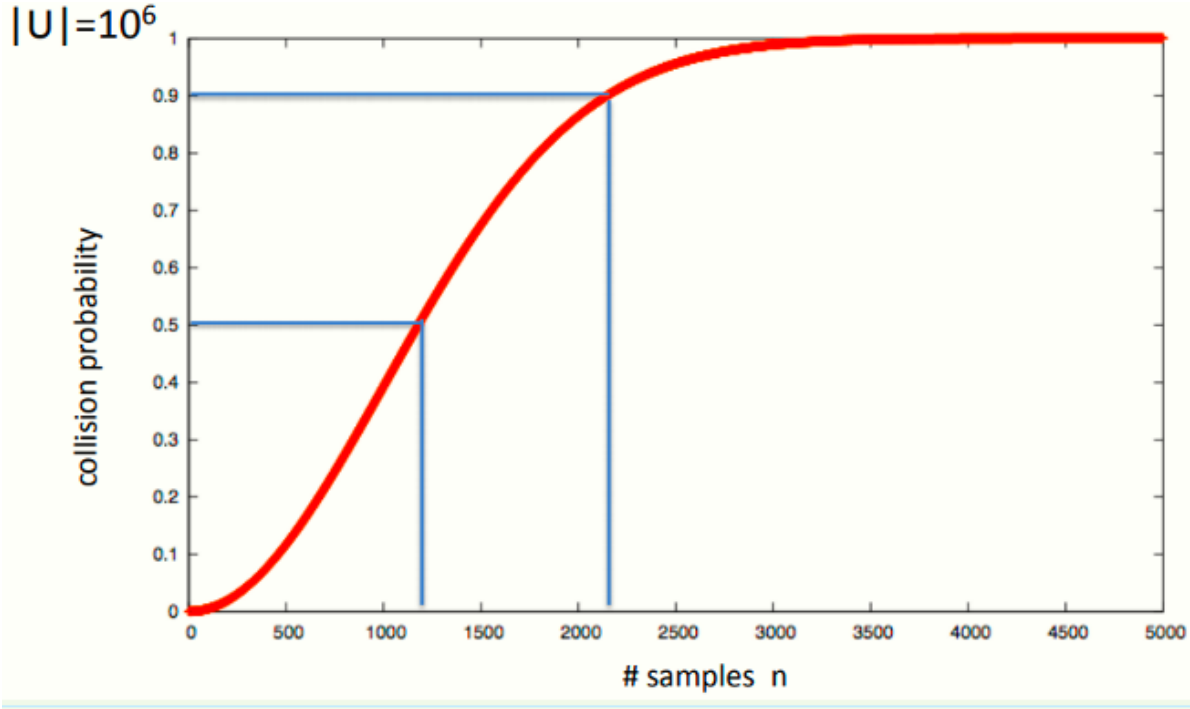
8、XOR

XOR of two strings in $\{0,1\}^n$ is their bit-wise addition mod 2

Thm:Y a rand. var. over $\{0,1\}^n$, X an indep. uniform var. on $\{0,1\}^n$ Then Z := Y⊕X is uniform var. on $\{0,1\}^n$

9、The birthday paradox

Let r1 , ..., rn ∈ U be indep. identically distributed random vars. Thm: when n= 1.2 × $|U|^{(1/2)}$ then Pr[∃i≠j: ri = rj] ≥ 1/2


|U|=10^6
collision probability
samples n
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000