Recalling Trigonometric Identities

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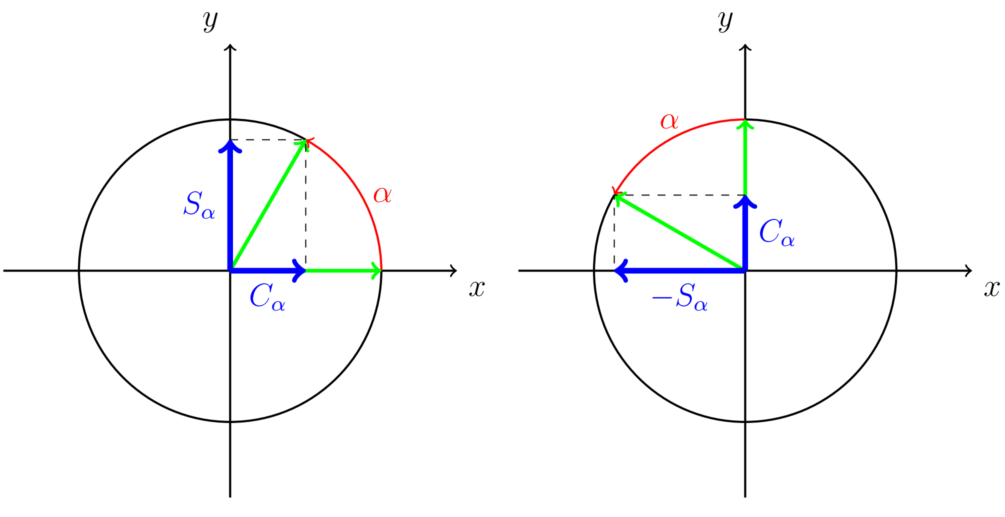
In this post I'll share an easy way to derive and remember trigonometric identities.

Surprisingly, all you need to know:

- $cos(\varphi) = \frac{hypotenuse}{adjacent\ side}$
- matrix multiplication

If you can remember the 2D rotation matrix you can jump through this section!
It can be easily constructed based on the rotation of the (1, 0) and (0, 1) vectors.

$$\big(1,0\big) \rightarrow \big(C_\varphi, S_\varphi\big) \qquad \big(0,1\big) \rightarrow \big(-S_\varphi, C_\varphi\big) \tag{1}$$



(a) (b)

$$\boldsymbol{rot}_\varphi = \begin{pmatrix} C_\varphi & -S_\varphi \\ S_\varphi & C_\varphi \end{pmatrix} \tag{2}$$

I won't give proper mathematical background or prove intuitive assumptions like rotating a vector by α and then by β around the same point gives the same result as a single rotation of $\alpha+\beta$.

$$\boldsymbol{rot}_{\alpha+\beta} = \boldsymbol{rot}_{\alpha}\,\boldsymbol{rot}_{\beta}$$

$$\begin{pmatrix} C_{\alpha+\beta} & -S_{\alpha+\beta} \\ S_{\alpha+\beta} & C_{\alpha+\beta} \end{pmatrix} = \begin{pmatrix} C_{\alpha} & -S_{\alpha} \\ S_{\alpha} & C_{\alpha} \end{pmatrix} \begin{pmatrix} C_{\beta} & -S_{\beta} \\ S_{\beta} & C_{\beta} \end{pmatrix} \tag{3}$$

$$C_{\alpha+\beta} = C_{\alpha} C_{\beta} - S_{\alpha} S_{\beta} \tag{4}$$

The addition formula for the sine can be calculated by the same way, just using the dot product of the corresponding rows and columns.

Have this one in the bag you can easily derive other identities, see the link below.

The post was written as an addition to 'The Two Key Trig Identities Worth Memorizing'