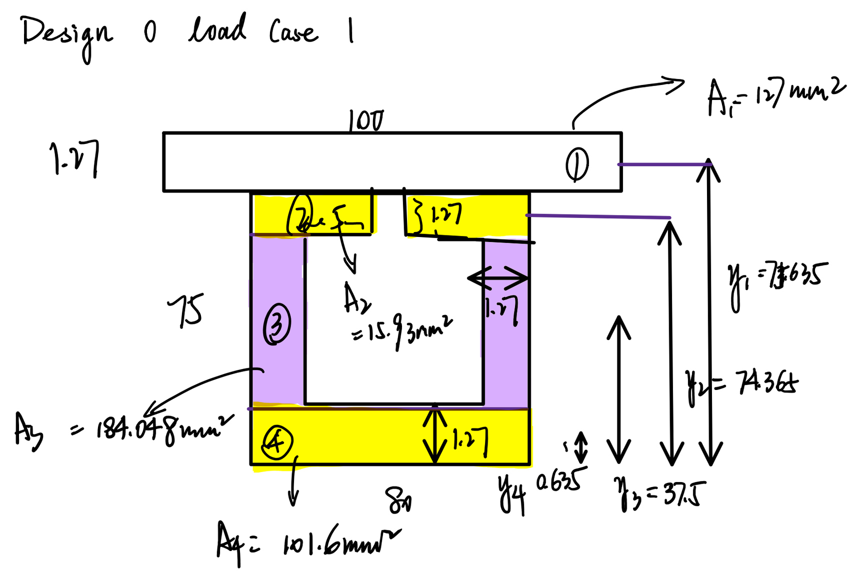
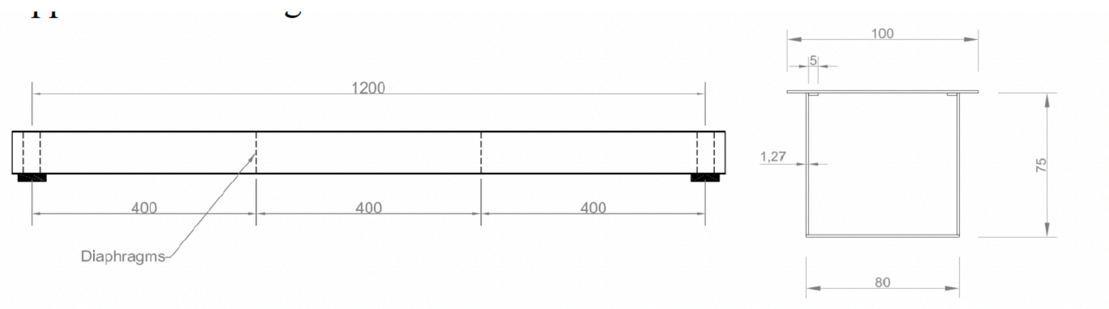
Report 1 Hand Calculation

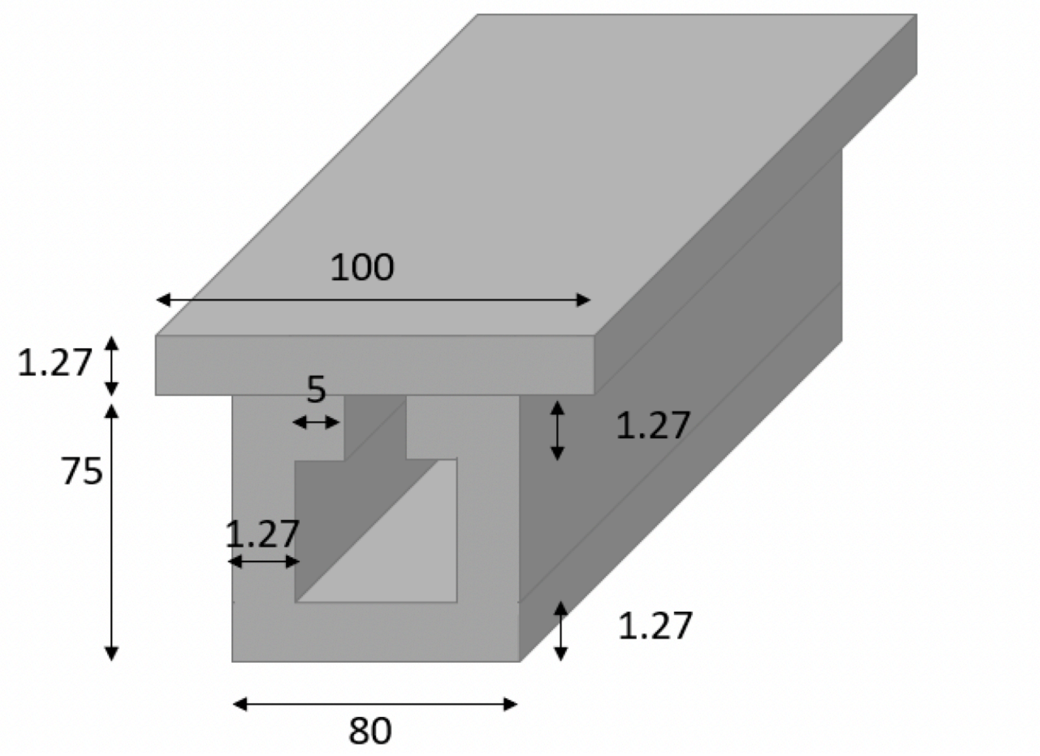
2023年11月25日 星期六 下午4:14

Design 0 load Case 1





Elevation view and cross-section view of Design 0 (same cross-section view along the entire span)



Moment of Area:

$I = I' + Ad^2 \qquad \bar{I} = \frac{bh^3}{12}$

$\bar{y} = \frac{\sum Ay}{\sum A}$

$= \frac{9605.65\,mm^3 + 1184.63\,mm^3 + 6901.8\,mm^3 + 64.516}{127\,mm^2 + 15.93\,mm^2 + 184.048\,mm^2 + 101.6\,mm^2}$

$= 41.3735\,mm$

$d_1 = 75.635\,mm - \bar{y} = 34.26\,mm \qquad \bar{I}_1 = \frac{100 \times 1.27^3}{12} = 17.07\,mm^4$

$d_2 = 74.365 - \bar{y} = 32.99\,mm \qquad \bar{I}_2 = \frac{12.54 \times 1.27^3}{12} = 2.146\,mm^4$

$d_3 = |37.5 - \bar{y}| = 3.874\,mm \qquad \bar{I}_3 = \frac{2.54 \times 72.46^3}{12} = 81479.2\,mm^4$

$d_4 = |0.635\,mm - \bar{y}| = 40.74 \qquad \bar{I}_4 = \frac{80 \times 1.27^3}{12} = 13.66\,mm^4$

$I = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 + \bar{I}_4 + A_1 d_1^2 + A_2 d_2^2 + A_3 d_3^2 + A_4 d_4^2$

$= 81512.1\,mm^4 + 149066\,mm^4 + 17337.3\,mm^4 + 2762.18\,mm^4 + 168630\,mm^4$

$= 419308\,mm^4 \approx 4.19 \times 10^5\,mm^4$

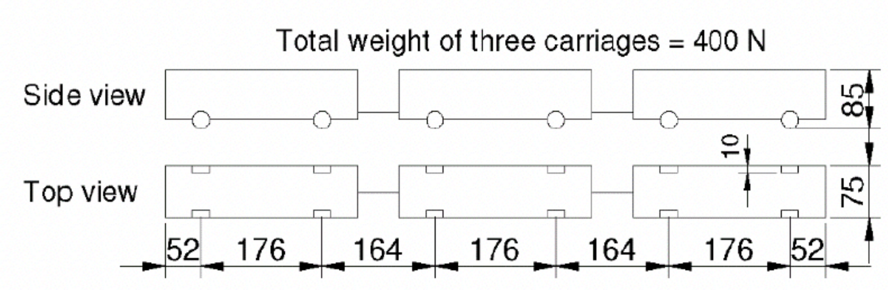


Figure 3. Train dimensions

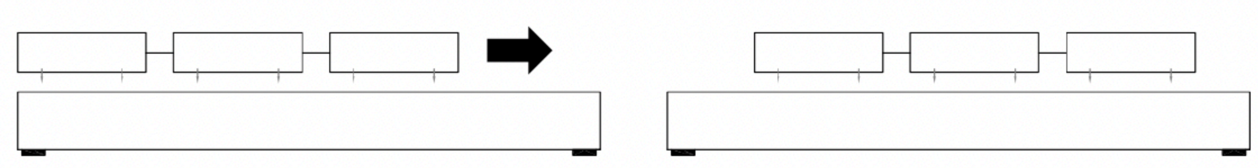
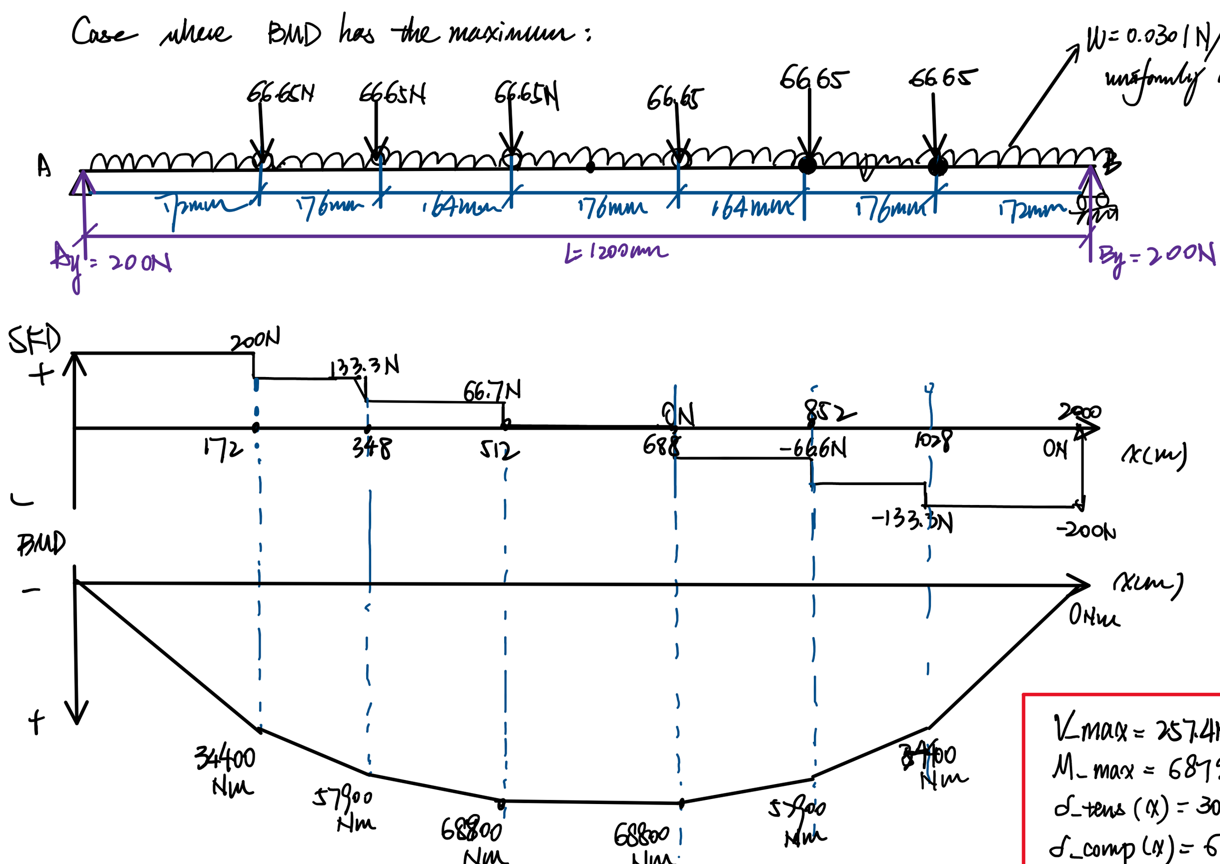


Figure 4. Loading schematic showing the movement of the train

Load Case 1

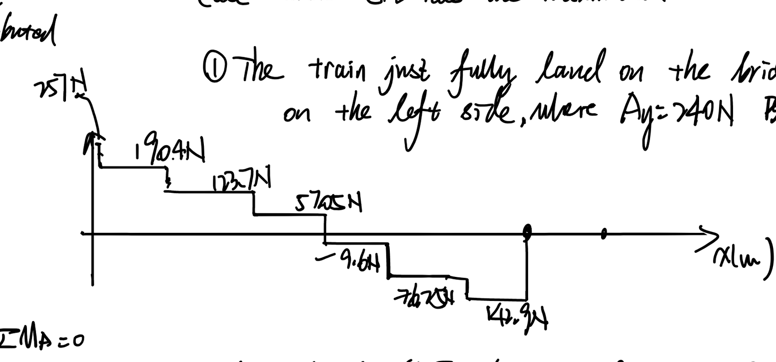
Initially, the train will not be loaded with any additional weight. This means the total train weight will be 400 N which will be evenly divided across the 6 sets of train wheels.

Case where BMD has the maximum:



Case where SFD has the maximum

① The train just fully land on the bridge, on the left side, where $A_y = 240N$ $B_y = 160N$



$\sum M_A = 0$

$1200 B_y = 66.65 \times 176 + 66.65 \times 340 + 66.65 \times 516 + 66.65 \times 680 + 66.65 \times 856$

$B_y = 142.63\,N$

$\sum F_y = 0$

$\therefore A_y = 257.4N$

$$\boxed{\begin{aligned} &V\_max = 257.4N \\ &M\_max = 68799.6\ N.m \\ &\sigma\_tens\ (x) = 30\ [MPa] \\ &\sigma\_comp\ (x) = 6\ [MPa] \\ &\tau\_shear = 4\ [MPa] \\ &T\_glue = 2\ [MPa] \end{aligned}}$$

① Tension always in the bottom

$y\_top = 75 + 1.27 - \bar{y} = 34.8965\,mm$

$M = 68799.6\ N \cdot m$

$\sigma\_top = \frac{M y\_top}{I} = 5.73\ MPa$

$y\_bottom = \bar{y} = 41.3735\,mm$

$\sigma\_bottom = \frac{M y\_bottom}{I} = 6.79\ MPa$

$$\boxed{\begin{aligned} &FOS\ tension = \frac{30\,MPa}{6.79\,MPa} = 4.12 \\ &FOS\ compression\ \frac{6\,MPa}{5.73\,MPa} = 1.05 \end{aligned}}$$

② Shear

$Q = Ad$

$= A_1 d_1 + 2 \times A_2 d_2 = 101.6 \times 41.065 + 2 \times [(41.7 - 1.27) \times 1.27][\bar{y} - \frac{(41.7 - 1.27)}{2} + 1.27] = 6246.263\,mm^3$

$T\_xy = \frac{VQ}{Ib} = \frac{240 \times 6246.263\,mm^3}{419308\,mm^4 \times (1.27 \times 2)} = 1.51\,MPa$

$\boxed{FOS\_shear = \frac{4\,MPa}{1.51\,MPa} = 2.65}$

③ Shear glue

$Q = Ad = 127 \times 33.63 = 4270.6\,mm^3$

$\tau\_xy\_glue = \frac{257.4N \times 4309.745}{4.16 \times 10^5 \times 12.54} = 0.211\ MPa$

$\boxed{FOS\_shear\_glue = \frac{2\,MPa}{0.211\,MPa} = 9.48}$

④: Buckling

Case ①: Center Buckling:

$\sigma\_xy_1 = \frac{4 \times \pi^2 \cdot 4000}{12(1 - 0.2^2)} [\frac{1.27}{80}]^2 = 3.45$

$\boxed{FOS\_b_1 = \frac{3.45}{5.73} = 0.602}$

Case ②: Plate Buckling Side

$\sigma\_xy_2 = \frac{0.425 \pi^2 E}{12(1 - \mu^2)} [\frac{1.27}{10}]^2$

$= 23.49\ MPa$

$\boxed{FOS\_b_2 = \frac{23.49}{5.73} = 4.10}$

Case ③: Support Buckling

$\sigma\_xy_3 = \frac{6\pi^2 E}{12(1 - N^2)} \times [\frac{1.27}{y.top}]^2 = 27.23\,MPa$

$\boxed{FOS\ b_3 = \frac{27.23\,MPa}{5.73} = 4.75}$

Case ④: Shear buckling

$\sigma\_xy_4 = \frac{5\pi^2 \times 4000}{12(1 - 0.2^2)} \times [(\frac{1.27}{400})^2 + (\frac{1.27}{75})^2]$

$= 5.086$

$\boxed{\therefore FOS\_shear.buck = \frac{5.086}{1.51\,MPa} = 3.37}$

$\therefore$ FOS_ Critical = FOS_Center.buck = 0.602

The failure load would be $p = 0.602 \times 66.66 = 40.1\,N$

which means a have a train of 241 N.