2019 Math 340:101. Quiz 2 Friday, 2019 September 20 IN CLASS

Last name ....

First name ....

Student number ....

Grade

1. (3 points) The following dictionary is obtained while solving a standard form LP problem by using the simplex method. What does this dictionary imply to the LP problem? **Explain your answer clearly.**

$$\begin{array}{rcrrrr} x_2 & = & 12 & -x_1 & +x_4 & +x_6 \\ x_3 & = & 1 & +x_1 & & +x_6 \\ x_5 & = & 4 & +x_1 & +x_4 & \\ \hline z & = & 8 & +x_1 & -2x_4 & +2x_6 \end{array}$$

Let $x_6 = t$. Then, while keeping $x_1 = x_4 = 0$, we have $x_2 = 12 + t, x_3 = 1 + t, x_5 = 4 + t$ is feasible and $z = 8 + 2t$, therefore, as $t \to +\infty$, $z \to +\infty$. This means that the LP problem is unbounded and has no optimal solution.

2. (3 points) **True/False** question. Circle the right choice.

   (1 mark for every 2 correct answers.)

   A) Every linear programming problem has an optimal solution. **True / False**

   B) If a dictionary of a standard form linear programming problem is feasible then the linear programming problem has a feasible solution. **True / False**

   C) If the feasible region of a linear programming problem is unbounded, then the linear programming problem does not have an optimal solution. **True / False**

   D) The union of two convex sets is always a convex set. **True / False**

   E) The auxiliary problem of the two phase method for a linear programming problem is always feasible. **True / False**

   F) If the auxiliary problem of the two phase method for a linear programming problem has a zero maximum value, then the original problem has an optimal feasible solution. **True / False**

   A) False

   B) True

   C) False

   D) False

   E) True

   F) False

3. (4 points) Suppose you want to solve the following LP problem by using the **two-phase simplex method**.

$$\begin{array}{lrrrcr}
\text{Maximize} & -x_1 & +2x_2 & +x_3 & & \\
\text{subject to} & -x_1 & +x_2 & & \le & -1 \\
 & x_1 & & -x_3 & \le & -2 \\
 & x_1 & +x_2 & +x_3 & \le & 3 \\
 & x_1, & x_2, & x_3 & \ge & 0.
\end{array}$$

The initial dictionary of **the auxiliary problem** for determining feasibility of the original LP problem is given by

$$\begin{array}{cclrrrr}
x_4 & = & -1 & +x_1 & -x_2 & & +x_0 \\
x_5 & = & -2 & -x_1 & & +x_3 & +x_0 \\
x_6 & = & 3 & -x_1 & -x_2 & -x_3 & +x_0 \\
\hline
z & = & & & & & -x_0
\end{array}$$

For this dictionary, find the entering and leaving variables and write down the next dictionary.

Note that this dictionary is for the auxiliary problem! The entering variable is obviously x_0, and we increase it until the first moment the basic variables become all nonnegative (while keeping the value of nonbasic variables at zero). From $x_4 = -1 + x_1 - x_2 + x_0$, we get $x_0 \ge 1$.

From $x_5 = -2 - x_1 + x_3 + x_0$, we get $x_0 \ge 2$

And from $x_6 = 3 - x_1 - x_2 - x_3 + x_0$, we get no restriction on x_0. Combined together, we get $x_0 \ge 2$ and x_5 is the leaving variable.

Now rewrite: $x_0 = 2 + x_1 - x_3 + x_5$, thus $x_4 = -1 + x_1 - x_2 + (2 + x_1 - x_3 + x_5) = 1 + 2x_1 - x_2 - x_3 + x_5$, and $x_6 = 3 - x_1 - x_2 - x_3 + (2 + x_1 - x_3 + x_5) = 5 - x_2 - 2x_3 + x_5$. So, the next dictionary is

$$\begin{array}{cclrrrr}
x_4 & = & 1 & +2x_1 & -x_2 & -x_3 & +x_5 \\
x_0 & = & 2 & +x_1 & & -x_3 & +x_5 \\
x_6 & = & 5 & & -x_2 & -2x_3 & +x_5 \\
\hline
z & = & -2 & -x_1 & & +x_3 & -x_5
\end{array}$$

Blank page