- You will *not* be able to use a calculator or computer for either the midterm or the final exam, so please do not use one for this assignment. You may use one to *check* your answer, but please do not use one to solve the problem.
- Only part of the problems may be graded. But, you have to submit all the problems.
- The deadline is *by 6pm on Friday*, Nov. 15. You have a grace period until 9am the next day. The grace period is to take care of any technical issues you have while submitting the file. The grace period should give ample time for handling any issues, so No late HW after the grace period will be accepted, **regardless** your technical issues.
- Submit only pdf files.

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1. $\boxed{\text{5 marks}}$ Solve the following LP problem using the revised simplex method:

$$\begin{array}{lrrrcl}
\text{Maximize} & 2x_1 & +3x_2 & +3x_3 & & \\
\text{subject to} & 3x_1 & +x_2 & & \leq & 60 \\
& -x_1 & +x_2 & +4x_3 & \leq & 10 \\
& 2x_1 & -2x_2 & +5x_3 & \leq & 15 \\
& x_1, & x_2, & x_3 & \geq & 0.
\end{array}$$

Hint: requires ≤ 3 iterations.

2. $\boxed{\text{5 marks}}$ Solve the following LP problem using the revised simplex method.

$$\begin{array}{lrrcl}
\text{Maximize} & 2x_1 & +3x_2 & & \\
\text{subject to} & x_1 & +2x_2 & \leq & 6 \\
& 2x_1 & +x_2 & \leq & 8 \\
& -x_1 & +x_2 & \leq & 1 \\
& & 2x_2 & \leq & 1 \\
& x_1, & x_2, & \geq & 0.
\end{array}$$

Hint: requires ≤ 3 iterations.

3. $\boxed{\text{5 marks}}$ Your colleague is working on the following LP problem:

$$\begin{array}{lrrrcl}
\text{Maximize } z = & -x_1 & +x_2 & +2x_3 & & \\
\text{subject to} & x_1 & +2x_2 & -x_3 & \leq & 2 \\
& 2x_1 & +3x_2 & +x_3 & \leq & 6 \\
& -2x_1 & +4x_2 & +2x_3 & \leq & 8 \\
& x_1, & x_2, & x_3 & \geq & 0.
\end{array}$$

He tells you that he has found an optimal solution when x_2, x_5, x_6 are non-basic and x_1, x_3, x_4 are basic.

(a) Write down the original dictionary and the dictionary that is thought to be optimal: Give $\mathbf{B}, \mathbf{N}, \vec{c_N}^T, \vec{c}_N^T, \vec{x}_B, \vec{x}_N, \vec{b}$, and remember to write in the basis headings (that is, label the columns of matrices with the corresponding variables).

(b) What are the values of the basic and non-basic variables in your colleague's dictionary? Is this a feasible solution?

(c) Is your colleague's dictionary optimal?

(d) What is the optimal value?

4. $\boxed{\text{5 marks}}$ [This is continuation of the problem 3] Your colleague is working on the following LP problem:

$$\begin{array}{lrrrcl} \text{Maximise } z = & -x_1 & +x_2 & +2x_3 & & \\ \text{subject to} & x_1 & +2x_2 & -x_3 & \leq & 2 \\ & 2x_1 & +3x_2 & +x_3 & \leq & 6 \\ & -2x_1 & +4x_2 & +2x_3 & \leq & 8 \\ & x_1, & x_2, & x_3 & \geq & 0. \end{array}$$

He tells you that he has found an optimal solution when x_2, x_5, x_6 are non-basic and x_1, x_3, x_4 are basic. **[You can use the results of Problem 3.]**

(a) Your colleague then decides to change the third constraint to

$$-2x_1 + 4x_2 + 2x_3 \leq 8 + \beta$$

Over what range of β is your colleague's dictionary feasible and optimal. What is the corresponding solution + optimal value?

(b) Your (increasingly indecisive) colleague then decides to leave the third constraint as it was originally, but instead changes the objective function to

$$z = (\gamma - 1)x_1 + x_2 + 2x_3$$

Over what range of γ is the dictionary feasible and optimal. What is the corresponding solution and optimal value?

5. $\boxed{\text{5 marks}}$ Consider the following LP problem in which the objective function depends on a parameter γ:

$$\max\ (4+\gamma)x_1 + 5x_2$$

$$\text{such that}\quad \begin{array}{rrcl} x_1 & +x_2 & \leq & 5 \\ x_1 & +2x_2 & \leq & 8 \\ 2x_1 & +x_2 & \leq & 8 \\ x_1, & x_2 & \geq & 0 \end{array}$$

When $-\frac{3}{2} \leq \gamma \leq 1$ this has optimal dictionary

$$\begin{array}{rcrrr} x_1 & = & 2 & -2x_3 & +x_4 \\ x_2 & = & 3 & +x_3 & -x_4 \\ x_5 & = & 1 & +3x_3 & -x_4 \\ \hline z & = & (23+2\gamma) & -(3+2\gamma)x_3 & +(\gamma-1)x_4 \end{array}$$

<u>Starting from this dictionary</u> find the optimal solution and the optimal value as a function of γ,

(a) when $-\frac{3}{2} \leq \gamma \leq 1$,

(b) when $-\infty \leq \gamma \leq -\frac{3}{2}$,

(c) when $1 \leq \gamma \leq 6$ and

(d) when $6 \leq \gamma \leq \infty$.

Do not solve the problem from scratch each time. You will find it easier to do this problem using dictionaries rather than the revised simplex method.

6. $\boxed{\text{5 marks}}$ Consider the following LP problem in which one of the constraints bounds depends on a parameter β:

$$\max\ 4x_1 + 5x_2$$

$$\text{such that} \begin{array}{llcr} x_1 & +x_2 & \leq & 5 \\ x_1 & +2x_2 & \leq & 8+\beta \\ 2x_1 & +x_2 & \leq & 8 \\ x_1, & x_2 & \geq 0 & \end{array}$$

When $-1 \leq \beta \leq 2$ the optimal dictionary is

$$\begin{array}{ccrrr} x_1 & = & 2-\beta & -2x_3 & +x_4 \\ x_2 & = & 3+\beta & +x_3 & -x_4 \\ x_5 & = & 1+\beta & +3x_3 & -x_4 \\ \hline z & = & 23+\beta & -3x_3 & -x_4 \end{array}$$

Starting from this dictionary find the optimal solution and the optimal value as a function of β,

(a) when $-1 \leq \beta \leq 2$,

(b) when $2 \leq \beta \leq \infty$,

(c) when $-4 \leq \beta \leq -1$,

(d) and when $-8 \leq \beta \leq -4$.

(e) What happens to this problem when $\beta < -8$?

Do not solve the problem from scratch each time. You will find it easier to do this problem using dictionaries and the dual-simplex method.