2019 Math 340:101. Quiz 3 Friday, 2019 October 4 IN CLASS

Last name . . . .

First name . . . .

Student number . . . .

Grade

1. (2 points) Consider the problem:

$$\begin{array}{rrrrl} \text{Maximize} & x_1 & +2x_2 & +3x_3 & \\ \text{subject to} & x_1 & & +x_3 & \leq 11 \\ & x_1 & -x_2 & & \leq 12 \\ & 3x_1 & & -x_3 & \leq 13 \end{array} \qquad x_1, x_2, x_3 \geq 0.$$

Write down its **dual** problem. (Do not solve it.)

Solution:

$$\text{Dual:} \quad \begin{array}{rrrrl} \text{Minimize} & 11y_1 & +12y_2 & +13y_3 & \\ \text{subject to} & y_1 & +y_2 & +3y_3 & \geq 1 \\ & & -y_2 & & \geq 2 \\ & y_1 & & -y_3 & \geq 3 \end{array} \qquad y_1, y_2, y_3 \geq 0$$

2. (4 points) **True/False** question. **Read carefully.** Circle the right choice. (1 mark for every 2 correct answers.)

   a. **True** / **False**. At a degenerate feasible dictionary, which is not the final dictionary, the next dictionary in the simplex iteration must be a different dictionary.

   b. **True** / **False**. At a degenerate feasible dictionary, which is not the final dictionary, the next dictionary in the simplex iteration must have the same basic solution.

   c. **True** / **False**. When a simplex algorithm left a corner point P_1 and moved to a different corner point P_2, then it might still be possible for it to come back to the corner point P_1 afterwards.

   d. **True** / **False**. If a standard form LP problem has no optimal solution, then it is either infeasible or unbounded.

   e. **True** / **False**. There are LP problems that have exactly two optimal solutions.

   f. **True** / **False**. For a given vector $\vec{a} \in \mathbf{R}^n$ with $\vec{a} \neq \vec{0}$, it must be true that

   $$\max_{\vec{x} \in \mathbf{R}^d} \vec{a} \cdot \vec{x} = +\infty.$$

   That is, there is no maximum and by choosing $\vec{x}$, the value $\vec{a} \cdot \vec{x}$ can be as large as possible.

   g. **True** / **False**. The function $f : \mathbf{R} \to \mathbf{R}$ given by $f(x) = \max[3x+1, 2x]$ is a convex function.

   h. **True** / **False**. If the primal problem is unbounded, then its dual problem is infeasible.

   a. True

   b. False

c. False

d. True

e. False

f. True

g. True

h. True

3. (4 points) Does there exist (x_1, x_2, x_3) that satisfies all of the following inequalities? Justify your answer.

$$\begin{array}{rrrrr} x_1 & +x_2 & +x_3 & \leq & 1 \\ 2x_1 & +2x_2 & -x_3 & \leq & -2 \\ -x_1 & +x_2 & +x_3 & \leq & -1 \end{array}$$
$$x_1, x_2, x_3 \geq 0$$

Solution. Use Phase 1 of the two phase method.

The initial dictionary of the auxiliary LP:

$$\begin{array}{rcrrrrr} x_4 & = & 1 & -x_1 & -x_2 & -x_3 & +x_0 \\ x_5 & = & -2 & -2x_1 & -2x_2 & +x_3 & +x_0 \\ x_6 & = & -1 & +x_1 & -x_2 & -x_3 & +x_0 \\ w & = & & & & & -x_0 \end{array}$$

Special pivot to feasibility: x_0 enters the basis, x_5 leaves.

$$\begin{array}{rcrrrrr} x_4 & = & 3 & +x_1 & +x_2 & -2x_3 & +x_5 \\ x_0 & = & 2 & +2x_1 & +2x_2 & -x_3 & +x_5 \\ x_6 & = & 1 & +3x_1 & +x_2 & -2x_3 & +x_5 \\ w & = & -2 & -2x_1 & -2x_2 & +x_3 & -x_5 \end{array}$$

Pivot: x_3 enters the basis, x_6 leaves.

$$\begin{array}{rcrrrrr} x_4 & = & 2 & -2x_1 & & & +x_6 \\ x_0 & = & 3/2 & +1/2x_1 & +3/2x_2 & +1/2x_5 & +1/2x_6 \\ x_3 & = & 1/2 & +3/2x_1 & +1/2x_2 & +1/2x_5 & -1/2x_6 \\ w & = & -3/2 & -1/2x_1 & -3/2x_2 & -1/2x_5 & -1/2x_6 \end{array}$$

This is an optimal dictionary, but the maximum value of w is $-3/2$, nonzero. This implies that the original inequalities do not have a solution.

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