Math 340 Practice Midterm Time 45min

This is a closed-book examination. ONLY pen/pencil/eraser are allowed.

1. (4 points) You are given an LP problem and are also given a feasible solution to it and a feasible solution to its dual. Explain (referring to relevant theorems) why both problems must have optimal solutions.

2. A colleague hands you the following LP problem

$$
\begin{array}{lrrrrcr}
\text{Maximize} & 9x_1 & +6x_2 & +x_3 & +9x_4 & & \\
\text{Subject to} & 2x_1 & +x_2 & +x_3 & +x_4 & \leq & 8 \\
 & 2x_1 & +x_2 & & +4x_4 & \leq & 12 \\
 & 10x_1 & +7x_2 & +4x_3 & -2x_4 & \leq & 45 \\
 & x_1, & x_2, & x_3, & x_4 & \geq & 0
\end{array}
$$

and tells you that they used the simplex method to get to the following dictionary:

$$
\begin{array}{rclrrrr}
x_2 & = & \frac{20}{3} & -2x_1 & -\frac{4}{3}x_3 & -\frac{4}{3}x_5 & +\frac{1}{3}x_6 \\
x_4 & = & \frac{4}{3} & +\frac{1}{3}x_3 & +\frac{1}{3}x_5 & -\frac{1}{3}x_6 & \\
x_7 & = & 1 & +4x_1 & +6x_3 & +10x_5 & -3x_6 \\
\hline
z & = & 52 & -3x_1 & -6x_3 & -5x_5 & -x_6
\end{array}
$$

(a) (2 points) Find the optimal solution.

(b) (3 points) Write down the dual of the original problem.

(c) (2 points) Find the optimal solution to the dual problem.

3. (6 points) Use the two-phase simplex method (and Anstee's rule) to solve the following linear programming problem:

$$\begin{array}{lrrcr} \text{Maximize} & 5x_1 & +3x_2 & & \\ \text{Subject to} & 2x_1 & +4x_2 & \leq & 4 \\ & -x_1 & -x_2 & \leq & -6 \\ & x_1, & x_2 & \geq 0 & \end{array}$$

Extra work space for Question 3

4. (8 points) Check whether each of the following statements is correct or not. Circle True or False, and you do not need to explain your answer. Each correct answer will earn 1 mark.

   (a) i. **True / False.** If a standard form linear programming problem is feasible but unbounded, then its dual problem is infeasible.

      ii. **True / False.** Given a linear programming problem, it is possible to have no optimal solution, while there is an optimal solution to its dual problem.

      iii. **True / False.** There is a linear programming problem for which both primal and dual problems are not feasible simulataneously.

      iv. **True / False.** For each vector $\vec{y} \in \mathbb{R}^n$, it holds that $\max_{\vec{x} \in \mathbb{R}^n} [\vec{y} \cdot \vec{x}] = +\infty$.

   (b) **(Read carefully.)** Suppose that Prof. Anstee is following his rule to perform the simplex method to solve an LP problem. At a certain step he gets a feasible dictionary D_1, and by continuing iterations (”pivotings”), he gets subsequent dictionaries D_2, D_3, D_4, D_5, D_6, and D_7.

      i. **True** / **False**. The dictionaries $D_2, D_3, D_4, D_5, D_6, D_7$ must be feasible.

      ii. **True** / **False**. In some cases, it is possible to have an optimal basic solution to D_1 but, non-optimal basic solution to D_3.

      iii. **True** / **False**. If D_7 is identical to D_1, then this LP problem has no optimal solution.

      iv. **True** / **False**. If D_7 is not identical to D_1, then the dictionaries D_1, D_2, D_3, D_4, D_5, D_6, and D_7 all have different basic solutions from each other.

5. A colleague hands you the following LP problem

$$
\begin{array}{lrrrcr}
\text{Maximize } z = & 2x_1 & +x_2 & +2x_3 & & \\
\text{Subject to} & x_1 & +x_2 & +2x_3 & \leq & 6 \\
 & 3x_1 & +6x_2 & +3x_3 & \leq & 18 \\
 & 4x_1 & +2x_2 & +2x_3 & \leq & 12 \\
 & x_1, & x_2, & x_3 & \geq & 0
\end{array}
$$

and tells you that they found the following optimal solution.

$$x_1^* = 2 \qquad x_2^* = 0 \qquad x_3^* = 2 \qquad z = 8$$

(a) (2 points) Check that their solution is feasible.

(b) (6 points) Check that their solution is optimal using complementary slackness.

Extra Page 1

Extra Page 2

Extra Page 3