Correction on Clicker question for Lecture 25

Given the LP problem $\max_{A\vec{x}\leq\vec{b},\vec{x}\geq\vec{0}} \vec{c}\cdot\vec{x}$, consider its primal and dual dictionaries determined by $B, N, \vec{b}, \vec{c}$:

$$\begin{array}{lrcll} \textit{primal} & z & = & \vec{c}_B^T B^{-1}\vec{b} & +[\vec{c}_N^T - \vec{c}_B^T B^{-1} N]\vec{x}_N \\ \hline & \vec{x}_B & = & B^{-1}\vec{b} & -B^{-1}N\vec{x}_N \end{array}$$

$$\begin{array}{lrcll} \textit{dual} & -w & = & -\vec{c}_B^T B^{-1}\vec{b} & -\left(B^{-1}\vec{b}\right)^T \vec{y}_B \\ \hline & \vec{y}_N & = & -[\vec{c}_N^T - \vec{c}_B^T B^{-1} N]^T & +\left(B^{-1}N\right)^T \vec{y}_B \end{array}$$

Let $\vec{x}^*, \vec{y}^*$ be the corresponding **primal/dual basic solutions**, respectively. Find a wrong statement:

A. If $\vec{x}^*$, $\cancel{\vec{y}^*}$ ~~are~~ is primal/~~dual~~ optimal, ~~respectively~~, then $\vec{x}_B^* \geq 0$ and $\vec{y}_N^* \geq 0$. **[I didn't mean to include $\vec{y}^*$ in this sentence. After this correction, A. is the answer.]**

B. If $\vec{x}_B^* \geq 0$ and $\vec{y}_N^* \geq 0$, then $\vec{x}^*, \vec{y}^*$ are primal/dual optimal, respectively.

C. $\vec{x}_N^* = \vec{0}$, and $\vec{y}_B^* = \vec{0}$.

D. One of A, B, and C is wrong. **[In the original problem before the correction, D was the answer.]**

Comparison with strong duality theorem

primal *dual*

$$\begin{array}{rcll} z & = & \vec{c}_B^T \vec{x}_B^* & -(\vec{y}_N^*)^T \vec{x}_N \\ \hline \vec{x}_B & = & \vec{x}_B^* & -B^{-1} N \vec{x}_N \end{array} \qquad \begin{array}{rcll} -w & = & -(\vec{y}^*)^T \vec{b} & -\left(\vec{x}_B^*\right)^T \vec{y}_B \\ \hline \vec{y}_N & = & \vec{y}_N^* & +\left(B^{-1} N\right)^T \vec{y}_B \end{array}.$$

- At the feasible and dual feasible dictionary (that is, $\vec{x}_B^* \geq \vec{0}, \vec{y}_N^* \geq \vec{0}$), the corresponding primal/dual basic solutions $\vec{x}^*, \vec{y}^*$ are primal/dual optimal, respectively.
- Optimality of the primal basic solution $\vec{x}^*$ **does not necessarily** imply dual optimality of the dual basic solution $\vec{y}^*$ as it may not be dual feasible (it may happen $\vec{y}_N^* \not\geq \vec{0}$).
- **Strong duality** says if there is a primal optimal solution then there is a dual optimal solution.
  - It **does not mean** that for an optimal basic solution, the corresponding dual basic solution is dual optimal. The corresponding dual basic solution may not be feasible, **unless** the dictionary is **final**, that is, both primal/dual feasible.
  - In the strong duality theorem, you can read off the dual optimal solution from the dictionary, **only when the dictionary is final, where the corresponding dual basic solution has $\vec{y}_N^* \geq \vec{0}$.**