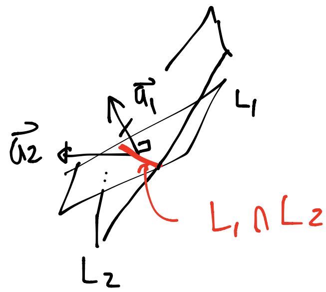
• Intersections of hyperplanes

e.g.

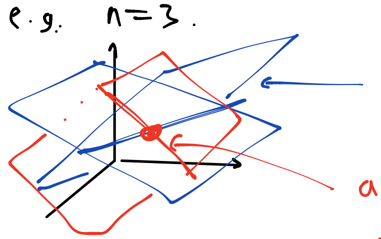
$\mathbb{R}^n$




$\dim(L_1 \cap L_2) = n-2$

if $\vec{a}_1$ & $\vec{a}_2$ are linearly independent.

e.g. $n=3$.



two planes meet to form a line

a third plane with "linearly independent" direction intersects to give a point.

In general

**Thm** Let $\vec{a}_1, \vec{a}_2, \cdots \vec{a}_m \in \mathbb{R}^n$, $b_1, \cdots, b_m \in \mathbb{R}$

n-dim'l vector space in general.

Let $n \geq m$.

Define hyperplanes: $L_k = \{\vec{x} \in \mathbb{R}^n \mid \vec{a}_k \cdot \vec{x} = b_k\}$

Suppose $\vec{a}_1, \cdots, \vec{a}_m$ are linearly independent.

(i.e. $y_1\vec{a}_1 + y_2\vec{a}_2 + \cdots + y_m\vec{a}_m = \vec{0}$ $\leftarrow$ $\begin{bmatrix}0\\0\\\vdots\\0\end{bmatrix}$ implies $y_1 = y_2 = \cdots = y_m = 0$ )

Then the intersection $L_1 \cap L_2 \cap \cdots \cap L_m$

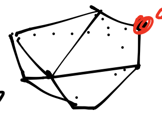
i.e. $\{\vec{x} \in \mathbb{R}^n \mid \vec{a}_k \cdot \vec{x}_k = b_k \ \forall k = 1, 2, \cdots, m\}$

is an $(n-m)$-dimensional subspace in $\mathbb{R}^n$.

So, in $\mathbb{R}^n$, $n$ linearly independent hyperplanes intersect to a point. ← 0-dimensional space

the same

e.g. $\mathbb{R}^n$



for a vertex point of this polyhedron there are $n$ linearly independent hyperplanes

out of those $\vec{a}_1 \cdot \vec{x} = b_1, \ldots, \vec{a}_\ell \cdot \vec{x}_\ell = b_\ell$

(some hyperplanes do not pass through at that point)

a polytope given by the set

$\{\vec{x} \in \mathbb{R}^n \mid \vec{a}_1 \cdot \vec{x}_1 \leq b_1, \ldots, \vec{a}_\ell \cdot \vec{x}_\ell \leq b_\ell\}$