**2019 Math 340:101. Quiz 1** Friday, 2019 September 13 IN CLASS

Last name ...............................

First name ..............................

Student number .........................

**Grade**

1. (3 points) Put the following linear programming problem in standard form, that is, standard inequality form. (Do not solve it.)

$$\begin{array}{rl} \text{minimize} & x_1 - 3x_2 \\ \text{subject to} & x_1 + x_2 = 2 \\ & x_1 \geq 3 \\ & x_2 \text{ unconstrained} \end{array} .$$

"minimize $x_1 - 3x_2$" $\Leftrightarrow$ "maximize $-x_1 + 3x_2$". "$x_1 + x_2 = 2$" $\Leftrightarrow$ "$x_1 + x_2 \leq 2$ and $-x_1 - x_2 \leq -2$". Also, let $x_2 = x_2' - x_2''$, $x_2', x_2'' \geq 0$ and $x_1 = x_1' + 3$. Then, the objective function becomes $-(x_1' - 3x_2' + 3x_2'' + 3) = -x_1' + 3x_2' - 3x_2'' - 3$ but $-3$ can be dropped.

$$\begin{array}{rl} \text{maximize} & -x_1' + 3x_2' - 3x_2'' \\ \text{subject to} & x_1' + x_2' - x_2'' \leq -1 \\ & -x_1' + x_2' - x_2'' \leq 1 \\ & x_1', x_2', x_2'' \geq 0 \end{array} .$$

2. (3 points) Let $S_1$ and $S_2$ are two convex sets in $\mathbb{R}^n$. Prove that their intersection $S_1 \cap S_2$ is again a convex set.

Let $x_1, x_2$ be two points in $S_1 \cap S_2$. Then, $x_1, x_2 \in S_1$. By convexity of $S_1$, $(1-t)x_1 + tx_2 \in S_1$ for any $t \in [0, 1]$. Similarly, $x_1, x_2 \in S_2$. By convexity of $S_2$, $(1-t)x_1 + tx_2 \in S_2$ for any $t \in [0, 1]$. Therefore, for any $t \in [0, 1]$, we see that $(1-t)x_1 + tx_2 \in S_1 \cap S_2$. This means that $S_1 \cap S_2$ is convex.

3. (4 points) The following dictionary is obtained from solving a standard form LP problem with the simplex method. Find the next dictionary in the simplex method.

$$\begin{array}{rcrrr} x_2 & = & 12 & +x_1 & -x_4 \\ x_3 & = & 5 & +x_1 & -x_4 \\ x_5 & = & 4 & +x_1 & -x_4 \\ \hline z & = & 7 & -x_1 & +2x_4 \end{array}$$

The current basic feasible solution is

$$x_1 = x_4 = 0 \qquad x_2 = 2 \qquad x_3 = 5 \qquad x_5 = 4.$$

We only have 1 choice of entering variable: $x_4$. It is constrained to be at most 12, 5, or 4, so $x_5$ is the leaving variable. From the corresonding row, that is $x_5 = 4 + x_1 - x_4$, we get $x_4 = 4 + x_1 - x_5$. We use this and get the next dictionary as follows:

$$\begin{array}{rcrrr} x_2 & = & 8 & & +x_5 \\ x_3 & = & 1 & & +x_5 \\ x_4 & = & 4 & +x_1 & -x_5 \\ \hline z & = & 15 & +x_1 & -2x_5 \end{array}$$

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