- You will *not* be able to use a calculator or computer for either the midterm or the final exam, so please do not use one for this assignment. You may use one to *check* your answer, but please do not use one to solve the problem.
- Only part of the problems may be graded. But, you have to submit all the problems.
- The deadline is *by 6pm on Friday*, Nov. 22. You have a grace period until 9am in the morning the next day. The grace period is to take care of any technical issues you have while submitting the file. The grace period should give ample time for handling any issues, so No late HW after the grace period will be accepted, **regardless** your technical issues.
- Submit only pdf files.

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1. $\boxed{\text{5 marks}}$ Consider the following LP problem.

$$\begin{array}{llrrl} \text{Minimise} & w = & 3x_1 & +2x_2 & \\ \text{Subject to} & & -x_1 & +2x_2 & \geq 4 \\ & & 2x_1 & -3x_2 & \geq 6 \\ & & x_1, & x_2 & \geq 0 \end{array}$$

   (a) Rewrite the above LP in standard form (*do not* give the dual of this problem).

   (b) What is dictionary corresponding to the LP in your answer to part (a)?

   (c) Your answer to part (b) should be a dictionary that is not feasible, but is dual-feasible. Use the dual-simplex method to find the optimal dictionary and so solve the original LP problem. (Remember to choose the most negative basic variable to be the leaving variable.)

2. $\boxed{\text{5 marks}}$ There are six independent questions below. What can you say about the dual if you already know: (answer for each of the following considered individually)

   a) a feasible solution to the primal exists?

   b) an optimal solution to the primal exists?

   c) an optimal solution to the primal exists with $x_1 > 0$?

   d) an optimal solution to the primal exists with $x_1 = 0$?

   e) there is no feasible solution to the primal?

   f) the primal is unbounded?

3. $\boxed{\text{5 marks}}$ Our famous game theory protagonists, Claude and Trucula, get sick of playing Morra and decide on a simpler game a bit like "rock-paper-scissors". Trucula being a gentleman, but not too bright, agrees to the following set of rules proposed by Claude:

- Trucula plays one of “rock” or “paper”.
- Claude has a coin and can choose one of “heads” or “tails”
- The payoff matrix (to Trucula) is

$$\mathbf{A} = \begin{array}{c c} & \begin{array}{cc} H & T \end{array} \\ \begin{array}{c} R \\ P \end{array} & \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} \end{array}$$

(a) What is Trucula’s optimal strategy?

(b) Verify that Claude can use a similar optimal strategy to that of Trucula — ie Claude plays “heads” with the same probability that Trucula plays “rock” and that Claude plays “tails” with the same probability that Trucula plays “paper”.

(c) How can the payoff matrix be altered so that the game is fair?

4. $\boxed{\text{5 marks}}$ Let $A = (a_{ij})$ be an $m \times n$ payoff matrix of a two person zero sum game.

a) Assume that the average entry in column j of A is at least 5 for each $j = 1, 2, \ldots, n$. Show that the assured (expected) winnings for the row player (player 1) is at least 5.

b) Find an example such that the average entry in row i of A is at least 5 for each $i = 1, 2, .., m$ and yet the game is fair.

5. [This problem is similar to Chvátal, page 237, Problem 15.2.]

(a) $\boxed{\text{5 marks}}$ Consider a ‘battleship’ type game played on a 4×4 board. Player 1 secretly chooses a location for a domino (there are 24 possibilities but that is not so crucial to answering this question). Player 2 secretly chooses a position (among the 16 different possibilities). Player 1 wins \$1 (and player 2 loses \$1) if the domino does not occupy a position chosen by player 2 else player 2 wins \$1 (and player 1 loses \$1). One would quess that the value of the game for Player 1 is $12/16 = 3/4$. Give a proof of this fact. Explicitly considering the 24×16 payoff matrix would probably be unproductive but you can use properties of the payoff matrix. **Here, the value of the game of Player 1 is the maximum possible outcome gain that Player 1 is to be able to make no matter what Player 2 does. It is also defined in Chvatal p233 or Vanderbei p155.**

(b) $\boxed{\text{5 marks}}$ Now for the same game, change the board to be an $N \times N$ board, where N is a large integer. What happens to the value of the game as $N \to \infty$? Justify your answer.