

Practice midterm questions — do not hand in

1. Use the two-phase method to solve the following LP problem

$$\begin{array}{llllll}
 \text{Maximise} & x_1 & +3x_2 & -6x_3 & & \\
 \text{Subject to} & x_1 & -x_2 & -x_3 & \leq & 2 \\
 & -x_1 & & +x_3 & \leq & -1 \\
 & & x_2 & -x_3 & \leq & 2 \\
 & x_1, & x_2 & & \geq & 0
 \end{array}$$

2. Consider the following LP problem:

$$\begin{array}{llllll}
 \text{Maximise} & 7x_1 & +5x_2 & +2x_3 & & \\
 \text{Subject to} & x_1 & +x_2 & +x_3 & \leq & 5 \\
 & x_1 & +3x_2 & +x_3 & \leq & 10 \\
 & 3x_1 & +x_2 & +x_3 & \leq & 6 \\
 & x_1, & x_2, & x_3 & \geq & 0
 \end{array}$$

Use complementary slackness to see if $(x_1^*, x_2^*, x_3^*) = (1, 3, 0)$ is an optimal solution.

3. (a) State the fundamental theorem of linear programming.
 (b) Explain (referring to relevant theorems) why if a primal problem is unbounded then the corresponding dual problem must be infeasible.
4. At Café Sunfrancs a cappuccino is made from one shot of espresso, three ounces of milk, and six ounces of foam. A latté is made from one shot of espresso, seven ounces of milk, and two ounces of foam. A café sells only cappuccinos and lattés, and makes one dollar profit on each drink it sells. Today the café has materials to produce 50 shots of espresso, 20 ounces of milk, and 30 ounces of foam.

Write down a linear program to maximize the profit the café will make. Write down the dual LP. For all variables involved (objective, decision, and slack, both in the primal and dual), state in what units they are given.

Extra practice questions — do not hand in

- Question 5.2 from Chvátal
- Question 5.3 from Chvátal
- Question 5.4 from Chvátal