SELF-STUDY Material (will be on Quiz 1).
: See also Anstee's online notes.

Standard form of LP

We will focus LP of the following form.

$$\text{maximize} \quad c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$\text{subject to} \quad * \begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\ a_{12}x_1 + a_{12}x_2 + \cdots + a_{2n}x_n \leq b_2 \\ \vdots \\ a_{m1}x_1 + a_{n2}x_2 + \cdots + a_{mn}x_n \leq b_m \\ x_1, x_2, \cdots, x_n \geq 0 \end{cases}$$

n variables

$\leftarrow m + n$ constraints

e.g. maximize $x_1 + x_2 + x_3$

subject to

$$x_1 + 3x_2 + x_3 \leq 1$$

$$x_1 - x_2 + x_3 \leq 2$$

$$x_1, x_2, x_3 \geq 0.$$

• Nonstandard form LP can be reduced to the standard form.

EX. min $3x_1 - 2x_2 + x_3 + 1$

subject to

$$-x_1 + x_2 \geq -3$$

$$2x_1 + x_2 \leq 2$$

$$x_1 + x_2 + x_3 = 4$$

$$x_1 \geq -2, \quad x_2 \leq 2$$

principles to use	changes
• $\min f = -\max(-f)$	$\max -3x_1+2x_2-x_3-1$
• $\max(f+\text{const}) \Longleftrightarrow \max f$	$\max -3x_1+2x_2-x_3$
• $x \geq a \Longleftrightarrow -x \leq -a$	$x_1-x_2 \leq 3$
• $x=a \Longleftrightarrow x\leq a$ & $-x \leq -a$	$x_1+x_2+x_3 \leq 4$, $-x_1-x_2-x_3 \leq -4$
• $x \geq a \Longleftrightarrow x-a \geq 0$	Replace x_1 with $x_1'-2$ then $x_1 \geq -2 \Leftrightarrow x_1' \geq 0$ Replace x_2 with $2-x_2'$ Then $x_2 \leq 2 \Leftrightarrow x_2' \geq 0$
• no restriction on x $\Longleftrightarrow x = x^+ - x^-$; $x^+, x^- \geq 0$	Replace x_3 with $x_3^+ - x_3^-$ & $x_3^+, x_3^- \geq 0$

$$\begin{aligned} -3x_1+2x_2-x_3 &= -3(x_1'-2)+2(2-x_2')-x_3^+ + x_3^- \\ &= -3x_1' - 2x_2' - x_3^+ + x_3^- + 10 \end{aligned}$$

can drop

Resulting standard form.

Note we dropped $+10$ from the objective function

$$\boxed{\begin{aligned} \max\ & -3x_1' - 2x_2' - x_3^+ + x_3^- \\ \text{subject to}\ & x_1' + x_2' \leq 7 \\ & 2x_1' - x_2' \leq 4 \\ & x_1' - x_2' + x_3^+ - x_3^- \leq 4 \\ & -x_1' + x_2' - x_3^+ + x_3^- \leq -4 \\ & x_1', x_2', x_3^+, x_3^- \geq 0 \end{aligned}}$$

$\leftarrow (x_1'-2) - (2-x_2') \leq 3$

$\leftarrow 2(x_1'-2) + (2-x_2') \leq 2$

$\leftarrow (x_1'-2)+(2-x_2') + x_3^+ - x_3^- \leq 4$

$-[(x_1'-2)+(2-x_2')+x_3^+ - x_3^-] \leq -4$

After finding optimal solution to this we get the optimal solution to the original problem, i.e. the values of (x_1, x_2, x_3) at the maximum

by using
$$x_1 = x_1' - 2$$
$$x_2 = 2 - x_2'$$
$$x_3 = x_3^+ - x_3^-$$

and

$$\min(3x_1 - 2x_2 + x_3 + 1) = -\max(-3x_1 + 2x_2 - x_3 - 1)$$
$$= 1 - \max(-3x_1 + 2x_2 - x_3)$$

$\blacksquare$