Lecture 6-self.

- Self-study material

Def A dictionary is feasible if the corresponding basic solution is feasible

e.g.

$$
\begin{aligned}
x_3 &= \boxed{+5} + x_1 \\
x_4 &= \boxed{1} \quad - x_2 \\
\hline
z &= x_1 + x_2
\end{aligned}
$$

$\geq 0.$

EX. Maximize $2x_2 + x_3$

subject to

$$
\begin{aligned}
x_1 - x_2 + x_3 &\leq \boxed{5} \\
-2x_1 + x_2 &\leq \boxed{3} \\
x_2 - 2x_3 &\leq \boxed{5} \\
x_1, x_2, x_3 &\geq 0.
\end{aligned}
$$

<sol>.

Initial dictionary:

$$
\begin{aligned}
x_4 &= \boxed{5} - x_1 + x_2 - x_3 \\
x_5 &= \boxed{3} + 2x_1 - x_2 \\
x_6 &= \boxed{5} \quad - x_2 + 2x_3 \\
\hline
z &= \quad 2x_2 + x_3
\end{aligned}
$$

The constants on the righthand side of the dictionary

Then the dictionary becomes a feasible dictionary

Exercise Finish this example by applying the simplex method and Anstee's rule.

$$x_4 = 5 - x_1 + x_2 - x_3$$
$$x_5 = 3 + 2x_1 - x_2$$
$$x_6 = 5 - x_2 + 2x_3$$

$$z = 0 \cdot x_1 + 2x_2 + x_3$$

non basic variables x_1, x_2, x_3

basic variables x_4, x_5, x_6

choose as an entering variable (following Anstee's rule) (x_3 can also be chosen in principle)

while $x_1 = 0$ & $x_3 = 0$

$x_4 = 0 \Rightarrow x_2 = -5$ not feasible so ignore

(In other words, x_4 is never zero as x_2 increases from 0.

leaving $x_5 = 0 \Rightarrow \boxed{x_2 = 3}$ the most strong restriction

$x_6 = 0 \Rightarrow x_2 = 5$

entering x_2, leaving x_5.

update the dictionary.

$x_5 = 3 + 2x_1 - x_2 \longrightarrow x_2 = 3 + 2x_1 - x_5$

$z = 2x_2 + x_3 = 2(3 + 2x_1 - x_5) + x_3 = 6 + \underbrace{4x_1 + x_3 - 2x_5}_{\text{not yet a good form.}}$

$x_4 = 5 - x_1 + x_2 - x_3 = 5 - x_1 + 3 + 2x_1 - x_5 - x_3$
$= 8 + x_1 - x_3 - x_5$

$x_6 = 5 - x_2 + 2x_3 = 5 - (3 + 2x_1 - x_5) + 2x_3$
$= 2 - 2x_1 + 2x_3 + x_5$

updated dictionary

$$x_4 = 8 + x_1 \quad - x_3 - x_5$$
$$x_2 = 3 + 2x_1 \quad\quad - x_5$$

gives no restriction on $x_1 \geq 0$

$$\boxed{x_6 = 2 - 2x_1 + 2x_3 + x_5}$$

gives restriction $x_1 \leq 1$.

leaving

$$z = 6 + 4x_1 + x_3 - 2x_5$$

choose for entering. (following Anstee's rule)

entering x_1, leaving x_6

Update the dictionary

leaving $x_6 = 2 - 2x_1 + 2x_3 + x_5 \longrightarrow x_1 = 1 + x_3 + \frac{1}{2}x_5 - \frac{1}{2}x_6$

entering

$$z = 6 + 4x_1 + x_3 - 2x_5 = 10 + 5x_3 \quad - 2x_6$$

$$\boxed{1 + x_3 + \tfrac{1}{2}x_5 - \tfrac{1}{2}x_6}$$

$$x_4 = 8 + (1 + x_3 + \tfrac{1}{2}x_5 - \tfrac{1}{2}x_6) - x_3 - x_5$$
$$= 9 \quad - \tfrac{1}{2}x_5 - \tfrac{1}{2}x_6$$

$$x_2 = 3 + 2(1 + x_3 + \tfrac{1}{2}x_5 - \tfrac{1}{2}x_6) - x_5$$
$$= 5 + 2x_3 \quad - x_6$$

Get

$$x_4 = 9 \quad\quad - \tfrac{1}{2}x_5 - \tfrac{1}{2}x_6$$
$$x_2 = 5 + 2x_3 - x_6$$
$$x_1 = 1 + x_3 + \tfrac{1}{2}x_5 - \tfrac{1}{2}x_6$$
$$z = 10 + \boxed{5x_3} - 2x_6$$

Basic variables
x_4, x_2, x_1

non-basic Variables
x_3, x_5, x_6.

Entering x_3.

While $x_5=0$, $x_6=0$,

$x_4 \geq 0 \rightarrow$ no restriction on x_3.

$x_2 \geq 0 \rightarrow$ no restriction on x_3 $\Rightarrow$ no restriction on x_3

$x_1 \geq 0 \rightarrow$ " " " but $x_3 \geq 0$.

No basic variable is leaving.

Let $x_3 = t \geq 0$.

& $x_5 = x_6 = 0$.

Then $x_1 = 1+t$, $x_2 = 5+2t$, $x_3 = 0$, $x_4 = 9$, $x_5 = 0$, $x_6 = 0$

is feasible for any $t \geq 0$!

Then $z = 10 + 5t$ is a possible value for any $t \geq 0$

As $t \to \infty$, $z \to \infty$,

So, the value of z is unbounded.

and no maximum exists. $\blacksquare$