- You will *not* be able to use a calculator or computer for either the midterm or the final exam, so please do not use one for this assignment. You may use one to *check* your answer, but please do not use one to solve the problem.
- Only part of the problems may be graded. But, you have to submit all the problems.
- The deadline is *by 6pm on Friday*, Nov. 1. You have a grace period until 9am the next day. The grace period is to take care of any technical issues you have while submitting the file. The grace period should give ample time for handling any issues, so No late HW after the grace period will be accepted, **regardless** your technical issues.
- Submit only pdf files.

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1. $\boxed{\text{5 marks}}$ Find all the solutions of the following LP problem:

$$\begin{array}{rllll} \text{maximise } z = 3x_1 + x_2 + 0x_3 & & & & \\ & x_1 & +2x_2 & & \leq 5 \\ \text{subject to} & x_1 & +x_2 & -x_3 & \leq 2 \\ & 7x_1 & +3x_2 & -5x_3 & \leq 20 \\ \text{and } x_1, x_2, x_3 \geq 0 & & & & \end{array}$$

2. $\boxed{\text{5 marks}}$ The company "Le chocolat délicieux" has you solve an LP problem, the objective being measured in dollars of profit. You solve the LP problem, arriving at a final dictionary that is non-degenerate. Currently they are getting 3000 kilos of cocoa ingredients from the contractor "Cocoa-good" at the price \$5 per kilo. The company wants to increase their production, but, the contractor "Cocoa-good" cannot provide additional amount. Then, another supplier "Cocoa-better" offers to provide a tiny bit of cocoa ingredients at the price \$7 per kilo. Everything else in the LP problem will remain the same. How do you determine whether the company should buy a tiny bit of such ingredients to make more production? In other words, using what criteria would you advise the company buy or not a tiny bit of such ingredient? Explain your answer clearly using relevant theorems.

3. $\boxed{\text{5 marks}}$ Consider the following dictionary from a standard form LP problem.

$$\begin{array}{rcrrrr} x_4 & = & 5 & -x_1 & -2x_2 & \\ x_5 & = & 0 & -x_1 & -x_2 & +x_3 \\ x_6 & = & 20 & -7x_1 & -3x_2 & +5x_3 \\ \hline z & = & & -3x_1 & -x_2 & -x_3 \end{array}$$

Find **all** the optimal **dual** solutions. (Hint: you may want to use the equation in Lecture 19 and complimentary slackness.)

4. Consider the standard form LP problem: Maximise $\vec{c} \cdot \vec{x}$ subject to $A\vec{x} \leq \vec{b}$ and $\vec{x} \geq \vec{0}$, where $c = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -2 & 1 \\ 0 & -1 & -1 \end{bmatrix}$, and $b = \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix}$. It has the final dictionary

$$\begin{array}{rcrrrr} x_2 & = & 4 & -x_1 & -x_3 & -x_4 \\ x_5 & = & 7 & & -3x_3 & -2x_4 \\ x_6 & = & 2 & -x_1 & & -x_4 \\ z & = & 8 & -x_1 & -x_3 & -2x_4 \end{array}$$

(a) $\boxed{\text{5 marks}}$ Let $\vec{x}^*$ be an optimal solution to this (primal) problem. Find the largest ϵ such that for any t, with $0 \leq t \leq \epsilon$, the vector $\vec{x}^*$ is still an optimal solution to the new primal problem obtained by changing $\vec{c}$ to $\vec{c} + \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix}$. (Hint: you may want to use the equation in Lecture 19 among others.)

(b) $\boxed{\text{5 marks}}$ * (This problem is more subtle than (a). Be careful.) Let $\vec{y}^*$ be an optimal solution to the dual problem. Find the largest ϵ such that for any t, with $-\epsilon \leq t \leq \epsilon$, the vector $\vec{y}^*$ is still an optimal solution to the new **dual** problem obtained by changing $\vec{b}$ to $\vec{b} + \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix}$. (Hint: First turn the dual problem into a standard form LP. You may also want to use the equation in Lecture 19 among others.)