

Communication Efficient Training of Federated Model Over Unbalanced Labels

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Abstract—xxx

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I. INTRODUCTION

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II. FORMULATION

A. Personalized Representation based on Similarity Network

Personalized models are built based on the similarity network. The similarity network measures the similarity of data distribution under data/feature/model space.

- **Data space.** In the case, local datasets of every node are used to construct a *kernel* matrix. The similarity of data distribution is measured by xxxxx.
- **Feature space.** In the case, local datasets of every node are used to construct a *covariance* matrix. It represents the dependence structure among features. The similarity of data distribution is measured based on the distance between covariance matrices.
- **Model space.** In the case, local model of every node is trained by using the local dataset. The similarity of data distribution is measured based on the distance between local models.

Based on similarity under those space, the similarity network $\mathcal{G} = \{\mathcal{N}, \mathcal{E}\}$ is built by using the KNN method [?]. $\mathcal{N} = \{1, 2, \dots, N\}$ represents the node set, consisting of N nodes. $\mathcal{E} = \{e_{i,j} : i \in \mathcal{N}, j \text{ is the node } i\text{'s neighbour}\}$ represents the edge set, consisting of M edges.

B. Final Formulation

Given the mapping matrix $\mathbf{M} \in \mathbb{R}^{d \times d_1}$ and $\mathbf{N} \in \mathbb{R}^{d \times d_2}$, the objective problem is formulated by

$$\min_{\{\mathbf{x}^{(n)}\}_{n=1}^N} \frac{1}{N} \sum_{n \in \mathcal{N}} f_n(\mathbf{x}^{(n)}; \mathcal{D}_n) + \lambda \sum_{\substack{e_{i,j} \in \mathcal{E}, \\ \forall i,j \in \mathcal{N}}} \left\| \mathbf{z}^{(i)} - \mathbf{z}^{(j)} \right\|_p, \quad (1)$$

subject to:

$$\mathbf{x}^{(n)} = \mathbf{M}\mathbf{x} + \mathbf{N}\mathbf{z}^{(n)}, \quad \forall n \in \mathcal{N}, \mathbf{x} \in \mathbb{R}^{d_1}, \mathbf{z}^{(n)} \in \mathbb{R}^{d_2}. \quad (2)$$

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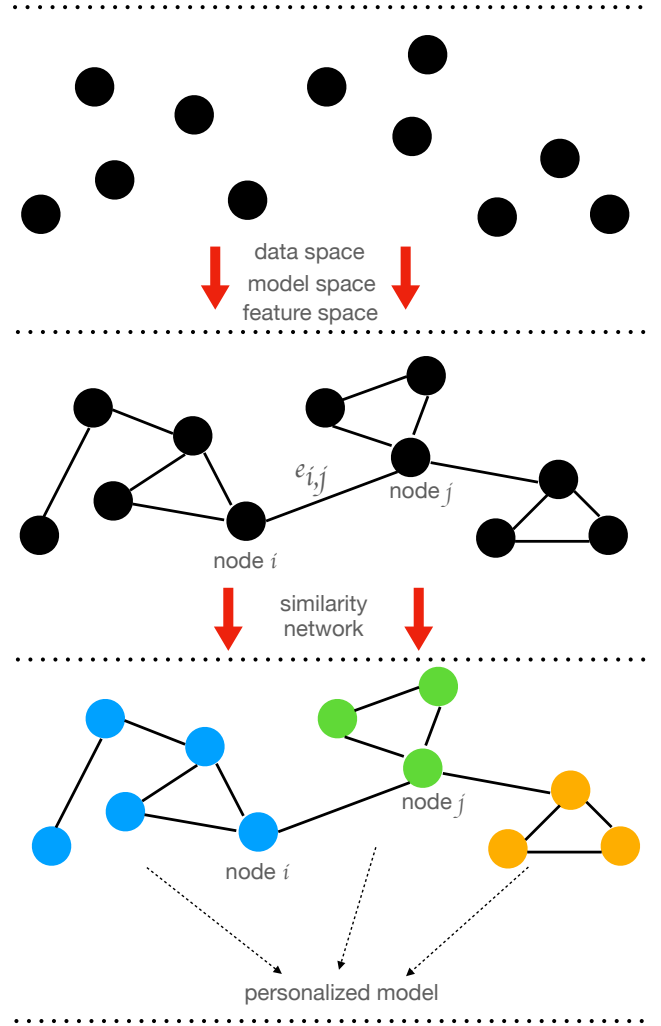


Fig. 1. Personalized representation based on similarity network

Here, $p \in \{1, 2, \infty\}$. \mathbf{M} and \mathbf{N} has special structure, where every row of them has at most one non-zero value, and the non-zero value is 1.

The formulation can be equally transformed as follows.

$$\min_{\{\mathbf{x}^{(n)}\}_{n=1}^N} \frac{1}{N} \sum_{n=1}^N f_n(\mathbf{x}^{(n)}; \mathcal{D}_n) + \lambda \|\mathbf{Z}\mathbf{Q}\|_{1,p}, \quad (3)$$

subject to:

$$\mathbf{x}^{(n)} = \mathbf{M}\mathbf{x} + \mathbf{N}\mathbf{z}^{(n)}. \quad (4)$$

Here, $\mathbf{Z} \in \mathbb{R}^{d_2 \times N}$, $\mathbf{Q} \in \mathbb{R}^{N \times M}$. N and M represent the total number of nodes and edges in the network \mathcal{G} , respectively. \mathbf{Z}

Algorithm 2 Training of personalized models on the server.

Require: The number of total iterations T , and the initial parameter \mathbf{x}_1 .

- 1: Deliver the parameter $\mathbf{y}_1 = \mathbf{M}\mathbf{x}_1 + \mathbf{N}\mathbf{z}_1^{(n)}$ to all client n with $n \in \{1, 2, \dots, N\}$.
- 2: **for** $t = 1, 2, \dots, T$ **do**
- 3: Collect stochastic gradient $\mathbf{G}_t = [\mathbf{g}_t^{(1)}, \mathbf{g}_t^{(2)}, \dots, \mathbf{g}_t^{(N)}]$ from all client n with $n \in \{1, 2, \dots, N\}$.
- 4: Update the global parameter \mathbf{x} by performing:

$$\mathbf{x}_{t+1} = \underset{\mathbf{x} \in \mathbb{R}^{d_1}}{\operatorname{argmin}} \frac{1}{N} \sum_{n=1}^N \left\langle \mathbf{M}^{-1} \mathbf{g}_t^{(n)}, \mathbf{x} \right\rangle + \frac{1}{2\eta} \|\mathbf{x} - \mathbf{x}_t\|^2.$$

- 5: Update the personalized parameter \mathbf{Z} by performing:

$$\begin{aligned} & \mathbf{Z}_{t+1} \\ &= \underset{\mathbf{Z} \in \mathbb{R}^{d_2 \times N}}{\operatorname{argmin}} \frac{\mathbf{1}_d^\top (\mathbf{G}_t \odot \mathbf{Z}) \mathbf{1}_N}{N} + \lambda \|\mathbf{Z}\mathbf{Q}\|_{1,p} + \frac{1}{2\eta} \|\mathbf{Z} - \mathbf{Z}_t\|_F^2. \end{aligned}$$

- 6: Deliver the parameter $\mathbf{y}_{t+1} = \mathbf{M}\mathbf{x}_{t+1} + \mathbf{N}\mathbf{z}_{t+1}^{(n)}$ to every client.
 - return** $\mathbf{x}_{T+1}^{(n)} = \mathbf{M}\mathbf{x}_{T+1} + \mathbf{N}\mathbf{z}_{T+1}^{(n)}$ with $n \in \{1, 2, \dots, N\}$.
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