VinUniversity ICPC Team Notebook (2025-26)

Contents

1 Mathematics

1.1 Combinatorics

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$\binom{n}{k} = \binom{n}{n-k}$

$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

$\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$

$\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

$\sum_{k=0}^{n} \binom{n}{k} = 2^n$

$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k}y^k$

$\sum_{k=0}^{n} \binom{k}{r} = \binom{n+1}{r+1}$

$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k$

$k\binom{n}{k} = n\binom{n-1}{k-1}$

1.2 Number theory (modular, linear Diophantine)

```cpp
// DONE!!

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
	return ((a%b) + b) % b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
	while (b) { int t = a%b; a = b; b = t; }
	return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
	return a / gcd(a, b)*b;
}

// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
{
	int ret = 1;
	while (b)
	{
		if (b & 1) ret = mod(ret*a, m);
		a = mod(a*a, m);
		b >>= 1;
	}
	return ret;
}

// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
	int xx = y = 0;
	int yy = x = 1;
	while (b) {
		int q = a / b;
		int t = b; b = a%b; a = t;
		t = xx; xx = x - q*xx; x = t;
		t = yy; yy = y - q*yy; y = t;
	}
	return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
	int x, y;
	VI ret;
	int g = extended_euclid(a, n, x, y);
	if (!(b%g)) {
		x = mod(x*(b / g), n);
		for (int i = 0; i < g; i++)
			ret.push_back(mod(x + i*(n / g), n));
	}
	return ret;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
	int x, y;
	int g = extended_euclid(a, n, x, y);
	if (g > 1) return -1;
	return mod(x, n);
}

// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2.  Here, z is unique modulo M = lcm(m1, m2).
// Return (z, M).  On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
	int s, t;
	int g = extended_euclid(m1, m2, s, t);
	if (r1%g != r2%g) return make_pair(0, -1);
```

```
        return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
}

// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i.  Note that the solution is
// unique modulo M = lcm_i (m[i]).  Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
        PII ret = make_pair(r[0], m[0]);
        for (int i = 1; i < m.size(); i++) {
                ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
                if (ret.second == -1) break;
        }
        return ret;
}

// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
        if (!a && !b)
        {
                if (c) return false;
                x = 0; y = 0;
                return true;
        }
        if (!a)
        {
                if (c % b) return false;
                x = 0; y = c / b;
                return true;
        }
        if (!b)
        {
                if (c % a) return false;
                x = c / a; y = 0;
                return true;
        }
        int g = gcd(a, b);
        if (c % g) return false;
        x = c / g * mod_inverse(a / g, b / g);
        y = (c - a*x) / b;
        return true;
}

int main() {
        // expected: 2
        cout << gcd(14, 30) << endl;

        // expected: 2 -2 1
        int x, y;
        int g = extended_euclid(14, 30, x, y);
        cout << g << " " << x << " " << y << endl;

        // expected: 95 451
        VI sols = modular_linear_equation_solver(14, 30, 100);
        for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";
        cout << endl;

        // expected: 8
        cout << mod_inverse(8, 9) << endl;

        // expected: 23 105
        //           11 12
        PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2, 3, 2 }));
        cout << ret.first << " " << ret.second << endl;
        ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
        cout << ret.first << " " << ret.second << endl;

        // expected: 5 -15
        if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" << endl;
        cout << x << " " << y << endl;
        return 0;
}
```

1.3 Chinese Remainder Theorem

```
// Official version
// Source: https://cp-algorithms.com/math/chinese-remainder-theorem.html

struct Congruence {
    long long a, m;
};

long long chinese_remainder_theorem(vector<Congruence> const& congruences) {
    long long M = 1;
    for (auto const& congruence : congruences) {
        M *= congruence.m;
    }

    long long solution = 0;
    for (auto const& congruence : congruences) {
        long long a_i = congruence.a;
        long long M_i = M / congruence.m;
        long long N_i = mod_inv(M_i, congruence.m);
        solution = (solution + a_i * M_i % M * N_i) % M;
    }
    return solution;
}
```

2 Combinatorial optimization

2.1 Sparse max-flow

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
//     O(|V|^2 |E|)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source and sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain actual flow values, look at edges with capacity > 0
//       (zero capacity edges are residual edges).

#include<cstdio>
#include<vector>
#include<queue>
using namespace std;
typedef long long LL;

struct Edge {
  int u, v;
  LL cap, flow;
  Edge() {}
  Edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};

struct Dinic {
  int N;
  vector<Edge> E;
  vector<vector<int>> g;
  vector<int> d, pt;

  Dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}

  void AddEdge(int u, int v, LL cap) {
    if (u != v) {
      E.emplace_back(u, v, cap);
      g[u].emplace_back(E.size() - 1);
      E.emplace_back(v, u, 0);
      g[v].emplace_back(E.size() - 1);
    }
  }

  bool BFS(int S, int T) {
    queue<int> q({S});
    fill(d.begin(), d.end(), N + 1);
    d[S] = 0;
    while(!q.empty()) {
      int u = q.front(); q.pop();
      if (u == T) break;
      for (int k: g[u]) {
        Edge &e = E[k];
        if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
          d[e.v] = d[e.u] + 1;
          q.emplace(e.v);
        }
      }
    }
    return d[T] != N + 1;
  }

  LL DFS(int u, int T, LL flow = -1) {
    if (u == T || flow == 0) return flow;
    for (int &i = pt[u]; i < g[u].size(); ++i) {
      Edge &e = E[g[u][i]];
      Edge &oe = E[g[u][i]^1];
      if (d[e.v] == d[e.u] + 1) {
```

```
        LL amt = e.cap - e.flow;
        if (flow != -1 && amt > flow) amt = flow;
        if (LL pushed = DFS(e.v, T, amt)) {
          e.flow += pushed;
          oe.flow -= pushed;
          return pushed;
        }
      }
    }
    return 0;
  }

  LL MaxFlow(int S, int T) {
    LL total = 0;
    while (BFS(S, T)) {
      fill(pt.begin(), pt.end(), 0);
      while (LL flow = DFS(S, T))
        total += flow;
    }
    return total;
  }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)

int main()
{
  int N, E;
  scanf("%d%d", &N, &E);
  Dinic dinic(N);
  for(int i = 0; i < E; i++)
  {
    int u, v;
    LL cap;
    scanf("%d%d%lld", &u, &v, &cap);
    dinic.AddEdge(u - 1, v - 1, cap);
    dinic.AddEdge(v - 1, u - 1, cap);
  }
  printf("%lld\n", dinic.MaxFlow(0, N - 1));
  return 0;
}

// END CUT
```

2.2 Min-cost max-flow

```
// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972).  This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]).  For a regular max flow, set all edge costs to 0.
//
// Running time, O(|V|^2) cost per augmentation
//     max flow:           O(|V|^3) augmentations
//     min cost max flow:  O(|V|^4 * MAX_EDGE_COST) augmentations
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - (maximum flow value, minimum cost value)
//     - To obtain the actual flow, look at positive values only.

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
  int N;
  VVL cap, flow, cost;
  VI found;
  VL dist, pi, width;
  VPII dad;

  MinCostMaxFlow(int N) :
    N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
    found(N), dist(N), pi(N), width(N), dad(N) {}

  void AddEdge(int from, int to, L cap, L cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
  }

  void Relax(int s, int k, L cap, L cost, int dir) {
    L val = dist[s] + pi[s] - pi[k] + cost;
    if (cap && val < dist[k]) {
      dist[k] = val;
      dad[k] = make_pair(s, dir);
      width[k] = min(cap, width[s]);
    }
  }

  L Dijkstra(int s, int t) {
    fill(found.begin(), found.end(), false);
    fill(dist.begin(), dist.end(), INF);
    fill(width.begin(), width.end(), 0);
    dist[s] = 0;
    width[s] = INF;

    while (s != -1) {
      int best = -1;
      found[s] = true;
      for (int k = 0; k < N; k++) {
        if (found[k]) continue;
        Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
        Relax(s, k, flow[k][s], -cost[k][s], -1);
        if (best == -1 || dist[k] < dist[best]) best = k;
      }
      s = best;
    }

    for (int k = 0; k < N; k++)
      pi[k] = min(pi[k] + dist[k], INF);
    return width[t];
  }

  pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
      totflow += amt;
      for (int x = t; x != s; x = dad[x].first) {
        if (dad[x].second == 1) {
          flow[dad[x].first][x] += amt;
          totcost += amt * cost[dad[x].first][x];
        } else {
          flow[x][dad[x].first] -= amt;
          totcost -= amt * cost[x][dad[x].first];
        }
      }
    }
    return make_pair(totflow, totcost);
  }
};

// BEGIN CUT
// The following code solves UVA problem #10594: Data Flow

int main() {
  int N, M;

  while (scanf("%d%d", &N, &M) == 2) {
    VVL v(M, VL(3));
    for (int i = 0; i < M; i++)
      scanf("%Ld%Ld%Ld", &v[i][0], &v[i][1], &v[i][2]);
    L D, K;
    scanf("%Ld%Ld", &D, &K);

    MinCostMaxFlow mcmf(N+1);
    for (int i = 0; i < M; i++) {
      mcmf.AddEdge(int(v[i][0]), int(v[i][1]), K, v[i][2]);
      mcmf.AddEdge(int(v[i][1]), int(v[i][0]), K, v[i][2]);
    }
    mcmf.AddEdge(0, 1, D, 0);

    pair<L, L> res = mcmf.GetMaxFlow(0, N);

    if (res.first == D) {
      printf("%Ld\n", res.second);
    } else {
      printf("Impossible.\n");
    }
  }

  return 0;
}
```

```
// END CUT
```

2.3 Global min-cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
//     - graph, constructed using AddEdge()
//
// OUTPUT:
//     - (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
        for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
        used[last] = true;
        cut.push_back(last);
        if (best_weight == -1 || w[last] < best_weight) {
          best_cut = cut;
          best_weight = w[last];
        }
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
      }
    }
  }
  return make_pair(best_weight, best_cut);
}

// BEGIN CUT
// The following code solves UVA problem #10989: Bomb, Divide and Conquer
int main() {
  int N;
  cin >> N;
  for (int i = 0; i < N; i++) {
    int n, m;
    cin >> n >> m;
    VVI weights(n, VI(n));
    for (int j = 0; j < m; j++) {
      int a, b, c;
      cin >> a >> b >> c;
      weights[a-1][b-1] = weights[b-1][a-1] = c;
    }
    pair<int, VI> res = GetMinCut(weights);
    cout << "Case #" << i+1 << ": " << res.first << endl;
  }
}
// END CUT
```

3 Geometry

3.1 Convex hull

```
// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm.  Eliminate redundant points from the hull if REMOVE_REDUNDANT is
// #defined.
//
// Running time: O(n log n)
//
//   INPUT:   a vector of input points, unordered.
//   OUTPUT:  a vector of points in the convex hull, counterclockwise, starting
//            with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>
// BEGIN CUT
#include <map>
// END CUT

using namespace std;

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }
  bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
#endif
}

// BEGIN CUT
// The following code solves SPOJ problem #26: Build the Fence (BSHEEP)

int main() {
  int t;
  scanf("%d", &t);
  for (int caseno = 0; caseno < t; caseno++) {
    int n;
    scanf("%d", &n);
    vector<PT> v(n);
    for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);
    vector<PT> h(v);
    map<PT,int> index;
```

```
    for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
    ConvexHull(h);

    double len = 0;
    for (int i = 0; i < h.size(); i++) {
      double dx = h[i].x - h[(i+1)%h.size()].x;
      double dy = h[i].y - h[(i+1)%h.size()].y;
      len += sqrt(dx*dx+dy*dy);
    }

    if (caseno > 0) printf("\n");
    printf("%.2f\n", len);
    for (int i = 0; i < h.size(); i++) {
      if (i > 0) printf(" ");
      printf("%d", index[h[i]]);
    }
    printf("\n");
  }
}

// END CUT
```

3.2 Miscellaneous geometry

```
// C++ routines for computational geometry.

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,   y*c  ); }
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};

double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  return os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p)   { return PT(-p.y,p.x); }
PT RotateCW90(PT p)    { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
      return true;
  return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
```

```
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion.  Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}

int main() {

  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;

  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;

  // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

  // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

  // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

  // expected: 1 0 1
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

  // expected: (1,1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

  vector<PT> v;
  v.push_back(PT(0,0));
  v.push_back(PT(5,0));
  v.push_back(PT(5,5));
  v.push_back(PT(0,5));

  // expected: 1 1 1 0 0
  cerr << PointInPolygon(v, PT(2,2)) << " "
       << PointInPolygon(v, PT(2,0)) << " "
       << PointInPolygon(v, PT(0,2)) << " "
       << PointInPolygon(v, PT(5,2)) << " "
       << PointInPolygon(v, PT(2,5)) << endl;

  // expected: 0 1 1 1 1
  cerr << PointOnPolygon(v, PT(2,2)) << " "
       << PointOnPolygon(v, PT(2,0)) << " "
       << PointOnPolygon(v, PT(0,2)) << " "
       << PointOnPolygon(v, PT(5,2)) << " "
       << PointOnPolygon(v, PT(2,5)) << endl;

  // expected: (1,6)
  //           (5,4) (4,5)
  //           blank line
  //           (4,5) (5,4)
  //           blank line
  //           (4,5) (5,4)
  vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

  // area should be 5.0
  // centroid should be (1.1666666, 1.166666)
  PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
  vector<PT> p(pa, pa+4);
  PT c = ComputeCentroid(p);
  cerr << "Area: " << ComputeArea(p) << endl;
  cerr << "Centroid: " << c << endl;

  return 0;
}
```

4 Numerical algorithms

4.1 Systems of linear equations, matrix inverse, determinant

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
//
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;
```

```
typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;

  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
  }

  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
  }

  return det;
}

int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
  double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  }

  double det = GaussJordan(a, b);

  // expected: 60
  cout << "Determinant: " << det << endl;

  // expected: -0.233333 0.166667 0.133333 0.0666667
  //           0.166667 0.166667 0.333333 -0.333333
  //           0.233333 0.833333 -0.133333 -0.0666667
  //           0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }

  // expected: 1.63333 1.3
  //           -0.166667 0.5
  //           2.36667 1.7
  //           -1.85 -1.35
  cout << "Solution: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
      cout << b[i][j] << ' ';
    cout << endl;
  }
}
```

4.2 Reduced row echelon form, matrix rank

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting.  This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxm matrix
//
// OUTPUT:   rref[][] = an nxm matrix (stored in a[][])
//           returns rank of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
  int n = a.size();
  int m = a[0].size();
  int r = 0;
  for (int c = 0; c < m && r < n; c++) {
    int j = r;
    for (int i = r + 1; i < n; i++)
      if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
    if (fabs(a[j][c]) < EPSILON) continue;
    swap(a[j], a[r]);

    T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
    for (int i = 0; i < n; i++) if (i != r) {
      T t = a[i][c];
      for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
    }
    r++;
  }
  return r;
}

int main() {
  const int n = 5, m = 4;
  double A[n][m] = {
    {16,  2,  3, 13},
    { 5, 11, 10,  8},
    { 9,  7,  6, 12},
    { 4, 14, 15,  1},
    {13, 21, 21, 13}};
  VVT a(n);
  for (int i = 0; i < n; i++)
    a[i] = VT(A[i], A[i] + m);

  int rank = rref(a);

  // expected: 3
  cout << "Rank: " << rank << endl;

  // expected: 1 0 0 1
  //           0 1 0 3
  //           0 0 1 -3
  //           0 0 0 3.10862e-15
  //           0 0 0 2.22045e-15
  cout << "rref: " << endl;
  for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 4; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }
}
```

4.3 Fast Fourier transform

```
#include <cassert>
#include <cstdio>
#include <cmath>

struct cpx
{
  cpx(){}
  cpx(double aa):a(aa),b(0){}
  cpx(double aa, double bb):a(aa),b(bb){}
  double a;
  double b;
  double modsq(void) const
  {
```

```
    return a * a + b * b;
  }
  cpx bar(void) const
  {
    return cpx(a, -b);
  }
};

cpx operator +(cpx a, cpx b)
{
  return cpx(a.a + b.a, a.b + b.b);
}

cpx operator *(cpx a, cpx b)
{
  return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}

cpx operator /(cpx a, cpx b)
{
  cpx r = a * b.bar();
  return cpx(r.a / b.modsq(), r.b / b.modsq());
}

cpx EXP(double theta)
{
  return cpx(cos(theta),sin(theta));
}

const double two_pi = 4 * acos(0);

// in:     input array
// out:    output array
// step:   {SET TO 1} (used internally)
// size:   length of the input/output {MUST BE A POWER OF 2}
// dir:    either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i * j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
  if(size < 1) return;
  if(size == 1)
  {
    out[0] = in[0];
    return;
  }
  FFT(in, out, step * 2, size / 2, dir);
  FFT(in + step, out + size / 2, step * 2, size / 2, dir);
  for(int i = 0 ; i < size / 2 ; i++)
  {
    cpx even = out[i];
    cpx odd = out[i + size / 2];
    out[i] = even + EXP(dir * two_pi * i / size) * odd;
    out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) / size) * odd;
  }
}

// Usage:
// f[0...N-1] and g[0..N-1] are numbers
// Want to compute the convolution h, defined by
// h[n] = sum of f[k]g[n-k] (k = 0, ..., N-1).
// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product).
// To compute h[] in O(N log N) time, do the following:
//   1. Compute F and G (pass dir = 1 as the argument).
//   2. Get H by element-wise multiplying F and G.
//   3. Get h by taking the inverse FFT (use dir = -1 as the argument)
//      and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.

int main(void)
{
  printf("If rows come in identical pairs, then everything works.\n");

  cpx a[8] = {0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0};
  cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};
  cpx A[8];
  cpx B[8];
  FFT(a, A, 1, 8, 1);
  FFT(b, B, 1, 8, 1);

  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", A[i].a, A[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx Ai(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
    }
    printf("%7.2lf%7.2lf", Ai.a, Ai.b);
  }
  printf("\n");

  cpx AB[8];
  for(int i = 0 ; i < 8 ; i++)
    AB[i] = A[i] * B[i];
  cpx aconvb[8];
  FFT(AB, aconvb, 1, 8, -1);
  for(int i = 0 ; i < 8 ; i++)
    aconvb[i] = aconvb[i] / 8;
  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", aconvb[i].a, aconvb[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx aconvbi(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
    }
    printf("%7.2lf%7.2lf", aconvbi.a, aconvbi.b);
  }
  printf("\n");

  return 0;
}
```

4.4 Simplex algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }
```

```
  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

5 Graph algorithms

5.1 Bellman-Ford shortest paths with negative edge weights (C++)

```
// This function runs the Bellman-Ford algorithm for single source
// shortest paths with negative edge weights.  The function returns
// false if a negative weight cycle is detected.  Otherwise, the
// function returns true and dist[i] is the length of the shortest
// path from start to i.
//
// Running time: O(|V|^3)
//
//   INPUT:   start, w[i][j] = cost of edge from i to j
//   OUTPUT:  dist[i] = min weight path from start to i
//            prev[i] = previous node on the best path from the
//                      start node

#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool BellmanFord (const VVT &w, VT &dist, VI &prev, int start){
  int n = w.size();
  prev = VI(n, -1);
  dist = VT(n, 1000000000);
  dist[start] = 0;

  for (int k = 0; k < n; k++){
    for (int i = 0; i < n; i++){
      for (int j = 0; j < n; j++){
        if (dist[j] > dist[i] + w[i][j]){
          if (k == n-1) return false;
          dist[j] = dist[i] + w[i][j];
          prev[j] = i;
        }
      }
    }
  }

  return true;
}
```

5.2 Dijkstra and Floyd's algorithm (C++)

```
#include <iostream>
#include <queue>
#include <cmath>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

// This function runs Dijkstra's algorithm for single source
// shortest paths.  No negative cycles allowed!
//
// Running time: O(|V|^2)
//
//   INPUT:   start, w[i][j] = cost of edge from i to j
//   OUTPUT:  dist[i] = min weight path from start to i
//            prev[i] = previous node on the best path from the
//                      start node

void Dijkstra (const VVT &w, VT &dist, VI &prev, int start){
  int n = w.size();
  VI found (n);
  prev = VI(n, -1);
  dist = VT(n, 1000000000);
  dist[start] = 0;

  while (start != -1){
    found[start] = true;
    int best = -1;
    for (int k = 0; k < n; k++) if (!found[k]){
      if (dist[k] > dist[start] + w[start][k]){
        dist[k] = dist[start] + w[start][k];
        prev[k] = start;
      }
      if (best == -1 || dist[k] < dist[best]) best = k;
    }
    start = best;
  }
}
```

```
// This function runs the Floyd-Warshall algorithm for all-pairs
// shortest paths.  Also handles negative edge weights.  Returns true
// if a negative weight cycle is found.
//
// Running time: O(|V|^3)
//
//   INPUT:  w[i][j] = weight of edge from i to j
//   OUTPUT: w[i][j] = shortest path from i to j
//           prev[i][j] = node before j on the best path starting at i

bool FloydWarshall (VVT &w, VVI &prev){
  int n = w.size();
  prev = VVI (n, VI(n, -1));

  for (int k = 0; k < n; k++){
    for (int i = 0; i < n; i++){
      for (int j = 0; j < n; j++){
        if (w[i][j] > w[i][k] + w[k][j]){
          w[i][j] = w[i][k] + w[k][j];
          prev[i][j] = k;
        }
      }
    }
  }

  // check for negative weight cycles
  for(int i=0;i<n;i++)
    if (w[i][i] < 0) return false;
  return true;
}
```

5.3 Fast Dijkstra's algorithm

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
//
// Running time: O(|E| log |V|)

#include <queue>
#include <cstdio>

using namespace std;
const int INF = 2000000000;
typedef pair<int, int> PII;

int main() {

        int N, s, t;
        scanf("%d%d%d", &N, &s, &t);
        vector<vector<PII> > edges(N);
        for (int i = 0; i < N; i++) {
                int M;
                scanf("%d", &M);
                for (int j = 0; j < M; j++) {
                        int vertex, dist;
                        scanf("%d%d", &vertex, &dist);
                        edges[i].push_back(make_pair(dist, vertex)); // note order of arguments here
                }
        }

        // use priority queue in which top element has the "smallest" priority
        priority_queue<PII, vector<PII>, greater<PII> > Q;
        vector<int> dist(N, INF), dad(N, -1);
        Q.push(make_pair(0, s));
        dist[s] = 0;
        while (!Q.empty()) {
                PII p = Q.top();
                Q.pop();
                int here = p.second;
                if (here == t) break;
                if (dist[here] != p.first) continue;

                for (vector<PII>::iterator it = edges[here].begin(); it != edges[here].end(); it++) {
                        if (dist[here] + it->first < dist[it->second]) {
                                dist[it->second] = dist[here] + it->first;
                                dad[it->second] = here;
                                Q.push(make_pair(dist[it->second], it->second));
                        }
                }
        }

        printf("%d\n", dist[t]);
        if (dist[t] < INF)
                for (int i = t; i != -1; i = dad[i])
                        printf("%d%c", i, (i == s ? '\n' : ' '));
        return 0;
}
```

```
/*
Sample input:
5 0 4
2 1 2 3 1
2 2 4 4 5
3 1 4 3 3 4 1
2 0 1 2 3
2 1 5 2 1

Expected:
5
4 2 3 0
*/
```

5.4 Strongly connected components

```
// Official version

#include <vector>
#include <algorithm>
using namespace std;
vector<bool> visited; // keeps track of which vertices are already visited

// runs depth first search starting at vertex v.
// each visited vertex is appended to the output vector when dfs leaves it.
void dfs(int v, vector<vector<int> > const& adj, vector<int> &output) {
    visited[v] = true;
    for (auto u : adj[v])
        if (!visited[u])
            dfs(u, adj, output);
    output.push_back(v); // This is used to record the t_out of each vertices
}

// input: adj -- adjacency list of G
// output: components -- the strongy connected components in G
// output: adj_cond -- adjacency list of G^SCC (by root vertices)
void strongly_connected_components(vector<vector<int> > const& adj,
                                   vector<vector<int> > &components,
                                   vector<vector<int> > &adj_cond) {
    int n = adj.size();
    components.clear(), adj_cond.clear();

    vector<int> order; // will be a sorted list of G's vertices by exit time

    visited.assign(n, false);

    // first series of depth first searches
    for (int i = 0; i < n; i++)
        if (!visited[i])
            dfs(i, adj, order);

    // create adjacency list of G^T
    vector<vector<int> > adj_rev(n);
    for (int v = 0; v < n; v++)
        for (int u : adj[v])
            adj_rev[u].push_back(v);

    visited.assign(n, false);
    reverse(order.begin(), order.end());

    vector<int> roots(n, 0); // gives the root vertex of a vertex's SCC

    // second series of depth first searches
    for (auto v : order)
        if (!visited[v]) {
            std::vector<int> component;
            dfs(v, adj_rev, component);
            components.push_back(component);
            int root = *min_element(begin(component), end(component));
            // actually, we can choose any element in the component!!!.
            for (auto u : component)
                roots[u] = root;
        }

    // add edges to condensation graph
    adj_cond.assign(n, {});
    for (int v = 0; v < n; v++)
        for (auto u : adj[v])
            if (roots[v] != roots[u])
                adj_cond[roots[v]].push_back(roots[u]);
}

int main() {}
```

5.5 Bridges And Articulation Points

```
#include <bits/stdc++.h>

using namespace std;

const int maxN = 10010;

int n, m;
bool joint[maxN];
int timeDfs = 0, bridge = 0;
int low[maxN], num[maxN];
vector <int> g[maxN];

void dfs(int u, int pre) {
    int child = 0;
    num[u] = low[u] = ++timeDfs;
    for (int v : g[u]) {
        if (v == pre) continue;
        if (!num[v]) {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] == num[v]) bridge++;
            child++;
            if (pre != -1 && low[v] >= num[u]) joint[u] = true;
        }
        else low[u] = min(low[u], num[v]);
    }
    if (pre == -1) {
        if (child > 1) joint[u] = true;
    }
}

int main() {
    cin >> n >> m;
    for (int i = 1; i <= m; i++) {
        int u, v;
        cin >> u >> v;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    for (int i = 1; i <= n; i++)
        if (!num[i]) dfs(i, -1);

    int cntJoint = 0;
    for (int i = 1; i <= n; i++) cntJoint += joint[i];

    cout << cntJoint << ' ' << bridge;
}
```

5.6 2-SAT

```
// Official 2-SAT implementation using Kosaraju's algorithm
struct TwoSatSolver {
    int n_vars;
    int n_vertices;
    vector<vector<int>> adj, adj_t;
    vector<bool> used;
    vector<int> order, comp;
    vector<bool> assignment;

    TwoSatSolver(int _n_vars) : n_vars(_n_vars), n_vertices(2 * n_vars), adj(n_vertices), adj_t(
          n_vertices), used(n_vertices), order(), comp(n_vertices, -1), assignment(n_vars) {
        order.reserve(n_vertices);
    }
    void dfs1(int v) {
        used[v] = true;
        for (int u : adj[v]) {
            if (!used[u])
                dfs1(u);
        }
        order.push_back(v);
    }

    void dfs2(int v, int cl) {
        comp[v] = cl;
        for (int u : adj_t[v]) {
            if (comp[u] == -1)
                dfs2(u, cl);
        }
    }

    bool solve_2SAT() {
        order.clear();
        used.assign(n_vertices, false);
        for (int i = 0; i < n_vertices; ++i) {
            if (!used[i])
                dfs1(i);
        }

        comp.assign(n_vertices, -1);
        for (int i = 0, j = 0; i < n_vertices; ++i) {
            int v = order[n_vertices - i - 1];
            // Assign according to topological sort order
            if (comp[v] == -1)
                dfs2(v, j++);
        }

        assignment.assign(n_vars, false);
        for (int i = 0; i < n_vertices; i += 2) {
            if (comp[i] == comp[i + 1])
                return false;
            assignment[i / 2] = comp[i] > comp[i + 1];
        }
        return true;
    }

    void add_disjunction(int a, bool na, int b, bool nb) {
        // na and nb signify whether a and b are to be negated
        a = 2 * a ^ na;
        b = 2 * b ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
        // if na = 1 -> a = 2*a + 1 , neg_a = 2*a
        // if na = 0 -> a = 2*a, neg_a = 2*a + 1
        adj[neg_a].push_back(b);
        adj[neg_b].push_back(a);
        // We define the adj_t to identify the component!!!!
        adj_t[b].push_back(neg_a);
        adj_t[a].push_back(neg_b);
    }

    static void example_usage() {
        TwoSatSolver solver(3); // a, b, c
        solver.add_disjunction(0, false, 1, true);  //     a  v  not b
        solver.add_disjunction(0, true, 1, true);   // not a  v  not b
        solver.add_disjunction(1, false, 2, false); //     b  v      c
        solver.add_disjunction(0, false, 0, false); //     a  v      a
        assert(solver.solve_2SAT() == true);
        auto expected = vector<bool>(True, False, True);
        assert(solver.assignment == expected);
    }
};
```

5.7 Kruskal's algorithm

```
// Official version
vector<int> parent, rank;

void make_set(int v) {
    parent[v] = v;
    rank[v] = 0;
}

int find_set(int v) {
    if (v == parent[v])
        return v;
    return parent[v] = find_set(parent[v]);
}

void union_sets(int a, int b) {
    a = find_set(a);
    b = find_set(b);
    if (a != b) {
        if (rank[a] < rank[b])
            swap(a, b);
        parent[b] = a;
        if (rank[a] == rank[b])
            rank[a]++;
    }
}

struct Edge {
    int u, v, weight;
    bool operator<(Edge const& other) {
        return weight < other.weight;
    }
};

int n;
vector<Edge> edges;

int cost = 0;
```

```
vector<Edge> result;
parent.resize(n);
rank.resize(n);
for (int i = 0; i < n; i++)
    make_set(i);

sort(edges.begin(), edges.end());

for (Edge e : edges) {
    if (find_set(e.u) != find_set(e.v)) {
        cost += e.weight;
        result.push_back(e);
        union_sets(e.u, e.v);
        // if we want, we can add a trace here!!!
    }
}
```

6 Data structures

6.1 Suffix array

```
// Suffix array construction in O(L log^2 L) time.  Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:   string s
//
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//          of substring s[i...L-1] in the list of sorted suffixes.
//          That is, if we take the inverse of the permutation suffix[],
//          we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<int,int>,int> > M;

  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
        P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
    }
  }

  vector<int> GetSuffixArray() { return P.back(); }

  // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
        len += 1 << k;
      }
    }
    return len;
  }
};

// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
  int T;
  cin >> T;
  for (int caseno = 0; caseno < T; caseno++) {
    string s;
    cin >> s;
    SuffixArray array(s);
    vector<int> v = array.GetSuffixArray();
    int bestlen = -1, bestpos = -1, bestcount = 0;
    for (int i = 0; i < s.length(); i++) {
      int len = 0, count = 0;
      for (int j = i+1; j < s.length(); j++) {
        int l = array.LongestCommonPrefix(i, j);
        if (l >= len) {
          if (l > len) count = 2; else count++;
          len = l;
        }
      }
      if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) > s.substr(i, len)) {
        bestlen = len;
        bestcount = count;
        bestpos = i;
      }
    }
    if (bestlen == 0) {
      cout << "No repetitions found!" << endl;
    } else {
      cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
    }
  }
}

#else
// END CUT
int main() {

  // bobocel is the 0'th suffix
  //  obocel is the 5'th suffix
  //   bocel is the 1'st suffix
  //    ocel is the 6'th suffix
  //     cel is the 2'nd suffix
  //      el is the 3'rd suffix
  //       l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArray();

  // Expected output: 0 5 1 6 2 3 4
  //                  2
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
  cout << endl;
  cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
// BEGIN CUT
#endif
// END CUT
```

6.2 Disjoint Set Union

```
#include <vector>
#include <algorithm>
using namespace std;
struct DSU{
    vector<int> r;
    int n;

    DSU (int n) : n(n) {
        r.assign(n+1, -1);
    }

    int root (int u) {
        if (r[u] < 0) return u;
        return r[u] = root(r[u]);
    }

    void mergetree (int ru, int rv) {
        ru = root(ru);
        rv = root(rv);
        if (ru == rv)
            return;
        if (r[ru] > r[rv])
            swap(ru, rv);
        r[ru] += r[rv];
        r[rv] = ru;
    }
};
```

6.3 SegmentTree + Lazy Propagation

```
// Official version
// Source: https://wiki.vnoi.info/algo/data-structures/segment-tree-basic.md

#include <bits/stdc++.h>
```

```cpp
using namespace std;

const int inf  = 1e9 + 7;
const int maxN = 1e5 + 7;

int n, q;
int a[maxN];
long long st[4 * maxN], lazy[4 * maxN];

void build(int id, int l, int r) {
    if (l == r) {
        st[id] = a[l];
        return;
    }
    int mid = l + r >> 1;
    build(2 * id, l, mid);
    build(2 * id + 1, mid + 1, r);
    st[id] = max(st[2 * id], st[2 * id + 1]);
}

void fix(int id, int l, int r) {
    if (!lazy[id]) return;
    st[id] += lazy[id];

    if (l != r){
        lazy[2 * id] += lazy[id];
        lazy[2 * id + 1] += lazy[id];
    }

    lazy[id] = 0; // already reset here
}

void update(int id, int l, int r, int u, int v, int val) {
        // Update the whole range [u, v]
    fix(id, l, r);
    if (l >  v || r <  u) return;
    if (l >= u && r <= v) {
        lazy[id] += val;
        fix(id, l, r);
        return;
    }
    int mid = l + r >> 1;
    update(2 * id, l, mid, u, v, val);
    update(2 * id + 1, mid + 1, r, u, v, val);
    st[id] = max(st[2 * id], st[2 * id + 1]);
}

long long get(int id, int l, int r, int u, int v) { // in this code, we reduce l, r such that it
     matches u, v
    fix(id, l, r); // luon luon fix truoc khi lay!!!
    if (l >  v || r <  u) return -inf;
    if (l >= u && r <= v) return st[id];

    int mid  = l + r >> 1;
    long long get1 = get(2 * id, l, mid, u, v);
    long long get2 = get(2 * id + 1, mid + 1, r, u, v);
    return max(get1, get2);
}
```

6.4 SparseTable

```cpp
// Official
// Sparse Table implementation for range minimum queries (RMQ), and range sum queries (RSQ)
// Preprocessing time: O(n log n)
// Query time: O(1)

int a[N], st[LG + 1][N];
void preprocess() {
    for (int i = 1; i <= n; ++i) st[0][i] = a[i];
    for (int j = 1; j <= LG; ++j)
        for (int i = 1; i + (1 << j) - 1 <= n; ++i)
            st[j][i] = min(st[j - 1][i], st[j - 1][i + (1 << (j - 1))]);
            // st[j][i] -> the smallest element from i to i + 2^j. the j represents for the length
                // ~ the smallest element from [i to 2^{j-1}] and from [i + 2^{j-1} to 2^j]
                //                                st[j-1][i],              st[j-1][i + (1 << (j-1))]
}
int queryMin(int l, int r) {
    int k = __lg(r - l + 1); // this is the length
    return min(st[k][l], st[k][r - (1 << k) + 1]); // query from l -> k , r traces back to k such that
          they overlap
}

int querySum(int l, int r) {
    int len = r - l + 1;
    int sum = 0;
    // for each interval [l, r], we divide it into log_2(r - l + 1) interval whose length is 2^x
    // example: [3, 15] -> length = 13 = 1 + 4 + 8 ~ [3], [4, 7], [8, 15]
    // we can check the activated bits of length, lets call them j_1, j_2, j_3
    // obviously, the result is sum += st[j_1][l] + s[j_2][l+j_1] + s[j_3][l + j_2] (we can ensure
        this sequence match r)
    // from here, we can sum st[l][]

    for (int j = 0; (1 << j) <= len; ++j)
        if (len >> j & 1) { // check whether j-th bit is 1 or not.
            sum = sum + st[j][l];
            l = l + (1 << j);
        }
    return sum;
}

// 2D Sparse Table

int a[M][N], st[LGM + 1][M][LGN + 1][N];
void preprocess() {
    for (int k = 0; k <= LGM; ++k) {
        for (int i = 1; i + (1 << k) - 1 <= m; ++i) {
            for (int l = 0; l <= LGN; ++l) {
                for (int j = 1; j + (1 << l) - 1 <= n; ++j) {
                    if (k == 0) {
                        if (l == 0) {
                            st[0][i][0][j] = a[i][j];
                        }
                        else {
                            st[0][i][l][j] = min(st[0][i][l - 1][j], st[0][i][l - 1][j + (1 << (l - 1)
                                )]);
                        }
                    }
                    else {
                        st[k][i][l][j] = min(st[k - 1][i][l][j], st[k - 1][i + (1 << (k - 1))][l][j]);
                    }
                }
            }
        }
    }
}

int getMin(int x, int y, int a, int b) {
    int k = __lg(a - x + 1);
    int l = __lg(b - y + 1);
    return min({ st[k][x][l][y],
                 st[k][x][l][b - (1 << l) + 1],
                 st[k][a - (1 << k) + 1][l][y],
                 st[k][a - (1 << k) + 1][l][b - (1 << l) + 1] });
}
```

6.5 Binary Indexed Tree

```cpp
#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
  while(x <= N) {
    tree[x] += v;
    x += (x & -x);
  }
}

// get cumulative sum up to and including x
int get(int x) {
  int res = 0;
  while(x) {
    res += tree[x];
    x -= (x & -x);
  }
  return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
  int idx = 0, mask = N;
  while(mask && idx < N) {
    int t = idx + mask;
    if(x >= tree[t]) {
      idx = t;
      x -= tree[t];
    }
```

```
    mask >>= 1;
  }
  return idx;
}
```

7 Miscellaneous

7.1 Longest increasing subsequence

```
// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
//   INPUT: a vector of integers
//   OUTPUT: a vector containing the longest increasing subsequence

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

#define STRICTLY_INCREASNG

VI LongestIncreasingSubsequence(VI v) {
  VPII best;
  VI dad(v.size(), -1);

  for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
    PII item = make_pair(v[i], 0);
    VPII::iterator it = lower_bound(best.begin(), best.end(), item);
    item.second = i;
#else
    PII item = make_pair(v[i], i);
    VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
    if (it == best.end()) {
      dad[i] = (best.size() == 0 ? -1 : best.back().second);
      best.push_back(item);
    } else {
      dad[i] = it == best.begin() ? -1 : prev(it)->second;
      *it = item;
    }
  }

  VI ret;
  for (int i = best.back().second; i >= 0; i = dad[i])
    ret.push_back(v[i]);
  reverse(ret.begin(), ret.end());
  return ret;
}
```

7.2 Knuth-Morris-Pratt

```
/*
Finds all occurrences of the pattern string p within the
text string t. Running time is O(n + m), where n and m
are the lengths of p and t, respecitvely.
*/

#include <iostream>
#include <string>
#include <vector>

using namespace std;

typedef vector<int> VI;

void buildPi(string& p, VI& pi)
{
  pi = VI(p.length());
  int k = -2;
  for(int i = 0; i < p.length(); i++) {
    while(k >= -1 && p[k+1] != p[i])
      k = (k == -1) ? -2 : pi[k];
    pi[i] = ++k;
  }
}

int KMP(string& t, string& p)
{
  VI pi;
  buildPi(p, pi);
  int k = -1;
  for(int i = 0; i < t.length(); i++) {
    while(k >= -1 && p[k+1] != t[i])
      k = (k == -1) ? -2 : pi[k];
    k++;
    if(k == p.length() - 1) {
      // p matches t[i-m+1, ..., i]
      cout << "matched at index " << i-k << ": ";
      cout << t.substr(i-k, p.length()) << endl;
      k = (k == -1) ? -2 : pi[k];
    }
  }
  return 0;
}

int main()
{
  string a = "AABAACAADAABAABA", b = "AABA";
  KMP(a, b); // expected matches at: 0, 9, 12
  return 0;
}
```

7.3 Longest common subsequence

```
#include<bits/stdc++.h>
using namespace std;
int main() {
    int n; cin >> n; int m; cin >> m;
    vector<int> a(n); for (int &x:a) cin >> x;
    vector<int> b(m); for (int &x:b) cin >> x;

    vector< vector< int > > f(n+1, vector<int> (m+1, 0));
    for (int i=0; i<n; ++i) {
        for (int j=0; j<m; ++j) {
            if (a[i] == b[j]) {
                f[i+1][j+1] = f[i][j] + 1;
            } else {
                f[i+1][j+1] = max(f[i][j+1], f[i+1][j]);
            }
        }
    }

    cout << f[n][m] << endl;
    int x=n,y=m; vector<int> trace;
    while (x>0&&y>0) {
        if (a[x-1] == b[y-1]) {
            trace.push_back(a[x-1]);
            x--;y--;
        } else if (f[x][y] == f[x-1][y]) x--;
        else y--;
    }
    for (int i=trace.size()-1; i>=0; --i) cout << trace[i] << " ";
}
```